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ABSTRACT

The first volume of this proceedings contains the papers presented at the 21st annual meeting of the Mathematics Education Research Group of Australasia. Papers include: (1) "Research, Rhetoric and Reality" (Lyn English); (2) "I Keep Six Honest Serving Men" (Peter Galbraith); (3) "The Intension/Intention of Teaching Mathematics" (Stephen Lerman); (4) "One Teacher's Problem Solving Beliefs and Practices: Influences and Coherence" (Judy Anderson); (5) "Towards the Study of the Processes and Effects of Internationalisation in Mathematics Education" (Bill Atweh, Phil Clarkson, and John Malone); (6) "The Use of Action Research To Assist the Transition into Teaching" (Bill Atweh and Ann Heirdsfield); (7) "The Effectiveness of a No Goal Approach in Solving Pythagorean 2-Stage Problems" (Paula Ayres); (8) "Factors Influencing Predictions about Randomly Generated Sequences" (Paula Ayres and Jenni Way); (9) "Towards a Framework for Analyzing Power Relationships in Small Group Discussions" (Mary Barnes); (10) "Year 5 Students' Available and Accessible Knowledge of Decimal-Number Numeration" (Annette Baturo); (11) "Mathematics Curriculum and Development" (Andy Begg); (12) "The Impact of An Early Number Project on the Professional Development of Teachers" (Janette Bobis and Peter Gould); (13) "Arithmetic, Pre-Algebra, and Algebra: A Model of Transition" (Gillian Boulton-Lewis, Tom Cooper, H. Pillay, and L. Wilss); (14) "The Invisible Wall Project: Problem Solving Processes of Pupils Who Work on Problems with a Goal Which Cannot Be Reached" (Birgit Burchardtz and Martin Stein); (15) "'Why Are We Assessed in Mathematics?': The Views of Students" (Ken Carr); (16) "Developing a Framework for Viewing Affective and Knowledge Factors in Teaching Primary Mathematics" (Jean Carroll); (17) "Teachers' Understandings of the Role of Learning Activities" (Min-Pyng Chen); (18) "Showing and Telling: Primary Students' Outcomes in Data Representation and Interpretation" (Helen Chick and Jane Watson); (19) "Representing the Connectedness of Mathematical Knowledge" (Mohan Chinnapan, Michael Lawson, and Rod Nason); (20) "Beginning

Teachers' Problems with Fundamental Mathematics" (Phil Clarkson); (21) "Investigating Students' Understanding of the Relationships among Quadrilaterals" (Penelope Currie and John Pegg); (22) "Rethinking What It Means To Understand" (Lyn English); (23) "New Times for Mathematics in Vocational Education and Training" (Gail FitzSimons); (24) "Exploring Avenues of Reflective Practice with Pre-Service Mathematics Teachers" (Sandra Frid, Ted Redden, and Chris Reading); (25) "Students' Perspectives on Using Computers and Graphing Calculators during Mathematical Collaborative Practice" (Vince Geiger); (26) "Identifying the Dilemmas in Early Mathematics Teaching" (Ann Gervasoni); (27) "'I Don't Know If I'm Doing It Right or I'm Doing It Wrong!' Unresolved Uncertainty in the Collaborative Learning of Mathematics" (Merrilyn Goos); (28) "Visual Reasoning and Teaching Styles in Mathematics Classrooms" (Kevin Hannah); (29) "Flexible/Inflexible: Clare and Mandy's Story" (Ann Heirdsfield); (30) "We Really Put Our Minds To It: Cognitive Engagement in the Mathematics Classroom" (Sue Helme and David Clarke); (31) "Maori Mathematics Education: The Challenge of Providing Immersion Programmes For Preservice Maori Primary Teachers" (Leeana Herewini); (32) "Towards an Explanation of Curriculum Control" (John Horwood); (33) "How Sixth Grade Students Explaining Connections between Common and Decimal Fractions" (Robert Hunting, Lauren Oppenheimer, Cath Pearn, and E. Nugent); (34) "Mathematics Curriculum Change in the Northern Territory: What Do Teachers Really Think?" (Rosemary Jacob and Sandra Frid); (35) "The Effective Use of Calculators" (Sonia Jones and Howard Tanner); (36) "New Knowledge/New Teachers/New Times: How Processes of Subjectification Undermine the Implementation of Investigatory Approaches to Teaching Mathematics" (Mary Klein); (37) "Affective Factors and the Improvement of Algebraic Problem Solving" (Saraswathi Kota and Mike Thomas); (38) "Tertiary Mathematics: Perceptions of School Levers and Mature-Age Students" (Gilah Leder and Helen Forgasz); (39) "There's More Than Meets the Eye: The Relationships between the User, the Teacher and the Technology" (Tom Lowrie); (40) "Pre-Service Teachers' Understandings of Word Problems" (Kevin Maguire); (41) "Protasis: A Technique for Fostering Professional Development" (John Mason); (42) "Towards a Psychology of Knowing-To" (John Mason and Mary Spence); (43) "School-Based Assessment in VCE Mathematics: Ten Years On" (Barry Mccrae, John Dowsey, and Max Stephens); and (44) "A New Trilemma" (D.F. Mckenzie). (ASK)

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Teaching Mathematics In New Times

Conference Proceedings
Volume 1

MERGA 21
1998

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Mathematics Education Research Group
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Teaching Mathematics In New Times

Volume 1

Clive Kanes
Merrilyn Goos
Elizabeth Warren
(Editors)

MERGA 21
1998

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Preface

The Twenty First Annual Conference of the Mathematics Education Research Group of Australasia (MERGA) was held on the Gold Coast, Australia, 5 - 8 July, 1998. The conference attracted papers from researchers in Australia, New Zealand, the Pacific Islands, the United States of America and Europe. Contributions from a large number of established researchers - many with substantial international reputations - as well as a significant number of beginning researchers were included in the program. Research studies reported at the conference encompassed a variety of mathematics education related topics and made use of a wide range of research methodologies and theoretical perspectives. Published presentations were organised into Keynote Addresses, Published Papers (including two symposiums), and abstracts of Short Communications.

In accepting presentations for the conference, every effort was made to be as inclusive as possible of the research needs and interests of members of the mathematics research community.

Refereeing processes for the conference were overseen by an Editorial Committee membership of which included Clive Kanes (Convener, Griffith University), Merrilyn Goos (University of Queensland) and Elizabeth Warren (Australian Catholic University).

Procedures adopted by this committee to select papers for presentation were as follows.

> ***Refereeing procedure for Published Papers***
> In this category, each paper was reviewed by at least two referees. If both made the same recommendation, then this decision was upheld. If the decision was split, the paper went to another referee. The majority decision was then upheld. Where referees made their acceptance of a paper conditional on certain revision of the paper, the Editorial Committee used its discretion to ensure that these tasks were appropriately completed. Referees were asked to heed guidelines for published papers consistent with those set out in the call for papers and provide their written comments in a standard format under the following suggested headings:
>
> 1. Statement of problem/issues and discussion of its significance
> 2. Literature review and theoretical framework
> 3. Methodology and data analysis (where applicable)
> 4. Discussion
> 5. Conclusions and implications
> 6. General comments on paper
>
> As far as was possible the research interests of referees were taken into account when papers were allocated for review.
>
> ***Refereeing procedures for Short Communications***
> Abstracts for Short Communications were reviewed by the Editorial Committee. The aim of these reviews was to ensure the relevance of the proposed communication for the conference.

The aims of the Editorial Committee in adopting the methods indicated above were to facilitate high quality feedback to authors and achieve best possible assurance that the conference proceedings conformed to MERGA research guidelines It was especially hoped that beginning researchers would benefit from the feedback received.

The Committee wishes to thank those who generously agreed to act as referees for the conference proceedings - without the dedication abundantly evident in many of the reviews both the conference, and the mathematics education research community, would have been the poorer.

All effort has been made to present authors' work as it was intended - apologies are made where this has not been successful.

Lastly, on a personal note, I wish to thank my colleagues, especially Merrilyn Goos and Elizabeth Warren, for the help and assistance they provided in the production of this work.

Clive Kanes
Editorial Committee, Convener
Brisbane, Australia
June, 1998

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The 3 Rs in new times: Research, rhetoric, and reality

Lyn D. English
Queensland University of Technology

In this paper I argue that we need to analyse critically our currently popular theories of mathematical learning and instruction. I initially raise a number of issues warranting attention and begin by reviewing briefly a selection of theories, confining my discussion to socioconstructivist perspectives, situated cognition, and cognitive psychology /cognitive science. I then undertake a critical analysis of these theories, with a focus on the curriculum examples that have been used to support these theories. Finally, I propose a working model that might assist us in advancing mathematics education and research into new times.

Introduction: Imminent Upheaval

> Learning involves asking, investigating, formulating, representing, reasoning, and using strategies to solve problems, and then reflecting on the mathematics being used. (Romberg & Collins, in press)

Ideally, learning in the mathematics classroom would incorporate all of the above features. Reality, however, is another issue. I anticipate a major upheaval in the international mathematics education scene, ignited largely by concerns over students' performance on the recent Third International Mathematics and Science Study (TIMSS; National Research Council, 1996). Global reactions to these test results will have far-reaching implications for mathematics education research and curriculum development. Researchers need to be proactive, rather than reactive, in responding to this emerging upheaval in our field, especially since our work and our theories are receiving much of the blame for the TIMSS findings (McKeown, 1998).

I thus propose that we need to analyse critically our currently popular theories of mathematical learning and instruction. In particular, we need to address these questions:

- Are our current theories adequately informing us of students' learning in the mathematics classroom?
- How well coordinated are the theories and the learning experiences they recommend?
- Do these theories effectively support our development of rich curriculum experiences?

I would argue that it is difficult to respond positively to each of these questions and will attempt to demonstrate why in the course of this paper. My concerns regarding these issues are reflected in a forthcoming editorial by Judith Sowder (1998), in which she quotes a recent Californian project commissioned to synthesise K-12 Mathematics Education Research (Carnine, 1997). The guidelines for the project stipulated that a minimum identification requirement for a piece of research to be included in the analysis was that it be an experimental study. I will leave the reader to ponder the implications of this, especially in light of Carnine's (1997) comments on his study:

> The stakes surrounding this work are extremely high in California. Not only will the quality of mathematics education of hundreds of thousands of California students be affected by standards, curriculum frameworks, and instructional materials associated with this synthesis of experimental mathematics studies, but quite likely, the quality of mathematics education for millions of other students around the country and beyond (p. 2).

Theories Of Mathematics Learning And Instruction

Socioconstructivism

During the past decade or so, constructivist and sociocultural theories of learning have had a major impact on mathematics education research and curriculum reform (e.g., Cobb & Bauersfeld, 1995; Cobb & Yackel, 1996; Hickey, 1997; National Council of Teachers of Mathematics, 1989). Strong support for socioconstructivist approaches to mathematics education has come from studies documenting how students develop conceptions that deviate quite dramatically from those the teacher intended (Cobb & Bauersfeld, 1995).

It is not the intention here to review the various interpretations of socioconstructivism. The reader is referred to a recent article by Hickey (1997) for an overview of the different paradigms. Of relevance here is the predominant socioconstructivist view of mathematics education, one in which the "inquiry mathematics microculture" is at the core of students' learning and communication processes (Cobb, Yackel, McClain, & Whitenack, 1997).

The Inquiry Mathematics Microculture: At the heart of the socioconstructivist paradigm is the "inquiry mathematics microculture," defined as "a classroom culture in which explanations and justifications carry the significance of acting on mathematical objects." These explanations involve "specifying instructions for manipulating symbols that do not necessarily signify anything beyond themselves" (Cobb & Bauersfeld, 1995, pp. 295-296). (In other words, mathematical constructs are operated on as entities in their own right, having moved beyond the processes or operations with which they were associated).

From the socioconstructivist perspective, inquiry mathematics has two dimensions, one cognitive and the other, sociological. In cognitive terms, inquiry mathematics entails creating and operating on "experientially real mathematical objects" (i.e., object-like structural conceptions), while the sociological component involves "participating in the development of a taken-as-shared mathematical reality" (Cobb, 1995, p. 104; Cobb, Boufi, McClain, & Whitenack, 1997). Learning within this inquiry microculture involves a process of self-organization, which occurs as students participate in, and contribute to, the development of mathematical practices established in the classroom. The mathematical concepts that each student individually constructs are seen as relative to their participation in these practices as well as constrained by it. The individual does not just internalize the meanings that were constructed during the social activity, rather, she reflects on the meanings, "replicating at a personal level what has occurred and is occurring on the social level" (Prawat, 1996, p. 220). As they participate in their classroom activities, students are considered to undergo a process enculturation into the mathematical practices of the wider community.

Students' beliefs about their own role, the role of others, and the general nature of school mathematical activity are important social norms that support the inquiry microculture (Cobb & Bauersfeld, 1995; Cobb & Yackel, 1996; Richards, 1991). Equally important are the sociomathematical norms which facilitate students' mathematical constructions and activities. These norms include: (a) established argumentation in which teacher and students challenge explanations that merely describe symbol manipulation, (b) acceptable explanations which take significance of acting on taken-as-shared mathematical objects, and (c) teacher and students acting in a taken-as-shared mathematical reality, enriching that reality as they negotiate mathematical meanings (Cobb & Bauersfeld, 1995).

Situated cognition, addressed next, shares some characteristics of the socioconstructivist approach, but has some special features.

Situated Cognition

In simple terms, situated cognition argues that a good deal of what is learned is specific to the situation in which it is learned. An indication of the theory's popularity is evident in the recent publication, *Situated cognition: Social, semiotic, and psychological perspectives* (Kirschner & Whitson, 1997). As the editors noted in their preface to this volume, the main impetus for situated-cognition theory has been a dissatisfaction with other paradigms for exploring learning and knowledge "as processes that occur in a local, subjective, and socially constructed world" (p. vii). Situated cognition, as we know it today, has arisen from a number of prominent studies, including those of Lave (e.g., 1988), Walkerdine (e.g., 1988), Davydov and his colleagues (e.g., Davydov & Radzikhovskii, 1985), and, of course, the classic works of Vygotsky (e.g., 1981).

On the other hand, as Bereiter (1997) has pointed out, the notion of situated learning was prominent in the first half of this century. Albeit, the concept was derived from a "psychology of situated rat behavior," but it nevertheless had the important tenet that "animals do not simply learn responses, they learn their environments" (Bereiter, 1997, p. 281). Rats will fathom a maze under carefully controlled conditions, learning a fixed route to reach a goal. However, they soon become lost when conditions are changed. On the other hand, rats will readily explore on their own (as we all know), and they soon work out an efficient way of getting from point A to point B. In other words, rats interact actively with their environment. As Bereiter further noted, we tend to forget that animal cognition is also situated, and, for humans, the situatedness of our cognition has a biological basis, despite strong cultural influences. What we need to do, according to Bereiter, is overcome the "situatedness of cognition" (Bereiter, 1997, p. 283).

Nevertheless, there have been numerous studies in the past decade that have lent support to the theory of situated cognition in students' mathematical learning (e.g., Carraher, Carraher, & Schleimann, 1985; Nunes, Schleimann, & Carraher, 1993; Saxe, 1991). It is beyond the scope of this paper to address these studies in detail. It will suffice to say that the work of situated-cognition theorists has included a focus on: (i) the problems associated with the transfer of "school mathematical knowledge" to the outside world, (ii) the influence of social and cultural practices in the classroom on students' mathematical learning, and (iii) the question of abstraction processes in mathematics instruction (Anderson, Reder, & Simon, 1996; Lerman, 1998). Probably the most frequently cited arguments arising from the situated-cognition perspective pertain to the mismatch between students' mathematical learning in typical school situations and their learning in the real world, such as the students' home environment or their place of part-time employment, where they need to apply mathematical knowledge. Lave's (1988) explanation for such a mismatch is that:

> . . . arithmetic practices are made to fit the activity at hand, and there are discontinuities between the techniques used to solve arithmetic problems in school-like situations and in the situations of shopping, selling produce, cooking, making and selling clothes, and assembling truckloads of dairy products. Place-holding algorithms do not transfer from school to everyday situations, on the whole. On the other hand, extraordinarily successful arithmetic activity takes place in these chore and job settings. (p. 149)

In concluding this section, I quote Lave's (1997) comment that mathematics education researchers adopting the situated-cognition perspective would likely describe conventional school mathematics learning:

> . . . as the all too mechanical transmission of a collection of facts to be learned by rote, a process devoid of creative contributions by the learner" (p. 17).

Cognitive Psychology

> Mathematics . . . is the study of the structures that we use to understand and reason about our experience -- structures that are inherent in our preconceptual bodily experience and that we make abstract via metaphor. (Lakoff, 1987, p. 355)

One of the challenges that face research and development in mathematics education is how students mentally structure their mathematical experiences, how they reason with these structures in learning and problem solving, and how they demonstrate to us that they do understand (Davis & Maher, 1997; English, 1997a; Hiebert, 1998). Such challenges are of particular interest to researchers who draw upon theories derived from cognitive psychology and, more recently, cognitive science.

In contrast to the socioconstructivist perspective, one of the core tasks of cognitive psychology is modelling the nature of hypothesized knowledge structures and cognitive processes underlying the learning and application of mathematics (e.g., Anderson, 1990; Greeno, 1989; Ohlsson & Rees, 1991; Resnick & Singer, 1993). Cognitive processes may be seen as actions on mental representations, these being internal mental structures that correspond to a segment of the world. Mental representations are often viewed in terms of networks of interrelated ideas, with the degree of understanding determined by the number and strength of the connections (English & Halford, 1995; Hiebert & Carpenter, 1992).

Cognitive researchers generally agree that we need to implement learning experiences that foster the construction of mental models or representations, which comprise the important relations and principles of a mathematical domain (e.g., Davis & Maher, 1997; English, in press a; Fuson, 1992; Goldin, 1992; Greeno, 1991; Hiebert, 1998; Romberg, 1998; Sfard, 1994). There has been considerable debate however, on how students construct these mental representations, on what forms these take and how they change with development, and on how students apply these representations in different mathematical situations.

Although each of the three main theoretical perspectives reviewed here recognize the importance of engaging children in mathematical problem solving, the cognitive psychologists have a particular interest in this issue. Some of our longstanding theories of problem solving were derived initially from the early information-processing models of human cognition, such as those of Newell and Simon (1972) and Simon (1978). Granted, though, these early models, which were tied closely to the computer metaphor, have some major limitations when applied to mathematical problem solving within and beyond the classroom (Greer, 1996).

Modern cognitive researchers generally agree that children learn mathematics best by solving problems, where they have the opportunity to work out the relationships between important ideas. Students develop understanding as they try to make sense of a problem, as they construct a mental model of it, and as they develop and refine methods of solution (Davis & Maher, 1997; English & Halford, 1995; Hiebert, 1998). As Hiebert (1998) emphasised, this understanding can be enhanced by engaging children in problems that are "nontrivial, multifaceted, and solvable using a variety of strategies" (p.142). Children's experiences with such problems should motivate them to "wonder why things are, to inquire, to search for solutions, and to resolve incongruities" (Hiebert, Carpenter, et al., 1996, p. 12).

In conjunction with this focus on problem solving has been an emphasis on the development of students' so-called "higher-order thinking skills" (e.g., Halpern, 1992; Paul, 1990; Peterson, 1988; Resnick & Resnick, 1992), and, more recently, on the development of their mathematical reasoning processes (e.g., analogical reasoning; English, in press b; English, 1997b). In recent times, cognitive psychology has been broadening into a cognitive science that draws upon several disciplines, including psychology, philosophy, computer science, linguistics, and anthropology (English, 1997a).

In the next section, I undertake a brief critical analysis of the key theories I have reviewed. In doing so, I highlight some of their perceived weaknesses, as indicated by others in the field. I follow this with a consideration of the curriculum activities used to support the theories.

Rhetoric and Reality: Time for Critical Analysis

> Research that aims to understand depends on the development of well-articulated theories . . . we need to think of theories not as grand global theories that unify the elements in mathematics education but rather as the products of making explicit our hypotheses and hunches about how things work . . . they [theories] provide explanatory solutions to the problems under study . . . it is well-recognized now that

> all observations, all data, are theory-laden. We do not make neutral observations; our observations are biased by the theories we use (Hiebert, 1998, pp. 144-145)

As Hiebert (1998) noted, theories are useful if made explicit, even if they are incorrect or inadequate. Theories need to be open to scrutiny and revised on the basis of constructive feedback. However, there is the danger that incorrect theories can interfere with developments in understanding, if the theories are retained in the face of conflicting evidence; this applies to the activities of both children and researchers. The problem is, as Kuhn and her colleagues have shown repeatedly, we find it very difficult to revise or even discard our theories, even in the face of conflicting evidence (e.g., Kuhn, Schauble, & Garcia-Mila, 1992). We would far rather confirm our theories than disconfirm them.

So, what is the message here for our popular theories of mathematics learning and teaching? Clearly, we need to ensure that we explicate our theories as best we can, that we reflect critically on our theories and practices, and that we remain open to new evidence and new ideas.We need to actively search for these new ideas. I argue that we cannot rest on our existing theories and assume they will carry us into new times, because they won't - - not even with a delicate face lift. Now is the time for us to act. We need to begin by critically evaluating our theories, our research, our progress, and, most importantly, our impact within the mathematics classroom.

A critical assessment of our research activities is essential for deepening our understanding of mathematics learning. As Hiebert (1998) commented, mathematics education is "subject to wildly oscillating opinions about courses of action" (p. 148). Without fully understanding the situations we are researching, we are prone to simply following the fashion of the day and not making informed and rational decisions. We need to step back and ask ourselves why we are adopting a particular theory to guide our investigations. Is it because it is the "accepted" theory to adopt ("flavour of the decade," so to speak), or because our supervisors or colleagues use it (one wouldn't dare use anything else!), or because the theory is so well established that no one will question our use of it? If so, it is time to step back and rethink our mathematics education, its research, and its theories.

To set the critical ball rolling, I present below some quotes from researchers who have commented on the present theories.

> For social constructivists, the negotiation of meanings in social situations, perhaps taking place implicitly, on a meta-level, is as important as the individual constructions of the learner. In my view, the process of that learning is not clearly elaborated. (Lerman, 1998)

> The term, "construct," . . . is unnecessarily vague and misleading . . . Just as past curriculum reform failures occurred when "discovery" was treated as if it were an end in itself, regardless of the quality of what was discovered, curriculum reformers today often treat "construction" as an end in itself, regardless of what gets constructed. In both cases, the means to an end is is treated as the end in itself, while more important ends receive too little attention. (Lesh, Hoover, Hole, Kelly, & Post, in press)

> Without that core [the world of immaterial knowledge objects], formal education becomes meaningless (as, indeed, some advocates of situated cognition seem to believe it is). Again, for better or worse, formal education is our individual escape route from the confines of situated cognition. (Bereiter, 1997, pp. 283-284).

> Although situated cognition researchers have taken a lively interest in learning, both in and out of school, they have not come up with anything that could be called a new educational vision. Instead, situativity theorists have tended to endorse various innovations of a social constructivist cast, interpreting them within their own

> frameworks. . . The main difficulty, I would suggest, is that situativity theory has not been able to provide a cogent idea of the point of schooling. (Bereiter, 1997, p. 295)

> The educational implications of research into situated cognition are not altogether clear . . . the implications of situated cognition research for mathematics curricula, and for the teaching and learning of school mathematics, need to be investigated in creative ways. (Ellerton & Clements, 1998, p. 165)

> The classic information-processing conception was based largely on research on human learning conducted in artificial laboratory environments. . . What was needed was a shift to the study of human cognition in more realistic contexts. In a sense, by holding too tightly to a literal view of the human-computer analogy, information-processing psychology took the field on a 20-year detour around a central challenge in cognitive science-- the explanation of human cognition in realistic settings. (Mayer, 1996, p. 159).

I now address some of the curriculum examples that have been used to illustrate the theories. I leave it to the reader to undertake a more critical analysis of these examples, and simply raise a few questions for further consideration.

The Inquiry Mathematics Microculture

The following is an example from Yackel and Cobb (1996), which they provided to illustrate their theory, in particular, to demonstrate sociomathematical norms in action. This episode occurred in a classroom of young children. The number sentences, 16 + 14 + 8 = __ and 78 - 53 = __ , were written on the chalkboard and posed as mental computation activities. The children were encouraged to generate their own meaningful ways of solving these examples, and to justify and explain their solutions. An excerpt from the teacher's and children's interactions is given below:

> Lemont: I added the two 1s out of the 16 and [the 14] . . . would be 20 . . . plus 6 plus 4 would equal another 10, and that was 30 plus 8 left would be 38.
> Teacher: All right. Did anyone add a little *different*? Yes?
> Ella: I said 16 plus 14 would be 30 . . . and add 8 more would be 38.
> Teacher : Okay! Jose? *Different*?
> Jose: I took two tens from the 14 and the 16 and that would be 20 . . . and then I added the 6 and the 4 that would be 30 . . . then I added the 8, that would be 38.
> Teacher: Okay! It's almost similar to -- (Addressing another student) Yes? *Different*? All right. (Yackel & Cobb, 1996, pp. 462-463)

In analyzing these interactions, Yackel and Cobb (1996) highlighted the teacher's emphasis on the children finding ***different*** ways of solving the problems. This is seen as important in developing the classroom sociomathematical norms, as Yackel and Cobb explained:

> . . . in responding to the teacher's requests for different solutions, the students were simultaneously learning what counts as mathematically different and helping to constitute what counts mathematically different in their classroom. It is in this sense that we say the meaning of ***mathematical difference*** was interactively constituted by the teacher and the children. The teacher's responses and actions constrained the students' developing understanding of mathematical difference and the students' responses contributed to the teacher's developing understanding. (Yackel & Cobb, 1996,. p. 462).

The authors also considered this classroom episode to be illustrative of how students develop particular mathematical beliefs and values, and consequently, how they develop intellectual autonomy in mathematics; in other words, how they develop a mathematical disposition.

The purpose of inquiry mathematics within the socioconstructivist paradigm is to foster students' ability to operate mentally on mathematical constructs as objects in their own right, that is, removed from the processes or operations with which the ideas were originally associated. In distinguishing between a traditional mathematics classroom and their inquiry microculture, Cobb and Bauersfeld (1995) referred to established practices of argumentation, where students and their teacher challenge explanations that merely describe symbol manipulation. The authors chose arithmetical examples from the early grades to illustrate these practices, primarily because the well-established, research-based models of children's arithmetical learning could guide the authors' instructional development. I raise some questions concerning this work:

1. What is really meant by the term, "inquiry," in describing the inquiry mathematics microculture?
2. Where is the evidence of students in this microculture developing a range of mathematical thinking, reasoning, and communication processes (e.g., thinking critically, creatively, and flexibly, posing mathematical questions and problems, and developing a commitment to the processes of mathematical inquiry and their improvement)?
3. What would be the key features of this microculture when other important mathematical domains are explored, such as geometry, probability, novel problem solving, and problem posing?
4. How would one describe an inquiry mathematics microculture operating at higher grade levels? [1]

Situated Cognition

In describing the learning of mathematics from a weight-watcher's perspective, Lave (1997) reported that the participants performed consistently better in solving isomorphic mathematics problems in other settings than in scholastic ones. In exploring their findings, Lave and her co-researchers tried to invoke the dieters' school mathematical knowledge during the course of their meal preparations. Given their failure to do so, Lave raised what she considered to be a central question: "Wherein lies the motivation for generating and solving problems in settings where there are not set tasks imposed on the problem solver?" (p. 24). I think the answer to this question is rather obvious. As part of her explanation, Lave explained that mathematical problem solving is not an end in itself for chefs, especially the weight-watching types. The mathematical relations they encounter when preparing their meals are important and meaningful to them in their efforts to reach a desired weight; that is, they have a sense of ownership over their problems. In dealing with the mathematical relations that arise, these weight-watching chefs would rarely employ algorithmic procedures taught in school. As Lave explained, "algorithmic math gets in the way" (Lave, 1997, p. 26). Instead, the chefs would invent a myriad of units of measurement and procedures for working out the required portions of food (e.g., using part of a design on a drinking glass to measure their milk allowance).

In support of her theory, Lave (1997) argued that mathematics education that focuses on detailed procedures for applying algorithmic knowledge to problems around the globe is not the answer:

> The problem is that any curriculum intended to be a specification *of* practice, rather than an arrangement of opportunities *for* practice . . . is bound to result in a teaching of misanalysis of practice . . . and the learning of still another. At best, it can only induce a new and exotic kind of practice contextually bound to the 'educational' setting. . . . In the settings for which it is intended (in everyday transactions), it will appear out of order and will not in fact reproduce good practice. (pp. 32-33).

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[1] Cobb (in press) addresses, in part, some of the issues raised here.

As previously noted, the message from situated-cognition theory for mathematics education is not clear, at least for the Australian curriculum that we have achieved (that is, before the Benchmarks hit the scene). In addition to requesting that situated theorists become more familiar with contemporary mathematics education, I raise the following issues for consideration:

1. Is the theory of situated cognition one that can be reasonably adopted by mathematics educators? (see Sfard's, 1998)
2. If so, how might we apply the theory effectively to classroom practice?
3. Can theories of situated cognition help us to overcome the problems of transfer of learning? If so, how?

With respect to the above questions, it is worth noting the comments of Bereiter (1997) and Anderson, Reder, and Simon (1996). According to Bereiter, the main weakness of situated cognition is "precisely its situatedness" (p. 286). This poses transfer problems in both out-of-school and in-school learning situations. The proponents of situated cognition tend to highlight the latter. However, as Anderson et al. pointed out, the claims of these theorists "demonstrate at most that particular skills practiced in real-life situations do not generalize to school situations. They assuredly do not demonstrate the converse" (p. 6).

Cognitive Psychology

In a forthcoming article, Fischbein (in press) analyses the relationship between cognitive psychology, as a general theoretical framework, and "the psychology of mathematics education." He argues that the latter is not obtained by applying cognitive psychology to mathematics education, but, rather, by "identifying genuine psychological problems in mathematical activities." Fischbein elaborates by claiming that this can be achieved by "creating adequate specific concepts which psychology does not provide, by devising adequate specific research strategies, and by formulating safe interpretations that are meaningful for both cognitive psychology and the didactics of mathematics." I think these comments nicely illustrate the limitations in applying any one theoretical perspective to understanding and advancing students' mathematical learning. I elaborate on this point in the final section of this paper.

The other issue I wish to highlight with respect to the limitations of cognitive psychology is the narrow focus on mathematical problem solving adopted by many advocates of cognitive theories. These researchers rarely seem to move beyond the traditional computational problems (e.g., *Kerry-Ann has saved $29. Justin has saved $42. How much more has Justin saved than Kerry-Ann?;* e.g., Fennema, Carpenter, et al., 1996; Mayer & Hegarty, 1996; Verschaffel & De Corte, 1997). Albeit, the focus now is on exploring the invented strategies that children use in solving such problems. However, while cognisant of the continued difficulties these word problems present children, and the need for us to further our research in this area, it is of concern that cognitive studies are not branching out into other problems. These include novel problems that develop a broad range of reasoning processes, which are fundamental to students' mathematical development (e.g., Bransford et al., 1996; English, in press c). Indeed, there have been recent calls for mathematics curricula to place a greater focus on problems of this nature (Mathematical Association of America, 1998).

In proposing that cognitive research needs to broaden its perspectives on mathematics learning, I refer the reader to the work of Bransford et al. (1996). They initially investigated children's solutions to the typical word problems and simply reproduced the findings of numerous earlier studies. These included the finding that "mathematical thinking," for many children, is the procedures used for solving numerical problems (often a search for key words). As a consequence, Bransford et al. decided to explore other ways of designing problem experiences so that children would not only construct meaningful mental models, but also would develop a range of important concepts, reasoning, and communication processes. Bransford et al.'s development of specially designed video supports is well documented (e.g., Cognition and Technology Group at Vanderbilt, 1996). By immersing children in a diverse range of problem situations, they were able to move the children's problem-solving experiences (and their own research) beyond "well-defined word problems" to include problem generation, problem projects, and ultimately, "mathematics-driven adventures" (p. 211-216). Likewise, my own research has highlighted the importance of students creating their own problems, sharing their problems with peers, and providing each other with constructive feedback for problem improvement and extension (English, 1998a, 1998b, 1997 c, 1997d). These problem-posing activities warrant further attention from mathematics educators and researchers.

In the final section of this paper, I draw upon some of the issues I have addressed and outline some suggestions as to how we might advance mathematics education and research into new times.

Moving into New Times: What Now for Mathematics Education and Research?

> . . . in accounting for the development of mathematical cognition in a child, an analysis of a series of participations in social situations needs to be complemented by some account of the coherent development and restructuring of that individual's knowledge and conceptualizations over extended periods of time. (Greer, 1996, p. 185)

A suggested working model for addressing mathematics education and research appears in Figure 1. I use the term, "working," to indicate that the model is an evolving one, open to modification and refinement from ongoing theoretical and empirical analyses. In proposing this psychological and socio-philosophical model, my sentiments reflect those of Greer (1996), cited above, and below:

> Improving mathematics education is a massively complex human problem, in the cause of which all relevant forms of knowledge need to be mobilized. In pursuit of the (perhaps unattainable) goal of a comprehensive theory of learning mathematics, multiple contributions are required. (Greer, 1996, p. 191).

As can be seen in Figure 1, the model comprises multiple components, all of which I consider to be fundamental to the design and implementation of rich and meaningful learning experiences in mathematics. I argue that these elements must be addressed in conjunction, not in isolation, if we are to make the progress in mathematics education and research that is needed for the new millennium.

The Psychological and Socio-philosophical Components of Mathematics Education and Research

Psychological Components:

- Consideration of the nature and structure of mathematics;
- Critical analyses of theories of cognition and cognitive development whose primary focus is on the individual's mental models and processes (cf. Greer, 1996);
- Analyses of different forms of mathematical representations (concrete and abstract);
- A focus on different processes of mathematical reasoning and thinking;
- Analyses of the relational complexity of mathematical tasks (English, in press c);
- A focus on problem structures, problem finding, problem generation, and problem critiquing (English, Cudmore, & Tilley, in press; English, 1998b).

Socio-philosophical Components: Individual and Collective Mathematical Inquiry:

- Promoting connected dialogue on mathematical issues, including conceptual and affective issues (English & Cudmore, 1998);
- Fostering critical, constructive, and creative analyses of individual and shared ideas;
- Conducting philosophical debate on mathematical viewpoints, meanings, and dispositions;
- Establishing a community atmosphere of trust and respect for participants and their ideas.

I will not elaborate on all of the above, rather, I will simply address a few key points pertaining to the socio-philosophical components. My notion of mathematical inquiry extends beyond the socioconstructivist perspective of reasoning with computational objects; it permeates all areas of mathematical learning. The development of a shared spirit of mathematical inquiry hinges critically on the reconstruction of the entire classroom as an environment in which open questioning predominates, and where both students and their teachers take responsibility for the asking and answering of questions (cf. Splitter & Sharp, 1995). Members of the class build on, shape, and modify one another's ideas, offer and analyse reasons for arguments put forward, help one another formulate questions and pose mathematical problems, and clarify and justify their mathematical ideas. For such constructive communication to occur readily, students need the confidence in offering their ideas and points of view about mathematics, and in correcting their own reasoning and that of others, if such reasoning appears faulty. We need to engender that confidence in our students.

This model of an inquiring community builds on the links between modern philosophy and mathematics. Effective mathematics education utilises many of the strategies, processes, dispositions, and understandings that are directly addressed in philosophy (English, in preparation). Furthermore, philosophical inquiry provides criteria for examining effective thinking and reasoning, while, at the same time, strengthening the dispositions and skills associated with forming reasoned judgements and decisions (Splitter & Sharp, 1995). These common links between philosophy and mathematics have the potential to enhance not only students' mathematical development, but also, that of their teachers. For example, teachers who bring a philosophical dimension to their subject matter are likely to be more reflective, critical, and self-correcting in their instructional approaches; likewise they can transfer this disposition to the students within the classroom community (Splitter & Sharp, 1995). Such a critical and reflective approach is important in teachers' understanding of the conceptual structure of their discipline. Teaching which reflects this understanding is critical in mathematics education, especially as we face some turbulent times ahead.

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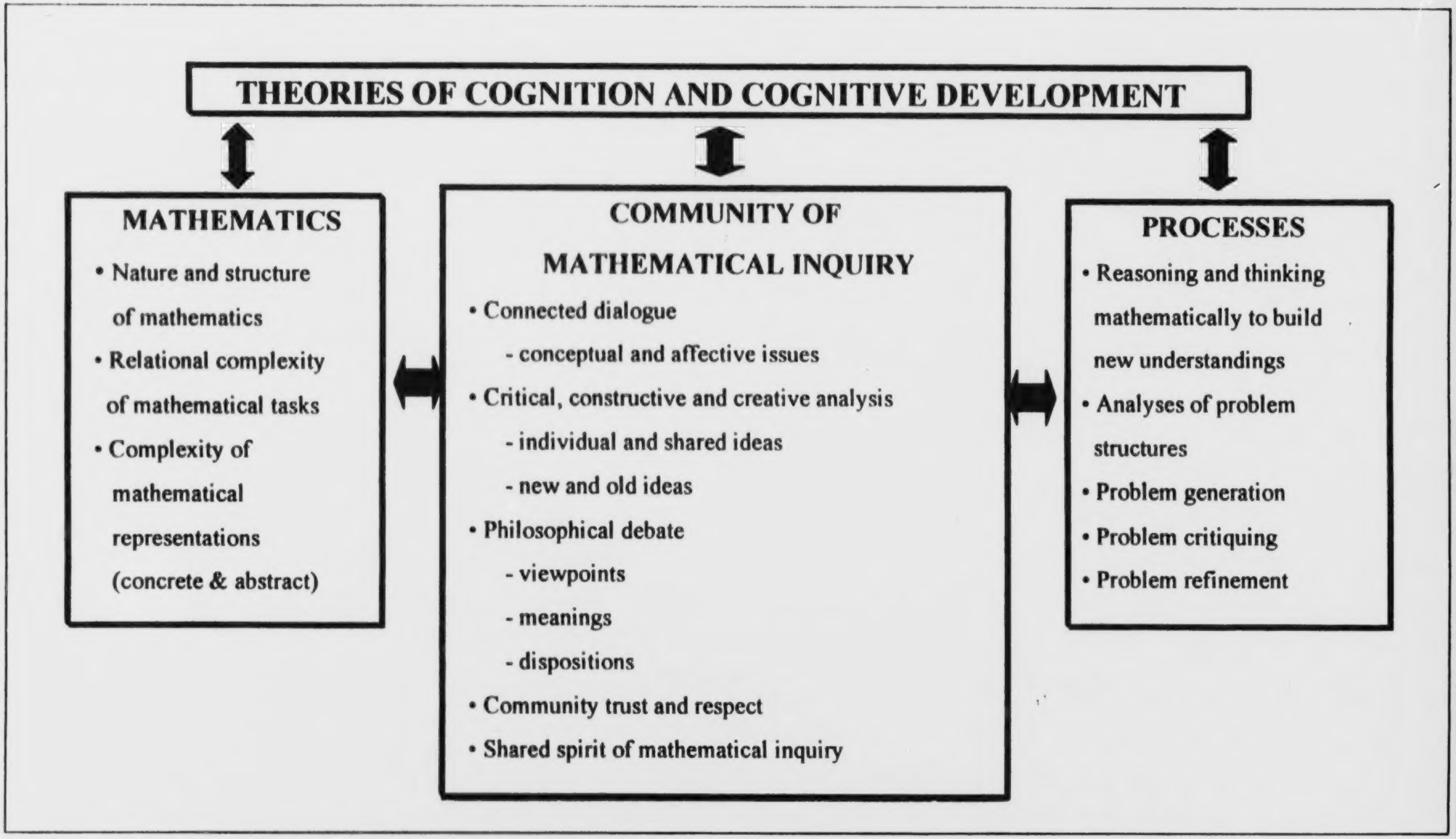
THEORIES OF COGNITION AND COGNITIVE DEVELOPMENT
MATHEMATICS
• Nature and structure of mathematics
• Relational complexity of mathematical tasks
• Complexity of mathematical representations (concrete & abstract)
COMMUNITY OF MATHEMATICAL INQUIRY
• Connected dialogue
- conceptual and affective issues
• Critical, constructive and creative analysis
- individual and shared ideas
- new and old ideas
• Philosophical debate
- viewpoints
- meanings
- dispositions
• Community trust and respect
• Shared spirit of mathematical inquiry
PROCESSES
• Reasoning and thinking mathematically to build new understandings
• Analyses of problem structures
• Problem generation
• Problem critiquing
• Problem refinement

Figure 1. A Psychological and Socio-philosophical Working Model of Mathematics Education and Research.

I keep six honest serving men

Peter Galbraith
Graduate School of Education, The University of Queensland

> " I keep six honest serving- men
> (They taught me all I knew);
> Their names are What and Why and When,
> And How and Where and Who. "
> (*Rudyard Kipling*)

In compiling thoughts for a plenary address such as this, a variety of personal choices present themselves. If I was to choose my own work as a focus then that would probably mean that ninety percent of any audience would be marginalised as far as their individual interests were concerned. Those interested in that work know where to find it and for present purposes I am happy for it to rest there, and in related conversations. I will, however, draw from experiences to illustrate some of the issues I hope to engage.

A paper such as this should reflect to some degree the joy of living and share light hearted aspects of our common endeavour as we face the challenge of teaching mathematics in new times. It should also draw the presenter to elements that make him uneasy and concerned, and these aspects also should be shared with the listenership/readership.

There are two risks with this approach. One is that avoiding specialization runs the risk of superficiality. The other is the risk of sounding like a preacher when the intention is rather to interrogate and question. We shall do our best.

Before commencing the text proper I should like to introduce an unseen presence. This presence is the 'ghost' of a child of the future. She is 10 or 15 or 20 years old, the year is 2010 or 2030 or 2050, the detail does not matter. What does matter is the single question she asks of each of us:

> *"What did you do in your academic lifetime that has helped make my mathematical education better than it would otherwise have been?"*

We shall return to her from time to time, but now to work:

> *"In the High and Far-Off Times O Best Beloved ..... there was an Elephant's child – who was full of 'satiable curtiosity', and that means he asked ever so many questions."*
> (Kipling, 1903)

Following this theme from one of the Just-So stories, I want to structure our excursion around Rudyard Kipling's six honest serving men. That is, I intend to share a personal response to the reflections generated when these questions are put within the field of Mathematics Education research and practice as we look forward at this point in time.

'What' Questions

To obtain a sense of 'what' is exercising our global community at the present time, I sought to summarize and classify the Short Communications written in English(500+) that were presented at ICME 8 in 1996. Because such work represents grass roots activity, it may be more indicative of total action than more selective journal publications. Of course, decisions on classification are to a degree subjective, and many presentations involve more than one focus but the following distribution emerged with the categories decided empirically from the content of the material, not pre-determined.

A. Major categories by number (influenced by my decisions to combine types)

- Teacher Education and Professional Development

- Technology
- Problem Solving/Modelling and Applications
- Metacognition/Constructivism/Collaborative Learning
- Gender/Social Context/Ethnicity
- Affect/Special Needs
- Geometry/Proof and Reasoning
- Number/Algebra/Calculus/Functions

B. Categories with Substantial number of entries

- Assessment issues
- Competitions/Materials/Texts/Games
- National Contexts/Political Content
- Probability and Statistics
- Tertiary Mathematics Topics
- History/Philosophy/Aesthetics

It is interesting to note that the two most populated categories were in fact Teacher Education (the human element in teaching) and Technology (the non-human element). Beyond that observation, what seems to be clear is that there is continued and intensive action somewhere on all the fronts that have provided recent foci for research and development activities, and as are represented for example in Research in Mathematics Education in Australia 1992-1995.

So what are some of the sub-themes that emerge from such an exercise. One is linked to the number of studies that continue to be generated in such basic learning areas as number, fractions, decimals and beginning algebra? The number of individual studies that can be designed appears to know no bounds. A question for our community is the extent to which studies on a given topic lead to isomorphic results – that is they say the same thing but are dressed in different clothes according to context. What might meta-analysis show that could provide stepping stones to a new level rather than adding to a maze of paths at the same level?

Secondly, what needs to be done to extend the power and applicability of conceptual frameworks such as information processing theory? Some few years ago I was talking with Gila Hanna when she lamented along the lines that:

> *"In the late seventies we were excited about understanding applications to learning such as 6 + 8 = 14 etc. What are we doing now (well over a decade later) – still looking at 6 + 8 = 14!"*

The point was not to talk down the importance of research in early number learning – it was to lament that we continue to take the easy option. That is- to look for mathematics that easily fits a theory rather than engaging the more challenging problem of extending theory to domains that lack the convenient discrete matching that operations, whole numbers, and simple symbols provide for processing models. And how many different IP theories are themselves isomorphic?

Thirdly, what tensions remain either unidentified or avoided within our research field? A clue is obtained by reviewing the substantial range of studies that fit within the description of Tertiary Topics. It was observed that with some few exceptions there appeared to be minimal interaction between the two components of our discipline; Mathematics and Education. Most tertiary topics were presented apparently free of learning contexts or implications. On the other hand, some of the presentations on learning mathematical topics at other levels, seem to be relatively innocent of concerns for the quality of the mathematics. So we are still in a socio-cultural situation where parts of our community are not communicating with each other. Put another way, the challenge remains to spell our discipline with a capital M and a capital E, not with one or the other or both in lower case. That this issue is of continuing importance was emphasised again as recently as April through an e-mail received from Mogens Niss:

> *"I have been invited to give a 45 minute talk in August at the International Congress of Mathematicians, in Berlin, on the nature and state of research in*

> *Mathematics Education, in particular as far as results are concerned.... It was not an easy matter at all to make ICM place Mathematics Education research in the program. I am planning to present a number, say 10, of major (and substantiated) findings in Mathematics Education research in a way that allows me to also discuss why a finding is neither a theorem, nor an experimentally verified result in the sense of, say Physics or Chemistry, but nevertheless something that represents deep reasoning and empirical analysis giving rise to insights of considerable significance to our understanding and development of teaching and learning of mathematics."*

Interestingly within this vein a hopeful note sounded as a result of the ICME analysis. This was the significant number of professional development and teacher education partnerships that involved collaborative contributions from mathematicians and/or mathematics pedagogy specialists and/or professional teachers.

So to summarize: What constitutes new knowledge and what represents re-packaged or isomorphic knowledge in well trodden research domains?

What is needed to enable theories such as information processing to reach the next level of application in complex mathematics learning?

What steps are necessary to increase the power of the partnership between Mathematics and Education in forums where one tends to dominate the other?

'Why?' Questions

Asking why? questions can help clarify reasons for shifts of emphasis within the field such as: Why is that area being abandoned or scaled down? Why are these new directions emerging at this point in time?

Reasons may broadly be classified as social, political, or personal and of course combinations of all three. Examination of some of the 1996 ICME projects is illuminating in understanding the social component. For example, Vandenberg points out the incompatibility of individualised Western approaches to learning within a group oriented African culture; Mel'Nyk from the Ukraine describes why the need to subsist is diverting teachers and students from mathematical study in a traditional centre of mathematical learning; Bhattycharyya notes the incongruence, and questions the relevance with which western computer technology can be superimposed in an Indian society rich in traditional culture!

Political reasons are more obvious, need little elaboration but rather more soul searching. Whether generated as a result of 'colonial echo' (Clements and Ellerton, 1996) or more locally generated by vocational or other contemporary needs as viewed by government they are centrally a locus for 'moral dilemmas' at system and personal levels. Do I accept the edict of political masters and help to make their interventions "work" whatever that means, or do I take a stand on principle when such is at stake, work for the higher ideals which I value, and risk marginalisation as a consequence?

Personal reasons may be subtler but no less influential. Why do I do what I do? To what extent am I swayed by the need for peer approval? I recall reading somewhere for example that "the '80s was the era of metacognition, the '90s the era of constructivism". Does this mean it is no longer fashionable in the 'best' research circles to espouse that which someone has declared yesterday's focus? In Thomas Kuhn's (1970) *Structure of Scientific Revolutions*, he refers to the social circumstances that arise in a community of scholars when a new paradigm challenges the old. While the emerging paradigm creates excitement through the new avenues it presents, a vast amount of "mopping up" activity continues within the earlier approach. This mopping up includes the solution of puzzles as yet unresolved and continues until by consensus no new amenable problems remain. While Kuhn was referring to methods of scientific enquiry, much of the logic translates to the content of activity. A new urgency arrives on the scene – but that does not mean that unfinished business should be set aside when much remains to be done. Our ghost child asks accusingly, "Why did you stop working in an area so central to understanding more about learning and in which so much remained to be done? Why did you abandon us?"

In asking Why? questions the fundamental driving force needs to remain the total enterprise of better learning, wherever that causes us to erect our tent, against the temptation that given the gift of a brand new hammer to see everything as a nail.

'When?' Questions

When do we stand back, reflect on our sometimes-frenetic activity, and ask our colleagues or ourselves uncomfortable questions about assumptions, values, and where our research is heading? For some time I have been involved in project activity associated with the use of technology in both secondary and tertiary settings, and I would like to draw on this orientation for purposes of illustration. Getting a grip on what is happening in technology is rather like trying to control a newly active volcano, as noted by Fey (1989) some years ago and it remains as true (or more so) today. In reviewing reports of technology-assisted instruction, it quickly becomes clear that the meaning given to such activity is fuzzy-so fuzzy and broad in some instances as to be almost useless if not actually misleading when trying to infer effectiveness, as distinct from reading descriptions of practice. The results of a valuable survey of Graphical Calculator research for example, by Dunham & Dick (1994), allowed inferences that were at most equivocal as to effectiveness, with the range of usage reported varying from limited access to integrated activity. Other studies in making very general claims for improved 'attitudes' triggered an 'alert' that it was time to ask targeted questions about possible interactions between attitudes to mathematics and attitudes to computers. In a paper in press (Galbraith and Haines, 1998) we found that among beginning undergraduates, mathematics confidence and mathematics motivation were strongly associated, as were computer confidence and computer motivation but that correlations between the mathematics and computer scales were much weaker. The scores on a computer-mathematics interaction scale correlated much more strongly with those on the computer scales than with those on the mathematics scales. This outcome provides empirical support for maintaining distinct scales for mathematics and computing when attitude measures are of interest, has resulting implications for the introduction of computer based activity into mathematics learning, and for our further research. The 'when' component arises because it is the accumulation of reported outcomes that seem 'not quite right' that eventually trigger the mental stocktake and altered direction. And of course our own findings are no more exempt than earlier ones from similar critique seeking to clarify future research priorities.

Other questions arise when time is taken to reflect on statements and claims made in relation to the impact of technology. One that comes to mind concerns claims made for computational technology in terms of rendering certain types of learning redundant. A case in point is the ability to produce complex graphs using graph plotters or graphical calculators. A popular type of traditional question for a year 12 test paper was to sketch the graph of a quotient of polynomials e.g. $y = \dfrac{x^2 - 5x + 6}{x^2 - 5x + 4}$

Now of course this can be produced immediately by computational means so clearly the purpose of such a question type is obsolete – or is it?

It is arguably so if the purpose was to be able to draw such graphs purely as an end – but no-one would ever suggest that the earlier goal was to prepare a student for a future in which such proficiency was a valued and essential vocational or life skill. Rather, the purpose was to test the ability to integrate information concerning domains, axial intercepts, discontinuities, asymptotic behaviour, and maxima and minima into a coherent whole, for which the graph provided the visible display of understanding. The question here is not whether such a goal is worthwhile (that is a separate debate) – but that if such a goal is valued then the use of computer technology is irrelevant to its purpose, and the resulting product of no educational consequence in this context. The situation is like the

Victor Borge story about the doctor who discovered a cure for which there was no disease!

A third stimulus to the act of review can occur when noticeably different actions and problems emerge in settings, which in other respects contain similar features. This has been vividly illustrated from data currently emerging from two projects involving respectively the use of technology in an undergraduate mathematics course, and in a course located at senior secondary level, in which the author has found it useful to use the descriptors of *power tool* and *learning tool* to describe two different ways in which students use technology. When used as a power tool, students use their computational aid as a self-generated means of solving existing problems and addressing new ones, demonstrating both mastery and versatility e.g. the effective use of spreadsheets. In learning tool mode students may, for example, be seeking to learn new concepts and calculus based problem solving techniques by working through workshop material designed around the use of symbolic algebra packages such as Derive, Mathematica or Maple.

Just how different these properties are has begun to emerge – students struggling with a learning tool are battling a plethora of difficulties, which are cascading and multiplying. Students mastering computing as a power tool are not only using it effectively in problem solving but in some cases re-inventing it as a learning tool for acquiring new skills and knowledge beyond the initial context. And evidence is accumulating that apparent power tool proficiency can flatter to deceive as when superb project work assisted by symbolic algebra software is compared with mediocre (or worse) performance on conceptual material closely aligned to the project content. Computer based learning has traditionally been reported in terms of variations in the computer role; it is now time to give greater attention to the different characteristics that human participants display in terms of the way they interact with given technological environments. Sociocultural theory is promising here as we consider not only the transforming effect of new technologies on human action, but that human action may in turn stimulate new learning possibilities for such technologies.

Questions can also be triggered when accumulating unease about perceived over simplification reaches some threshold level such as misgivings about the over-interpretation latent in the use of metaphor. I recall reading a piece on problem solving in which the game of tennis provided the metaphor. The argument was for immediate involvement in problems without attempting the thorough development of associated skills. The reasonable grounds were that it is possible to play and have some success and motivation with tennis, without requiring that the component shots be perfected first. A balancing observation would be that the substitution of a military or medical metaphor would result in the reverse argument -survival on the battlefield or performance of difficult surgery cannot be left to exploratory chance or semi-developed skill. It is not a question of being right or wrong but a recognition that the use of metaphor is a concession to our inability (due to limited processing capacity) to keep in mind all relevant aspects of a topic or field. What this does at one level is illustrate why different perspectives on a common issue (such as the development of problem solving expertise) continue to attract their respective and at times vehement proponents. What it does in general is to remind us of the potential abuse of metaphor- that is the imputation of objective truth such that the metaphor becomes "fact" and inferences are ascribed the status of logical consequences.

'How?' Questions

Here I would like to ask a different kind of question. How can we, as members of the Mathematics Education community of practice, make our own unique contribution to methodological critique?

As Educators, we are part of a Social Science community, co-existing with or engaging in a world of deconstruction, post-modernism and a variety of other 'isms' that characterize the field. Some of us feel pressured, or obliged, or act by conviction to engage to a greater or lesser degree with these concepts. From time to time I am tempted to remind over-enthusiastic friends that concepts such as post-modernism have the same level of reality as Puff the Magic Dragon! That is not to say that such concepts are not useful or important, but to recognise that they are but preferred frameworks through which individuals choose to interpret their world.

Inevitably, most of us have been involved at some stage with debates about research paradigms, e.g. the validity of quantitative versus qualitative methods, either as research students or as supervisors concerned for the 'academic safety' of thesis students. Interestingly perhaps, the post-modern view has been instrumental in bringing about the acceptance of a wide range of methodological approaches. Whatever the reason, it now seems true that both qualitative and quantitative approaches are accepted, including in studies in which they serve complementary purposes. However, it is also true that this acceptance remains grudging in some circles and that hard-nosed positivists and 'purely' qualitative researchers do little more than tolerate the presence of the other. Against this background complemented by those peopling some middle ground, we can ask how those who specialise in both Mathematics and Education can add a positive voice. Firstly, it is of significance that the term is 'Mathematics' not 'Statistics', although some may legitimately claim both titles. It is significant because for much of the methodological world the terms are regarded synonymously.

There are three aspects within which I believe we can contribute conceptually.Firstly, to continue to refute the erroneous assumption that the use of mathematical representation is necessarily associated with positivism. The fundamental tenets of positivism are the separation of fact and value and a belief in the universality of 'laws' irrespective of context. That these lead to and support the design of experiments and thence the use of statistics does not mean that the latter implies the former. Such an assumption represents one of the most fundamental mathematical mistakes – the confusion between necessary and sufficient conditions. Statistical representation and calculation can provide productive illumination within many contexts. But this is essentially a defensive response and my argument here is that we should be using our Mathematics background in a positive sense – to occupy the high ground as it were. To do this we can invoke one of the 'favourite sons' of contemporary social science viz deconstruction. Just as the post-structural view claims the validity of deconstructing and re-constructing the meaning of text, and rejects the concept of single imposed meanings, so we can invoke the commensurable 'right' to deconstruct the meaning others may traditionally have imputed to the use of statistics in research, and to reconstruct it, for example, in terms of a descriptive illuminating representation of case study data. Put another way, we are importing mathematics as a form of communication into the discourse of research reporting, and claiming no more nor less than that the accepted norms of discourse analysis apply.

Secondly, we can contribute a balancing dimension to the important principle that value free observations do not exist. Typically presented as an argument that every quantitative measure involves a qualitative aspect (i.e. the set of values or the theory that prompted the decision to obtain that particular measure), we can likewise argue that no purely qualitative research is possible. This has been a fond claim of some of the more vehement opponents of quantitative measurement. Quite simply, every qualitative decision involves a quantitative element. The decision to ask this person rather than that one for an interview, to include this question but not that one on an interview schedule etc. means that more 'units of worth' have been attached to some alternatives than to others. Behind every qualitative design there lies (at least) one ordinal scale. A purely qualitative study does not exist either.

Thirdly, our Mathematics should equip us to contribute to debates that invoke arguments claiming support from broad mathematical theories. A contemporary example would involve Chaos Theory with appeal to sources such as Gleick (1987). An argument I have seen in various forms runs along lines that typically appeal to some version of the "butterfly effect". Essentially, it argues that because chaotic modes have been shown to exist in many complex non-linear systems, and because social systems such as educational organisations are examples of such systems, we can expect unpredictable outcomes (chaotic regimes) no matter what programs we put in place. This latter inference is derived from the property that for chaotic regimes different outcome patterns are obtained from initial conditions that vary even minutely. My objection in spirit is that the view is defeatist and pessimistic, and on the basis of reason that it is erroneously incomplete. When confronted by such reasoning, there are two responses that can be given. Firstly, that chaotic behaviour occurs generally across a very small volume of parameter space – that is the vast majority of behaviour modes will be non-chaotic anyway. Secondly, that for modes that do exhibit chaotic behaviour the settling time (time to become independent of initial conditions) is often significantly long. For interest I ran a model of a small social system, and the time taken for chaos to set in was what we might agree could be regarded as substantial – about the same time as between the present day and the battle of Waterloo! Given the normal timescales associated with curricula, schools, universities, and other learning centres it is safe to say that initial conditions will usually retain much greater influence on outcomes over the relevant time-period than the potential of chaos. What we try to achieve does matter! Our child of the future will not thank us for giving up on the grounds that chaos is likely to emerge anyway.

Finally, in a methodological vein it is worth noting that Mathematics Education is unique in that its theory and practice involves separately, and in various combinations logico-deductive knowledge, empirical scientific knowledge, and knowledge derived from the naturalistic (Lincoln & Guba, 1990) and critical (Gibson, 1986) paradigms. That methodological issues continue to arise is not surprising, noting that even the first two of these (often combined in both professional and popular circles) are not necessarily viewed as close companions within the scholarly community; for example as noted by Snow (1948) in a celebrated essay on G.H. Hardy. We should continue to expect to be exercised by methodological challenges and in finding how to inject our perspective into debate.

Where? and Who? Questions

One of the most frustrating of all searches is for points of influence and impact? Where and on whom should we concentrate our efforts? Where and to whom should we present our findings? Where can we find forums and tools of communication to reach and convince our necessary audiences? Who are these audiences and whose arguments carry the greatest weight?

These examples include at least two meanings of 'where' – political content and physical location. The latter is fairly obvious – as an example of the former consider the use of the terms such as *accountability* that are increasingly part of the education agenda. A word such as *accountability* can be construed as, following Wolf (1996):

(i) being able to *explain* actions; or
(ii) being responsible *for* some actions; or
(iii) being responsible *to* someone for actions taken.

These interpretations have substantially different meanings in practice where for example (i) allows considerable teacher autonomy while (iii) could mean the withholding of funds unless prescribed standards and methods (as specified by major stakeholders) are followed.

So in addressing questions of *where?* and *to whom?* the matter of the various "interests" served by concepts, procedures, and individuals is brought into focus.

The question of "interests" as applied in education generally has been associated with the 'critical theory' movement from which it has drawn its conceptual basis (Habermas, 1971; Gibson, 1986; Carr and Kemmis, 1986). Three levels of interest have been articulated, viz technical, practical, and emancipatory, and much has been written concerning these in the general teacher education literature, see for example van Manen

(1977) and particularly Zeichner and his associates (Zeichner and Liston, 1987; Zeichner, 1993). For completeness a brief summary of the characteristics of these interest positions follows.

Technical interests refer to the interest in gaining knowledge for the purpose of efficient and effective application in controlling the environment defined in a broad sense. This may be taken to include the attainment of prescribed educational objectives requiring technical skill, and mastery approaches to learning and assessment provide an exemplar for the achievement of technical interests in mathematics. Technical interests are not bad in themselves, however they do not represent the only kind of knowledge to be sought.

Practical interests refer to a conception of action that involves explicating and clarifying assumptions behind alternative actions and evaluating the ultimate consequences of the actions, such that actions and decisions are linked to value positions and those who are actors in the system must consider the worth of the various alternatives.

This type of knowledge is concerned with interpretive understanding. However, the subjective meanings embedded in aspects of this knowledge are controlled (limited) by the context in which it is enacted, and hence govern the extent of what can be achieved. Reflection plays a significant role in developing the scope of *practical interests, which* might encompass, for example, acceptance of a range of teaching approaches and assessment types as necessary to meet and evaluate varied aims, and types of mathematical activity.

Emancipatory interests refer to issues of justice, equity, and fulfilment, and can only be served by a 'critical approach' that identifies the restrictions referred to above and reveals how they may be eliminated. Thus they offer an awareness of how aims, purposes and possibilities have been repressed and distorted, and what actions are required to eliminate sources of inadequacy or frustration.

It is the distortions that offend this latter viewpoint not the pursuit of other interests as such. For example, technical interests may facilitate emancipation through the provision of mathematical power to learners, but they may also disempower through culturally inappropriate methods of teaching or assessing and thence accrediting competence.

In turning attention to Where? And Who? questions it is useful to invoke these conceptions of *interests* as a lens through which to view the content of Mathematics Education. Returning to the concept of *accountability* we consider how it might be interpreted within an Education system. If we define the term as meaning *being able to explain actions* (i.e. account for) we give freedom for teachers and schools to argue for or defend a variety of teaching and assessment practices based on corresponding value positions – practical interests at least are provided for. If we define the term as *being responsible for some action* we again locate the power and responsibility with those who design the learning and the assessment. Decisions made must be defensible in terms of the context of operations. If however the term is defined to mean being *responsible to someone (or some organization)*, then there is a major shift in the location of power which might involve a requirement to carry out a particular educational program with non-compliance punishable by withdrawal of funding. Interests served in this situation are likely to be narrowly technical, and those who value practical or emancipatory interests are similarly outraged by this position, an outrage which is often compounded by their exclusion from the design and rationale of curricular and assessment procedures. The former's outrage is essentially intellectual, concerned for the quality of mathematics mandated by the procedures; the latter's outrage extends to encompass the political – concern for both students and educational professionals, at the narrowly technical expertise they are offered, and concern that educational decisions have been usurped by ideological expediency. National testing and benchmarking systems are sources for such concerns.

The point at issue is that of involvement. At the outset terms like *accountability* tend to be used as buzzwords in as yet uncrystallised forms. Emancipatory interests would demand that the profession become involved at the outset, while there may still be time to influence the ultimate decision, and to fight where necessary to overthrow imposed structures and definitions deemed educationally or morally unsound or restrictive.

It is possible to go on to identify conflicting interests in almost every area of Mathematics Education practice. The question is what this does for the advancement of knowledge and process, for awareness and critique at one level can serve to harden and re-ify opposition and defences at another, or can remain simply as critique. One possible path to progress is to adopt a *dialectical strategy*, that is to specify potential conflicts of interest and subject them to informed rational debate, rather than polarize and entrench opposing viewpoints through verbal attack or to strive for an uneasy but superficial peace. To question rather than condemn is also consistent with the Habermasian approach exemplified in his promotion of the Ideal Speech Situation (Habermas, 1971; Gibson, 1986).

The ideal speech situation (ISS) requires that each individual communication possess four qualities. It should be
(a) comprehensible – e.g. made in a shared language
(b) true, i.e. matching what we perceive as reality
(c) correct, i.e. legitimate within the context of the topic
(d) sincere

Furthermore, the ISS requires that all speakers (or communicators) have equal rights to dispute, assert and question and, by inference, have equal access to relevant knowledge. Put differently, this requires that an ISS be free from domination and, given this condition and genuine goodwill, progress can be pursued by rational argument.

It is not however difficult to identify impediments to this ideal in our debates on issues in Mathematics Education, e.g. unequal 'rights of speech' afforded to political voices versus educational voices, administrators versus teachers, academic mathematicians versus professional educators, parent demands versus teacher ideals, competency advocates versus wider mathematical adherents. Witness for example the difficulty MERGA experienced in attempting to have its voice heard by DEETYA.

How might such an approach work? A simple beginning would involve selecting issues of significance and for each posing questions that direct debate to the intellectual content and power relations represented by identified alternative interests made public. The following examples illustrate the dialectic properties of this approach.

Interests of teachers in developing understanding	vs	Interests of students in achieving grades
Interests of administrators in placating parents	vs	Interests of teachers in using innovative assessment
Interests of schools in meeting numeracy benchmarks	vs	Interests of weaker students abandoned to enable others to reach that level
Interests of subject heads in controlling curriculum	vs	Interests of teachers in developing best practice
Interests of technocrats in proliferating technology	vs	Interests of professionals in requiring quality learning through technology
Interests of universities in circumscribing school	vs	Interests of schools in increasing professional

mathematics		autonomy
Interests of Testing Services in selling instruments	vs	Interests of teachers in designing assessment
Interests of the 'State' in competency measures	vs	Interests of the school in quality measures

And we might add a third column – the interests serving the question asked by our ghost child!

Such a listing is not intended to imply a "good" versus "bad" characterisation. *The purpose in establishing a dialectic is to create an agenda and a forum through which interests may be disclosed and addressed that are often present but not made public.* While there is no assurance that this will always work, public declarations of position and the relentless exposure of assumptions, enhance the possibility of progress and render a little less likely the use of vicarious means to avoid confronting issues and the use of managerial methods to overthrow educational goals.

A question of scale

So the six honest serving men have had a say. It is clear that they are versatile and able to overlap their functions. What? questions can be readily re-phrased as Why? or How? questions and so on. What is also clear is that their questions permeate all boundaries. The downside of this is that they do not in themselves provide measures of relative importance, scope, or urgency. Another way of approaching issues is to estimate the extent and type of their impact. This was attempted by Kaput and Thompson (1994) in their use of nautical metaphors to describe the perceived profundity of the impact of aspects of computer technology on mathematics learning. Borrowing their terminology with suitable adaptation we obtain the following classification:

Surface wave:	represents procedures and impacts at the level of the individual classroom, school, or institution.
Swell:	larger scale and more pervasive so there is impact at the local system level.
Tidal wave:	generated beyond local boundaries and frames of reference and requiring time-scales with larger orders of magnitude.
Sea level change:	fundamental reshaping, analogous to a change in sea level produced by global warming.

Using this broad classification we can begin to assign issues to the various levels – acknowledging that clearly a substantial element of subjective judgment is involved. Looking across the large range of studies represented by the ICME Short Communications it appears that the vast majority are at the level of *surface waves*. Examples might include variations of classroom organisation, studies on student learning, the development of contextualized assessment materials such as school-based projects, and the use of performance data alternatives such as innovative writing tasks, interviews, and various kinds of application tasks. Also included would be most technology based learning initiatives.

At the *swell* level we note moves to augment formal certificate examinations by a greater range of alternative performance measures including increased use of teacher judgments. Some noticeable activity at national levels is occurring in countries such as African states that is related to opening mathematics to groups previously unable to participate. In some Eastern European countries a reverse effect has been noted, in that talented youth are being targeted to mitigate potential fall-out from a loss of direction and increasing vagueness and uncertainty that is a legacy of the major political changes of the past decade. The impact of National Statements and Testing Programs would be appropriately included here.

At the *tidal wave* level we look for more pervasive change, identified beyond the boundary of a local system and represented and sustained by strong broadly based influences. Although not necessarily legislated into practice, such influences can gain their strength from internationally located movements that impact through professional exchange. Contemporary examples would be the transforming action of a new epistemological stance such as constructivism, the impact of Vygotskian psychology on collaborative learning approaches, and the changes wrought by some countries in response to outcomes of international studies of mathematics achievement. Perhaps also the movement to link mathematics with elements of local culture and real world applications, that has developed an increasing international presence. In a previous generation various versions of the "New Mathematics" movement would fit this classification as would the influence of Piagetian theory and practice.

Although tidal influences are profound, there may still be regions that remain isolated from their impact and for a *sea-level change* we envision uni-directional forces of massive impact and irresistible momentum. As a past example we might consider the development of communications, including the series of ICME conferences and the establishment of international journals in Mathematics Education that have changed forever the relationship between individual members of our international academic community; although whether there has been commensurable impact on practice could be contended. It would be a very wise or very foolish individual who felt confident to predict definitively on such matters so I will merely suggest three that seem to be potential candidates.

They are respectively the continuing development of mathematical software and associated technology, the communication properties and potential of the Internet, and globalisation as represented by an internationalisation or notions of competence and standards as a consequence of a developing international trade in qualifications and courses.

The nautical metaphor is introduced to address the problem of structure. The issues we consider, the research we undertake, and the reports we write tend to be important but undifferentiated, apart from classifications by topic or interest group that may be used (indeed productively) to organise presentations into cognate groupings for presentations at conferences such as this, or for publication in monographs or special journal issues. This method of organisation seems to resemble J.J. Thomson's plum pudding model of the atom with issues (clusters of electrons!) scattered like currants and raisins in an amorphous mass labelled Mathematics Education.

The nautical metaphor (primitive though it is) is intended to facilitate a classification closer to the Rutherford – Bohr model by separating issues and themes into 'energy levels' on the basis of their perceived location, pervasiveness, and ultimate influence. This is a crude device but may provide one way of structuring research content into 'family groups' that can then, if desired, be examined using other sets of dimensions such as 'interests served' or cognate topic grouping.

Conclusion

One of the frameworks we have found productive in current research is the concept of a community of practice. The mathematics fraternity has, over a period of centuries established norms and customs through which to evaluate and refine the standing of new mathematical claims- a socially constructed community of practice. Classrooms organised along community of practice guidelines aim to develop a similar environment for the exploration, testing, and refinement of ideas and conjectures, leading to authenticated mathematical knowledge (Goos, Galbraith, & Renshaw, 1998). Yet there is a potential irony in the dedicated application of constructivist principles to learning environments, in that while there is encouragement to construct, interpret, and re-construct understanding there is a noted exception." You (the student) are encouraged to challenge and reconstruct all aspects of classroom activities except one- the overall constructivist approach through which I am teaching you". Such is an example of the epistemological dilemmas that need to be addressed.

When we add (capital E) Education to (capital M) Mathematics we change the norms of our total professional activity in a radical way-particularly when embracing contemporary principles such as post-modern theory. The consequences I suspect are that the Mathematics Education community is plunged into an exercise in constructivism far more profound than that occurring in any classroom. Our community does not yet have an agreed knowledge base or set of procedural norms to which to appeal-for that is part of the construction in progress. To use an analogy from the history of mathematics, that I have applied elsewhere to issues in assessment, we can think of ourselves as growing through our Babylonian and Egyptian periods in which imperatives of pragmatism and urgency, arising through our responsibilities towards the human subjects involved in all our work, have seen us engineer and develop methods tailor made for the urgency of the moment and the location. To progress according to the analogy into and beyond the Greek period means attempting to set our practices within a consistent theoretical framework-mathematics would seek such order, post-modernism might argue it is a false goal.

In any case there is the need to ask searching questions on account both of the current influences in our field, and on behalf of our child of the future whose destiny we are helping to shape. In this address/paper I have suggested just two possible ways to interrogate our activities. I do think there is a need to look for structures and to estimate the potential future significance of topics and issues that at present are distinguished only by their content, and the nautical model is a beginning attempt to address that need. Ultimately the way our field develops depends upon the ongoing synthesis that results from the giant exercise in social constructivism that our community is engaged in. Within this activity two extreme responses threaten our collective progress. One is withdrawal from debate on the grounds of irreconcilable differences, the other is comfortable living within a closed community of like minds.

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The intension/intention of teaching mathematics

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In recent years mathematics educators have begun to draw on academic disciplines beyond mathematics and traditional, cognitive psychology as resources for theorising. Sociology, anthropology, and even psychoanalysis have formed the theoretical framework for some researchers. Mathematics education has become more conscious of culture, in all sorts of ways. But in cultural studies, writers have been addressing what it is to be and to communicate, and therefore to learn and to teach, in ways which of most of us in mathematics education have not been aware. In this paper I attempt to examine and use postmodern ideas to look, in particular, at mathematics teaching. (1)

Given the enormous changes in almost all aspects of life over the last two or three decades of the twentieth century, it is rather remarkable that the teaching of mathematics looks, for the most part, in most parts of the world, much the same as it did 50 years ago. From within the community it seems to suffer from an inertia that militates against substantial change in teaching styles or perceptions of learning. For instance, in the UK where we have national tests at ages 7, 11, 14 and 16, teachers say that they have no time for doing interesting mathematics, they have to just get on and teach the stuff. I understand and sympathise with classroom teachers for the pressure they are under but the point I wish to make is that it's as if the teaching/learning style doesn't make any difference: on the contrary the most efficient and time-saving is the familiar chalk-and-talk. Although the establishment of coursework as 30% of the marks in the national examinations in mathematics in the UK at age 16 in 1988 forced all teachers to integrate investigations and extended pieces of work into their mathematics teaching repertoire, for most this did not lead to a change in teaching style (Lerman, 1989). It rapidly became institutionalised into a set of standard procedures (Hewitt, 1992; Morgan, 1995) and in any case has now been reduced to 20% and become optional. From outside the community, governments, business, industry and parents hold onto an image of good mathematics teaching from a mythical past which is about rote learning, drill and practice, plenty of algebra, no calculators, and so on.

But if we look around us at what the world is like today we have to admit that we are in a time of rapid, almost breathtaking, change, what many call postmodern times: a new era of communications and access to information; of shifting employment patterns; potentially changing structures of government; and of global capitalism (Jameson, 1991). Culturally, socially and intellectually, the move to the postmodern has been rather like letting the genie out of the bottle. For the foreseeable future there seems no way of imagining how it might be possible to get it back into the bottle. Where one might have imagined that one could pin down, for example, objective educational aims, now one is forced to admit that there is no objectivity apart from a stance, a position, bringing with it a situation of powerfulness or powerlessness. Take the examples of culture and communication. They must be seen today as commodified and political. What constitutes culture is what commands high prices and is valued popularly, the constant search for the new whilst colonising the old, the debates about high and low culture; all these are about markets, success, fame and consequently power. They owe little to Gombrich's and Read's debates about Art with a capital A. Those who own the channels of global communication and control who has access to those media, also control what we know about the world. Baudrillard could claim that the Gulf War never 'happened', it was just a souped-up computer game. The anarchism of the World Wide Web may fragment late capitalism's increasing domination of communication and the media, but it is by no means certain. There seems no way of turning back from the condition of postmodernity (Harvey, 1989).

What about education in the postmodern world? "The very possibility of education depends on arriving at some view about how people and societies can and should be *represented*." (Kemmis, 1995, p. 136) If we are forced to recognise that all such representations will be partial, that they will be readings, and can even be described as simulacra, fictions which will give access to some and deny it to others, then there is a need to review what we mean by education, curricula and for our particular concern, the teaching of mathematics. I have used words of coercion, such as 'forced' and 'admit' when speaking of postmodernity because it feels for me and for many others as a wrench from a time of at least the possibility of certainty, of being convinced that one can speak of, in particular, class as a fundamental structure of society, but also of culture as something hard to define but nevertheless reachable. It appears not to be as painful for our children who are growing up in this era, and it has some interesting repercussions. For instance, in the UK in higher education, having gone through the 1980s where students chose courses for the secure financial future which these courses seemed to open up, security which often turned out to be a mere chimera, they are now changing to philosophy, psychology and sociology. I interpret that as being about styles but also as about students recognising the inherent unpredictability of their futures and the impossibility of determining what the economy wants and needs even three years hence, and so it becomes instead a choice of doing what looks interesting now, and facing whatever comes next when it arises. I feel sure that the uncertainty is heightened by the *fin de siècle*, indeed the end of the millennium.

Twelve months ago Shlomo Vinner (1997) warned the PME community that we ignore developments in children's lives, in education and in mathematics at our peril. Focusing here particularly on teaching, there is no doubt that we face major problems: in the UK we continue to fail mostly working class children and some ethnic groups in terms of acquiring certification in their mathematics; each year there are less interested and/or qualified people coming into mathematics teaching; the profession has a low status which goes together with generally low pay when compared to other professions, mathematics remains probably the most unpopular school subject; and in adult life people will happily own up to innumeracy, whereas illiteracy is something about which to be embarrassed. It has been suggested that the UK government's move back to basics in numeracy and literacy is a doomed attempt to plug the postmodern bottle again, to lay claim to the established verities of the foundation of knowledge as supplied by the three Rs, but unfortunately the genie has already escaped. It is certainly, however, an effective way of controlling and disempowering teachers. In the last year, since the election of a Labour Government, the Department for Education and Employment (does the the name implies the aims for education?) has issued pronouncements on: calculator use (or rather non-use) in primary schools; the required time for homework; the compulsory implementation of a numeracy hour (equal to 50 minutes!) every day for primary pupils; and the correct style of teaching (interactive whole-class) to be used. With the implementation of a National Curriculum for mathematics in primary and secondary teacher education courses next academic year the intention of many of us to present the image of mathematics teachers, educators and researchers as professionals will be undermined even further.

I want to take this opportunity to cast a concerned eye, from within the community, over the mathematics classroom. First I will review what postmodernists are saying about mathematics and about teaching and I will then take a sceptical look at what we are doing in much of our research on teaching and teacher education. Finally I will suggest some moves that we might make in response. I do not claim to be able to offer any answers: indeed it would be inappropriate to a paper on postmodernism even to try. My aim is merely to try to ask some useful questions about what might be *meant* by the term 'teaching mathematics', which is the sense of the term 'intension' in the title, and how one might encourage and enable the *becoming*-teacher, the sense of the word 'intention' in the title. I will play with these two words a little in this paper. A struggle with words and meanings brings a self-consciousness about language use to the surface, an important element in postmodernism. I am trying, also, to articulate and make explicit a struggle over what is taken to be 'teaching mathematics' (intension), which is usually implicit in mathematics education whilst it determines what people want to do or want done in teaching (intention).

Academic Mathematics

In the early years of my work in mathematics education I was very preoccupied with a characterisation of views of the nature of mathematics as either relativist or absolutist and the possible pedagogic consequences of the adoption of either of those two positions. Such a binary opposition is outdated in a time of postmodernism. I will try to sketch a view of the development of mathematics which owes more to Wittgenstein's notion of language games than to relativist/absolutist oppositions.

The invention of non-Euclidean geometries in the 19th century had a profound effect in mathematics and in philosophy. Euclid's fifth postulate appears to have worried him, and many other mathematicians, for centuries afterwards: its very length suggests that it is more problematic than the others. Euclid seems to have structured the Elements to start with as many theorems as possible, the first fifteen, which do not require that postulate (Kline, 1980). Centuries of attempts to resolve this concern followed, including rephrasing the postulate to present it as less open to doubt. One version of it, due to Playfair, is that through any point not on a given line there exists one and only one parallel. Others tried to prove it from the first four postulates. In attempting to prove the fifth postulate by *reductio ad absurdum* Bolyai, Lobachevsky and Gauss separately found that they could not reach an absurdity; the fifth postulate is an independent postulate. You can take it in Euclid's form, or another opposing version of it, such as that there is more than one parallel. Either way, the resulting geometry is quite consistent. Instead, therefore, of having a single geometry of nature, fitting empirical phenomena, as given by God the Great Geometer through the way the universe was constructed, we now have many geometries. It is not that one is right and the others wrong as far as the world is concerned: there is no way of saying which geometry one should use *a priori*. The geometry chosen structures the problem and the potential results. Einstein (1954) chose four-dimensional Riemannian geometry for relativity because it suited his hypotheses about matter and its distribution. "Hilbert spaces are symbolic abstractions, not geometric pictures. Although Hilbert spaces are rule-governed, movement within these spaces is more like a move in a game of chess than a knowledge-claim" (Ashley & Bettebenner, 1996, p. 140). In this period of what they term late modernity, using a framework given by Baudrillard (1983), mathematics had become a game which marked the absence of any discernible referent, a dissimulation. That is not to say that it is meaningless, merely foundationless. It is about rule-governed activities within which meanings are carried, meanings which certainly matter greatly to the people engaged in them. The dissimulation forces us to focus on the rule-governed discourse rather than on what and whether it represents. Beginning with non-Euclidean geometries and culminating in the twentieth century failures of the foundationalists' struggles, the Kantian image collapsed.

Moves into hypermathematics, starting perhaps with the reluctant acceptance into mathematics of Appel and Haken's 1977 proof of the four-colour theorem, can be said to have marked the postmodern. The problem, that any map can be coloured using four colours only, was reduced by them to just (!) 1,482 configurations, but the evidence of the validity of this reduction and the checking of those cases took 1,200 hours of computer time and 10,000 pages of paper copy (Tymoczko, 1986). Lakatos (1976) argued that mathematics is an *a posteriori* science, in the sense that examples can be counter-examples leading to revision at a local or global level; the trouble is that the evidence in the case of the four-colour problem is humanly unsurveyable. We could go further and claim that "If a whole branch of mathematics were to be based on hypermathematical computation, this sub-discipline would to some extent be unsurveyable, auto-constitutional, and self-referential" (Ashley & Bettebenner, 1996, p. 148). In chaos theory we have seen how the determinism of a set of rules is profoundly affected by unpredictable initial conditions, yet patterns almost miraculously emerge. Thus, in weather prediction meteorologists now produce perhaps 50 different forecasts, arising from different starting conditions, different vigour in the flapping of the butterfly's wings, and the pattern that emerges produces a reliable range of possibilities. Finally, there are only the rules of the language game, but following the grammar still leads to fascinating and often useful results.

I do not want to suggest that the transfer of this view of mathematics into the classroom is easy, or necessary. Clearly a recontextualisation is effected through principles whose nature must be addressed (Bernstein, 1996). I want to emphasise, though, that an argument can be made for a mathematics that is a very different animal to the one of popular perception. It is not a question of whether the animal has two or four legs, but perhaps that it has no legs at all, or body, but is merely a simulacrum.

The Individual Child

Mathematics teaching still focuses on facilitating the individual child's cognitive constructions, and much theorising in mathematics education goes into creating a language for describing and creating a child-centred, constructivist practice (for a critique of constructivism see Lerman, 1996). This is the dominant perspective even though the image of the individual as the source of sense-making and as the autonomous builder of her/his own subjectivity has been hardly a tenable claim since it was challenged by Marx and Durkheim in the nineteenth century. The extent to which the individual is constituted in and through economic, social and cultural forces has been the subject of much discussion and analysis outside of the mathematics education community, and some within (Evans, 1993; Evans & Tsatsaroni, 1994; Klein, 1997a; 1997b; Lerman, 1998a; Winbourne & Watson, 1998). Today we might want to talk of the individual as a fragmented self at the intersection of a unique collection of overlapping identities constituted in different practices, as lived out through class, race, ethnic, sexual, gendered, regional and other positions. Thus "we cannot fully specify the psychological subject/agent as an object whose nature can be defined in isolation from a context" (Harré & Gillett, 1994, p.26). This often seems almost contradictory, since the language available to us to talk about our minds is one which gives the appearance of a context-independent inner reality. However Wittgenstein's discussion of pain puts another perspective on language and reality, in particular the nature of our internal, mental reality. Bloor (1983) describes it thus:

> "One possibility, he *(Wittgenstein)* said, to explain how we learn the word 'pain', is that 'words are connected with the primitive, the natural, expressions of the sensation and used in their place' (*PI*, I, 244). A child who hurts himself and cries is comforted by adults who provide a vocabulary for expressing the pain. The word 'pain' does not mean 'crying' but takes over from it. The newly acquired verbal behaviour, he says, is itself a form of pain behaviour." (p. 51)

Labov's experiments from 1972, replicated and elaborated by Cole and colleagues in 1978, offer some powerful examples of children speaking very differently about a situation or task, depending on the social setting, the other persons in the interaction, and the perceived goals, demonstrating the fragmented nature of subjectivity. Labov describes an African-American child, thought to be deficient in language, demonstrating that 'deficiency' when talking with a white interviewer in a formal setting and with a black interviewer in the child's home. In a third situation the black interviewer sits with the child and his friend, eating snacks, and the interviewer introduces taboo words and topics. "The effect of these changes on Leon's speech was dramatic. Not only did he go beyond one-word replies to questions, but he actively competed for the floor... Labov concludes that Leon had no difficulty using the English language." (Cole, 1996, p. 236). Of course Nunes, Schliemann and Carraher's (1993) work in relation to the significance of the socio-cultural setting and the nature of the activity on mathematical performance, and the consequent misperception of children's abilites when judged soleley by mathematics in the school setting is well known, as is that of a number of others (Saxe, etc.).

The situation-specificity of children as persons, then, raises major questions for teaching and learning mathematics. What is it to learn? Can one speak at all of decontextualised knowledge of mathematics? How can one use the results of cognitive studies? Is it perhaps more useful to speak of aiming at children becoming school mathematics persons, in much the same way as children become part of other social and cultural groups, with their own language, rules and logics? I will return to this idea below.

Teaching

One of the most significant moves in thinking about teaching (one might say one's intention in teaching) in the last few decades has been that of reflective practice and its association with critical theory and action research. Reflective practice offers a view of how teachers act in the classroom as informed, concerned professionals and of how they continue to learn about teaching and about learning, about themselves as teachers and about their pupils as learners. It encourages and feeds the notion of the autonomous, emancipated teacher who is not dominated by government rhetoric in choosing the way s/he wishes to conduct the classroom interactions, nor by the self-interests of some university-based researchers in defining what constitutes valid research. At the same time its association with critical theory invites an engagement with the institutions of schooling in actions to change those structures and emancipate thinking and acting. It injects a relativism into what we can know about teaching: in this classroom, with these students, this learning material, certain things happened which might be explained thus and might be acted upon thus. It seeks to avoid the traps of extreme relativism, however, by emphasising principled thinking, reason, and critical judgement. The critical, reflective practitioner:

> "thinks and acts in accordance with, and values, consistency, fairness, and impartiality of judgement and action. Principled, critical judgement, in its rejection of arbitrariness, inconsistency, and partiality, thus presupposes a recognition of the binding forces of standards, taken to be universal and objective, in accordance with which judgements are made." (Siegel, 1988, in Parker, 1997, p. 44)

It is precisely here that the critique of the postmodern enters. First, there has to be a presupposition of autonomy in order to argue for autonomy. Unless one is free and emancipated, how can one recognise when the dominated teacher steps out from their chains, whether that teacher be oneself or someone else? From which Archimedian position can one identify that autonomy? Positivists appeal to the transcendental to resolve this, that the truth of the transcendental argument presupposes even being able to ask the question. Habermas uses the ideal speech situation in this way: "all speech, even intentional deception, is oriented towards the idea of truth... Insofar as we master the means for the construction of an ideal speech situation, we can conceive the ideas of truth, freedom and justice" (Habermas, 1970, in Parker, 1997, p. 58). Similarly, rationality is claimed to be a transcendental notion. To argue about rationality is to presuppose rationality. The problem is that reasoning is a process, a language game, whilst what counts as justification for reasons, or validity of reasoning, is specific to a social context or is overlapping across a range of social situations and is entirely part of the grammar of the particular language game. The force of a parent's statement as reason to a child, "Because I say so", is in the nature of the relationship. The child, and perhaps later in the day the parent too, may well recognise that it was not a reasonable reason, and certainly not one that had been justified. Such a statement would certainly not be considered reasoning in a debate. Rationality, then, is deconstructed, it needs to be placed under a sign of erasure, what Derrida calls *sous rature*, and written ~~rationality~~, which highlights the absence of a foundation whilst it claims to provide a foundation; so too with ~~autonomy~~.

Second, from where is the criticism, rejecting arbitrariness etc., to come? Views are positions, perspectives from where one is situated. A critical view is another view. Now the claim of critical theory is that one can become aware of how one's intentions may have become distorted by self-interest or by false ideology, and one can reject and remove these distortions:

> "a critical social science will seek to offer individuals an awareness of how their aims and purposes may have become distorted or repressed and to specify how these can become eradicated so that the rational pursuit of their real goals can be undertaken" (Carr & Kemmis, 1986, p. 136)

There is no doubting that each of us has aims, purposes and goals but, first, they are always context-specific and, second, what might it mean to describe them as distorted or repressed? Indeed the goals of critical theory have been seen by some as far from emancipatory. Reporting on a course designed to address experiences of students' sense of powerlessness on a US campus, which began with a critical theory approach but then adopted a post-structuralist one, Ellsworth (1989) made the following points:

(i) It appears that critical pedagogy draws on a notion of a decontextualised individual when encouraging teachers to help students to identify and choose between "sufficiently articulated and reasonably distinct moral positions" (Liston and Zeichner, 1987, in Ellsworth, *op cit.*, p. 304). The argument for choosing rationally between moral positions is again based on treating 'rationality' as a transcendental signifier. Further, it can be understood as operating to establish an irrational Other and therefore denying a voice to perspectives that question the positions guaranteed by rationalism (p. 306). In contrast, poststructuralist thought is not bound to reason, but "to discourse, literally narratives about the world that are admittedly *partial*. Indeed, one of the crucial features of discourse is the intimate tie between knowledge and interest, the latter being understood as a 'standpoint' from which to grasp 'reality'". (Arnowitz (1987/88), in Ellsworth, *op cit.*, p. 304)

(ii) Some critical pedagogues argue for the teacher seeing herself as re-learning with the students, thus equalising the teacher/learner imbalance. Nevertheless, Ellsworth suggests that teachers are expected to use this equality to liberate and empower their students, thus re-emphasising the imbalance in knowledge and power (p. 307).

(iii) The authority of the teacher is justified as 'emancipatory authority' in that it is to be continually questioned and critiqued by the students and the teacher, and it is maintained through the respect and trust of the students. Ellsworth points out that the multiple socio-cultural positionings of the actors in the classroom setting are characterised by shifting relationships of power and powerlessness, of voice and its lack, through the many overlapping and separate identities of gender, ethnicity, class, size, age, etc., to say nothing of the 'unknowable' elements of the unconscious (p. 307-8, 318). Unless these positionings are given voice, 'emancipatory authority' is as repressive as any other.

(iv) No matter how strong one's identification with other oppressed groups, one can never know what it is like to be an other.

> "My understanding and experience of racism will always be constrained by my white skin and middle-class privilege. Indeed, it is impossible for anyone to be free from these oppressive formations at this historical moment. Furthermore, while I had the institutional power and authority in the classroom to enforce "reflective examination" of the plurality of moral and political positions before us in a way that supposedly gave my own assessments equal weight with those of students, in fact my

> institutional role as professor would always weight my statements differently from those of the students." (p. 308)

Feminist research, as well as postmodernism, has encouraged us, researchers and writers, to recognise and speak out our positionings and our powerfulness and powerlessness. In this way we at least share the difference and the fragmented nature of our overlapping subjectivities (Lerman, forthcoming, a).

There is no time or space here to pursue further the lengthy debates about postmodernism and critical theory (see e.g. Kemmis (1995)), and in particular the responses to Ellsworth's paper (e.g. 'Correspondence', 1990). Ellsworth and others express powerful concerns about critical theory. I will argue below, however, that one can continue usefully to draw on notions of reflective practice but from a different position.

Klein (1997a) describes similarly a shift in her teaching in which she saw her "practice as being coercive in that students always had to arrive at the authoritative 'truth' as portrayed throughout my subject and discriminatory in that students not adhering to my construction of the 'autonomous' student were classified as unmotivated" (p. 291). In this case Klein's strongly held theory was not critical theory but constructivism, whose tenets she was attempting to encourage student teachers to learn and adopt. She found that a poststructuralist focus enabled her to critique her practice and offered her a language for reconceptualising the function of the support that teachers give to move away from coercion to a position of enabling students' voices.

Research on Teaching Mathematics

In previous decades, as may be seen for instance in the early years of PME proceedings, it was thought that the answers to research questions concerning the teaching of mathematics could be found within mathematics itself (either diachronically, as in the search for epistemological obstacles, or synchronically, as in establishing structural/logical connections between concepts) or in developmental and cognitive psychology. In the last ten years, however, research on teaching and learning mathematics has diversified considerably. I will review here some of these directions. I believe we will see the same kind of fragmentation and lack of foundations which characterises the previous sections.

Teachers' Beliefs

I will start with research on teachers' beliefs, that is their theories about teaching (one might say the intension of teaching for them), their theories about practice, and their teaching as observed by researchers, work in which I have played a part (1983, 1986, 1990). It has been argued (see, e.g. Thompson, 1984) that teachers' beliefs are critical factors determining how they teach. So-called mismatches between theories and practices have been discussed in the literature (Cooney 1985; Thompson 1982, 1984, 1992; Lerman 1986, 1990), although it is sometimes referred to as espoused and enacted theories of mathematics teaching (Ernest 1989). In her review of the research in this field, Thompson (1992, p. 138) suggests that the relationship between teachers' conceptions of mathematics and their practice is complex and argues for viewing the relationship as a dialectic one, citing the work of Cobb, Wood & Yackel (1990). Mason (1990) engages with this dialectic by arguing that through awareness the teacher can reconcile "the theorist and practitioner inside" (p. 178). Whilst I feel sure that the relationship is a dialectic one for each of us in our teaching, in terms of researching theories and practices I want to suggest that the problem lies elsewhere. Research which examines teachers beliefs and theories in one context and attempts to examine practice, or beliefs about practice, in another context is again based on a notion that the core of a subject's identity is somehow unified and decontextualised. It is as if the teacher brings theories (abstract mental objects) to bear on practice (an empirical domain of actions affected by emotions), although it may be described as theories influencing one's practice and practices influencing one's theories (Thompson's 'dialectic'). Such a description recalls the classical philosophical distinction between 'knowing how' and 'knowing that'. Given this separation, attitudes and beliefs about the teaching of mathematics can be examined by an instrument in one setting, interviews in a laboratory or questionnaire completion on one's own for example, and their impact examined in another setting, the classroom. The activity is seen as, in essence, the same. There is, then, the argument that whatever mismatches there appear to be result from the influence of one particular factor, in a school environment perhaps, that distorts or over-rides beliefs.

These researchers recognise that the shift from one setting to another allows the appearance of factors that significantly change teachers' actions from those they would profess to apply or would wish to apply but there is no clear sense of any mechanism or relationship between settings and actions and/or beliefs. I would argue that, whilst there is a 'family resemblance' between concepts, beliefs and actions in one context and another they are qualitatively different by virtue of those contexts. One cannot speak of decontextualised opinions or actions; the setting in which the questions are asked constitutes the conversation and is not separable from it. The activities of answering a questionnaire, talking with a researcher, or teaching, are not, in essence, the same. Essentialism is not viable as an underlying supposition of human social behaviour.

Mathematics Teacher 'Change'

Research on teaching has fed into teacher education courses, both in-service and pre-service and back into research on teacher education. Interestingly, it is usually termed research on teacher 'change' rather than education or learning (Lerman, 1997). There have been some very impressive projects aiming at teacher change, described and analysed in Clarke (1997), including the well-known Cognitively Guided Instruction project and the Purdue project in the USA. In the many reports I have read it is clear, from the interviews with teachers, that they consider that their practice has changed, sometimes dramatically. I include here also the smaller scale studies such as Davis (1997) and Clarke's own (1997) project. My comments are not to criticise this work but to suggest a different reading.

The notion of 'change' suggests a teleology, a direction from something to something else, a desirable goal. As mentioned above, it is not generally called 'learning', except perhaps in pre-service teacher education courses. It may be that, in projects such as these and on in-service courses, it might sound too prescriptive or even arrogant to claim that teachers may be learning something about teaching. Nevertheless projects, particularly those motivated by the current reform in the USA, do have a vision of better teaching, teaching that, it is believed, will enable more students to learn more mathematics. Is it surprising that teachers change their perceptions of their teaching and in many cases are seen by the researchers to have changed their practice? As alternative stories of what teaching is (intension) and how it might be are offered to teachers, who have taken on board the goals of the project at least at the level of a commitment to participate, it might be expected that they would appropriate some of these stories and 'become' different teachers. I have in mind an analogy with a different setting, that of psychoanalysis where, in some interpretations, the patient is helped to develop different stories for their behaviour and experiences, some of which become meaningful, that is to say the patient becomes the person in those stories. A particularly vivid illustration of this, for me, comes in Davis's (*op cit.*) study where the researcher and the teacher develop a way of characterising teaching, through the two years of the teacher's development or change, as different forms of listening, termed evaluative ("the complement of listening strategies" (p. 360)), interpretative ("being aligned with constructivist theories of learning" (p. 364)) and hermeneutic ("to reflect the negotiated and participatory nature of this manner of interacting with learners" (p. 369)). My reaction was to realise that the research offered me, as it did them, a new story for thinking about teaching as that teacher's intentions for her teaching changed. It is not that teaching is listening, or that what is being suggested, namely that it is helpful to focus on how one listens, prescribes good teaching.

Clarke's (*op cit.*) study reveals factors that influence change, as perceived by the participants in the project, emphasising in particular innovative curriculum materials and the relationship between the researcher as critical friend and the teacher. Another direction of research on teacher change is the growing interest in using Shulman's (1986) description of a teacher's knowledge being about both subject matter knowledge (SMK) and pedagogic content knowledge (PCK) (Even 1990; Ponte, 1994). They are not independent of each other and the most interesting research emerges from considering their interaction. Even, Tirosh and Markovits (1996) describe how they use this interaction between PCK and SMK in teacher education programmes. For example (p. 126), they offer pre-service teachers possible student answers to the addition of fractions and the teachers engage with issues of teaching at the same time as sorting out for themselves the why of fraction addition.

I began this section by referring to research projects whose goals are to reform teaching in a certain direction and I then moved to those from which I gain most as reader, those that study teaching development whilst theorising about teacher development, without an explicit notion of a better way to teach, offering new readings of teachers and teaching mathematics. It seems to me that many of these researchers are not seeking closure, the assumption that ultimately there are answers to what is good teaching of mathematics and that reasoning and/or research will reveal the truths. Postmodernism is characterised by the denial of the possibility of closure, of an ultimate correct account, and instead encourages the production of a multiplicity of accounts with their meaning and value established locally.

Recontextualisation

School mathematics is a different animal from academic mathematics. It is subject to a social process which Bernstein calls recontextualisation (1996, p. 24). It is also different from everyday mathematics which is often harnessed, as a resource, into school mathematics. We can identify and discuss different expressions of school knowledge, or we can call them different levels of school knowledge, such as the construction of the school curriculum, the production of textbooks for schools, or that of school practice, in our case teaching mathematics. There is also a need to distinguish between mathematics education as a field of inquiry (Zevenbergen, 1996; Dengate & Lerman, 1995; Ernest, 1998) and the practice of teaching school mathematics. They are strongly linked, of course, and it is certainly the case that when we are engaged in the field of mathematics education we hope that we will have some effect on practice. Bernstein calls them the field of production and the field of reproduction, with the field of recontextualisation between them (1996, p. 116). The point is to recognise that the principles which govern the process of recontextualisation, at any time and any place, are arbitrary and serve interests and purposes. Such a realisation enables a re-inscription of those principles, not in the expectation or demand that those principles will be emancipatory, free of bias and undistorted, but principles which arise from local commitments and beliefs shared by the participants in a community of practice.

A Postmodern Pedagogy - The Intension of Intended Teaching

It is not that postmodernism means a rejection of reflective practice or the possibility of change in one's classroom, school, university or region. It means a re-inscription of these activities into a language of the recognition of difference; of a rejection of the expectation of closure but not of a loss of meaning and values; of deconstruction of reason, rationality and emancipation. Ellsworth (*op cit.*) argues that a classroom based on a practice

> "grounded in the unknowable is profoundly contextual (historical) and interdependent (social). . . What remains for me is the challenge of constructing classroom practices that engage with the discursive and material spaces that (the removal of the privileged self-image of the critical pedagogue) opens up" (p. 323).

The grand meta-narratives have not served us well. We are subject to capitalism's manipulations more than ever: racism, sexism and other forms of prejudice are increasing; nationalisms which oppress and murder others proliferate. Education, certainly in the UK and I am sure in many other places, is ever more dominated by the language of the market: measurement, standards, competencies, performance criteria, raising standards by tests of children, teachers and schools and by threats to teachers and schools. Rather than aiming for equality and emancipation we may be better served by recognising that the multiplicity of the social and cultural identities which constitute each of us result in powerlessness and powerfulness as those fragmented identities surface. We can create and celebrate pedagogies, theories and styles which allow and encourage the expression of differences and lead to the recognition that change at the local level is both possible and, actually, all there is. For example, in an earlier paper (Lerman, 1994) I referred to the most commonly used set of British textbooks, the SMP (School Mathematics Project), in which there are 4 levels of books for levels of ability (see Dowling, 1998 for a detailed analysis). I suggested there that one could offer pupils examples of the 'same' topic from top and bottom ability books to engage them in a critique of the way that notions of ability are constructed, and of how they themselves are constructed as able or not by the texts (Dowling, *op cit.*). This is not, however, a claim that there can be such things as textbooks free of bias, but that local action and deconstruction of the texts that construct people's identities and delimit their futures are the only source of taking power.

The point is that doubt and criticism, the cornerstones of critical reflective practice, don't 'bottom-out' upon arrival at freedom from bias (Parker, 1997), but at the language games in which one validates 'true', 'acceptable' and 'good'.

> "I did not get my picture of the world by satisfying myself of its correctness; nor do I have it because I am satisfied of its correctness. No: it is the inherited background against which I distinguish between true and false." (Wittgenstein, 1969, section 94)

Thus rehabilitated reflective practice is local, an expression of belief and commitment of the teacher(s) for improvement in teaching and learning. It is not that the binary oppositions of rational/irrational, emancipated/biased, illusory/true are wrong, merely unaskable and not interesting (Parker, *op cit.*, p. 141) in postmodern pedagogy. The issues to be tackled are meaningful at the local level.

Teaching and Learning

There is a sense of the intension of teaching in which it is a precondition of learning. It seems to me that there is a common thread, from Durkheim and Marx, through Vygotsky, to Wittgenstein and postmodernism, whereby all of what one might understand as human consciousness is the product of communication, social interaction and the relations of historically developed, socio-cultural experience.

> "Grammar tells us what kind of object anything is." (Wittgenstein, 1958, 373)
>
> "*Essence* is expressed by grammar." (*ibid*, 371)
>
> "How do I know that this colour is red? - It would be an answer to say: 'I have learnt English'." (*ibid*, 381)

People will not learn things on their own, by which I mean that a child born without human contact, the mythical 'wolf child', will not become a conscious human being. To develop conscious attention, memory, awareness and to gain culture, knowledge etc. are all the results of learning from others. "Instruction and development do not meet for the first time at school age; rather, they are in fact connected with each other from the very first day of a child's life." (Vygotsky, 1956, in Wertsch, 1985, p. 71). Harré (1997, personal communication) gives an example from Bruner's work of how this process operates from the earliest days of a child's life:

> "Bruner was interested in the very early moment of the development of intentionality and causality. Here's an attractive object. And an infant, a very small infant, reaches for it. Causal relation. Object is there...infant reaches out like that. What happens? Mother gives it to him. At that point the relationship between the reaching and the grasping changes...the mother has introduced into this game, baby wants, baby intends. Baby just acted causally...didn't get, so mother says baby wants. Next step, so she's completing the action, she is giving him the interpretation of his action. Next step, baby reaches forward, and compresses the air in his little chest and goes, 'Ah.' So, mother gives it to him. We've now turned the intentionality into a babble, the first primitive verbal utterance. Causality turning into intentionality. And the mediating role is the caretaker, who has done the defining, and done the supplementing." (2)

The learning may be in everyday situations, or in school or other intentional learning situations. The distinctions between them are important but the similarities are just as important. In both cases there is a necessary imbalance, one that is inherently about power relations, whether it be the social group whose rules the child wants to acquire in order to belong, or the classroom in which there is the struggle between multiple social demands on the child, of which a major demand is that of the teacher. Of course at a later stage one can learn from books, which one apparently does alone, but books are the writing down of human life and the learner-from-books has also learnt how to learn from books.

Now I do not want to suggest that teaching necessarily leads to learning. In some recent analyses of classroom videos of young children in a nursery classroom (Meira & Lerman, forthcoming) we have been describing situations where teaching, in the sense of the teacher's intended actions in the classroom, does not always lead to learning. We have described this as the zone of proximal development (zpd) not being created. We interpret the zpd as a creation of the interaction, including the goals and needs of the actors, the social relations of the classroom, the texts, the framing of the task and so on. The zpd is the classroom's (3), if it is anyone's (it is often assumed to be a kind of force-field which the child carries around, the dimensions of which the teacher must gauge); hence the inherent unknowability to which Ellsworth refers. The teacher and child may well miss each other's meanings. Similarly in other work I have been carrying out analysing peer interactions (Lerman, forthcoming, b) students may not catch each other's ideas (Vile & Lerman, 1996) and hence not create a zpd. Creating a zpd is as much about mutual alignment of goals and desires as about the intended content of the interaction and paying attention to this allows the potential for the re-inscription of teaching and learning into a language of difference which I emphasised above.

Learning as Becoming

Studies of how people learn in out-of-school situations, particularly work environments (Lave & Wenger, 1991), have offered valuable perspectives of the nature of the alignment of goals and desires by the learner and the teacher, the latter as mentor or 'master' (I use this gendered term in the absence of a more suitable one). In order to bring those notions into her discussion of schooling Lave (1996) draws on Olsen's study of the way that schooling shapes the identities of newcomers to the USA in terms of the "racialization of social relations and identities" (p. 159). Describing learning in terms of students becoming, in our case, motivated participants in school mathematics, it seems to me, is where Lave's approach is particularly fruitful for us. Lave's focus on the shaping of identity in social practice emphasises the centrality of the social relationships constituted and negotiated during classroom learning. Lave talks of learning as "an aspect of participation in socially situated practices" (p. 150). Provided we do not expect those practices to be those of the teacher, in our case of mathematics, or the practices of the mathematician, but instead of the practices of the particular mathematics classroom culture, seen as the intersection of a multiplicity of practices, this interpretation of learning is very useful, as is shown by Winbourne (1997), and Winbourne & Watson (1998) for example. After all, for the most part, students do not want to become teachers of mathematics, nor mathematicians, and hence the direct transfer of the apprenticeship model into the classroom by some writers is perhaps too simplistic.

Learners come to the classroom as persons of multiple, overlapping subjectivities. Different aspects of those subjectivites are called up by different aspects of the practices of the classroom, and are expressed through identities of powerfulness or powerlessness. At the same time, new subjectivities are constituted in the social relationships and forms of communication which make up the activities of the classroom. Rather than the intension of teaching mathematics as the handing over, or the individual construction, of ultimately decontextualised mathematical concepts by the teacher or by the pupil respectively, teaching can be conceived of as enabling pupils to become mathematical actors in the classroom and beyond. The goals and needs of pupils, and the ways of behaving and speaking as mathematicians, become the focuses of the teacher's intentions. This does not translate into either side of a traditional-drill-and-practice/progressive-group-learning dichotomy, however. Both may well be appropriate at different times. That dichotomy, like so many others, are irrelevant and uninteresting; clearly so when seen from a postmodern perspective.

Social Semiotics of Mathematics

Objects, including concepts, have meanings only within relations of signification (Walkerdine, 1988). One of the familiar examples of this is workers not seeing their work practices as mathematical although a mathematician looking at those practices would wish to say that they can be seen as applications of mathematics (e.g. scaffolders using lengths of pipes which are Pythagorean triples). The issue is to examine and identify from within which practice one is observing, since those practices carry with them their relations of signification. An illustration of this might be the following: Take a class of young children for a walk around the neighbourhood with the instruction "Observe". Repeat the walk, this time with the instruction "Observe and identify as many examples as you can of circles, triangles, squares and rectangles". The second activity is different from the first because the children are becoming different actors, they are observing the 'same' objects with a different pair of spectacles, those of the mathematician. From the perspective of teaching and learning mathematics the research programme would therefore be to study empirically the semiotic mediation of those objects. The language of semiotic mediation, whereby the person and the world for that person are transformed by the acquisition or appropriation of cultural tools (analogous with Marx's thesis concerning physical tools) which are historically, socially and culturally constituted artefacts, is a resource which engages with the specificity of relations of signification (Lerman, 1998b). It offers a medium through which one can account for: cultural specificity, such as the understandings of the Aboriginal student described in Klein (1997b), both of mathematics and of appropriate teaching styles; the effects of particular IT tools, such as using a dynamic software package for learning geometry; the manner in which learning the place value system structures perceptions of number; and the importance of enabling the articulation of students' own understandings of ideas and concepts.

The Intention of Teaching Mathematics

I will conclude this paper with some remarks about mathematics teacher education, drawing again on the second spelling, intention, in the title. Many teacher educators will be familiar with the problem that, despite sometimes writing good theoretical essays on teaching and learning during a pre-service course, students often begin teaching in the same way that they would have without attending the course (Crawford & Deer, 1993; Lerman, 1997; Klein, 1997b). It appears that courses do not provoke students to confront their naive notions of teaching mathematics. It may better be expressed in Lave's terms, as I discussed above, that student teachers have a sense of who and how they will be as teachers before coming to the course. The ideas that we offer, and even the essays that the students write to which we give credit, do not impinge on that initial sense of being a teacher. At the same time, the messages that teacher educators may be attempting to convey, explicitly and implicitly, about what they consider is good teaching, or that they consider student teachers should know, will in some measure be oppressive, to the extent that they deny agency to student teachers. In an analogous way to a postmodern pedagogy for schools, a postmodern pedagogy in teacher education would: encourage the expression of difference; teach methods of critique of orthodoxies concerning mathematics and mathematics education; and encourage theorising about teaching and learning mathematics. It would find ways of confronting student teachers with their naive conceptions of teaching and with different theories about teaching which they might well have been espousing, through activities which bring these together (Crawford & Deer, 1993; Lerman, 1997). It would engage with the personal transitions in becoming a teacher, in terms of developing an identity as a person in that profession and with purposes appropriate to that role, by enabling them to constitute through articulation different ways of being, as a teacher. "Teaching is more difficult than learning; for only he who can truly learn - and as long as he can do it - can truly teach" (Heidegger, in Krell, 1993, p. 254).

Notes

(1) My thanks to Peter Winbourne, Adam Vile and Anna Tsatsaroni for comments on an earlier draft.

(2) This quote comes from the transcription by Anne Watson of a tape recording of a seminar with Rom Harré, held in Oxford University on December 6th 1997.

(3) Along with Newman & Holzman (1993) I take the zpd to be the explanatory framework for learning as a whole, both in intentional settings, such as schooling, and in informal settings; in other words all socio-cultural milieus. In relation to classroom studies, the zpd is the classroom's.

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One Teacher's Problem Solving Beliefs and Practices: Influences and Coherence

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This study explores the sources of influence on and the level of coherence of one teacher's beliefs and practices in relation to mathematical problem solving. Rose regularly uses problem solving approaches in her Year 2 class as she believes that this makes mathematics learning meaningful. Her beliefs and practices have been strongly influenced by her own experiences as a learner of mathematics as well as by advice received in postgraduate education and in curriculum documentation. Her beliefs are coherent when explored using a variety of data collection methods.

It is widely recognised that many factors influence teachers' beliefs about mathematics and how mathematics should be taught. Teachers' beliefs about the role of problem solving in the learning of mathematics are similarly influenced by a variety of factors. It is possible that teachers' classroom practices may reflect the latest trend or fad in education or they may be based on a coherent set of beliefs. Information on the sources of influence on beliefs and practices and the level of coherence of those beliefs would inform preservice and inservice education. Encouraging teachers to reflect on their experiences as learners of mathematics and to discuss their beliefs and practices may better prepare them to embrace new approaches.

This study attempts to identify factors that influence beliefs and the impact of these on classroom practices. Possible contributing factors of influence might include: early learning experiences in mathematics; experiences as a teacher of mathematics; advice about teaching mathematics from preservice and inservice education; and advice from curriculum documentation. In this paper, the sources of influence and level of coherence of problem solving beliefs and practices are investigated for one teacher, Rose. Data were collected using a questionnaire, interviews and classroom observations.

Overall Research

The research reported in this study is part of a larger investigation that aims to explore a range of beliefs about problem solving that primary school teachers hold as well as to describe their reported practices. In the first stage of the overall research, data were collected from 174 classroom teachers in New South Wales (NSW) using survey research methods. Teachers were identified as holding a wide variety of beliefs ranging from a more traditional perspective to a more contemporary perspective (Anderson, 1996). Teachers' espoused beliefs were ascertained by their level of agreement with particular statements that were presented to them in the questionnaire. These statements were constructed on the basis of reference to other studies on teacher beliefs by Van Zoest, Jones and Thornton (1994), and Perry, Howard and Conroy (1996). The questionnaire also asked teachers to identify the frequency of use of a variety of teaching strategies and problem types (Anderson, 1997).

The second stage of the overall research involved interviews and classroom observations. Nine teachers were chosen from the questionnaire responses so that a variety of beliefs were represented. These teachers were interviewed to gather more in depth information about espoused beliefs and reported practices. In each interview, teachers were presented with their completed questionnaires and discussion focused on their responses. This process also helped to confirm interpretation of items. From the interviews, two teachers were selected to participate in classroom observations. The combination of information collected from the questionnaires, interviews and observations provides a rich picture of the factors that have influenced beliefs and practices of these two observations provides a rich picture of the factors that have influenced beliefs and practices of these two teachers. The variety of data collection methods also provided an indication of the level of coherence of their beliefs and practices.

Many factors influence teachers' problem solving beliefs and practices. Teachers' espoused beliefs are influenced by their actual beliefs (Thompson, 1992), by their knowledge and interpretation of advice about teaching problem solving (Fennema, Carpenter & Peterson, 1989), by their use and understanding of curriculum documents, and by their own experiences as learners of mathematics as well as by their experiences in classrooms. Reported classroom practices are influenced by espoused beliefs, by actions in classrooms as well as by constraints such as parent' and student' beliefs and school culture. A proposed model of the relationship between these factors is presented in Figure 1. The usefulness of this model to describe the influences on espoused beliefs and reported practices will be examined for one of the teachers in the overall research. Indeed one of the purposes of this investigation is to explore the validity of this model.

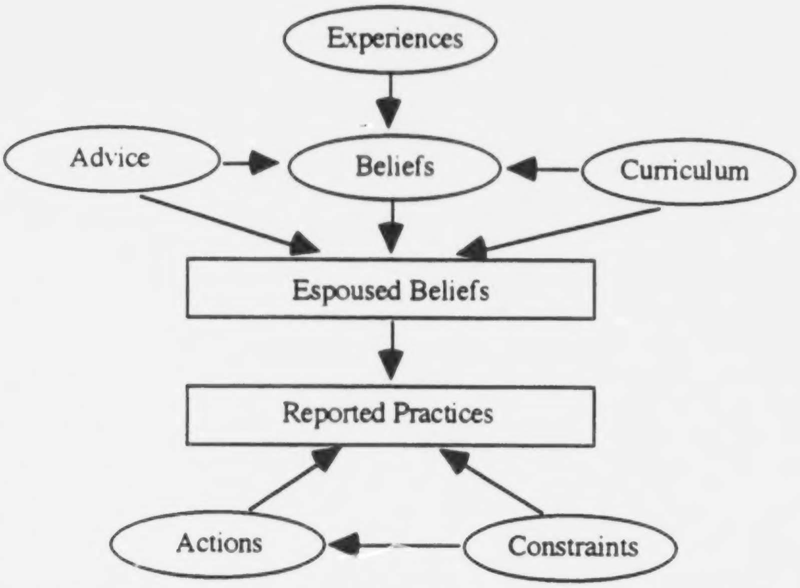

Figure 1. Factors that impact on espoused beliefs and reported practices.

Survey Responses

To facilitate a discussion about problem solving beliefs and practices, an artificial dichotomy about teaching and learning, described here as perspectives about the role of problem solving, is used. One end of this dichotomy is the belief that mathematics is a fixed body of facts to be delivered by teachers and internalised by students. This belief is often associated with classroom practices involving individual student work with rehearsal of routine questions and reliance on textbooks or worksheets. This view may be accompanied by a belief that problem solving is an 'end' (Wright, 1992) and that problems should be presented to students after they have mastered basic facts and skills. This perspective was described by Ernest (1989) and is referred to in this paper as a *traditional* teaching approach.

Another perspective, referred to as a *contemporary* teaching approach, has been described as representing a reformed classroom (Clarke, 1995). Such teachers believe that mathematics is a dynamic subject to be explored and investigated. Classroom practices associated with this perspective usually involve more group work and the use of non-routine questions that promote mathematical thinking and the development of problem solving skills. This view may be accompanied by a belief that problem solving is a 'means' (Wright, 1992) and that problems can be the focus of learning in mathematics lessons. These two perspectives represent end-points of beliefs about mathematical problem solving with many teachers holding beliefs that are situated somewhere between. These perspectives were used as the basis of the construction of the questionnaire.

Rose's questionnaire responses indicated that she supports a contemporary perspective as she agreed with all of the statements relating to this position. She disagreed with all of the traditional statements except for an item on language difficulty that stated "some students find problem solving difficult because of the language involved in the problems". In the interview, Rose was asked to comment on her responses to the traditional statements. The first statement was "students should learn basic number facts before they do *application* and *unfamiliar problems*". Rose read this and said:

> Well I think that's the wrong way around because I think the understanding of basic number facts comes from actually doing things ... That if you do things and if you've actually had a lot of hands on practice then the basic number facts are going to come so that's why I disagree with that.

After reading more of the statements, she commented that the traditional approach did not work when she herself was learning mathematics and so she does not believe that it is the best way to teach.

The teaching strategies Rose reported using "almost always" included group work, whole class discussion at the end of lessons, and concrete materials. The strategies that were "often" used included presenting problems with little indication of how they are to be completed, encouraging individual student recording of methods to problems, modelling problem solving processes, and using problems to show the usefulness of mathematics. Students working alone and full explanations of how problems were to be solved were less frequently used strategies. Rose reported that she only "sometimes" used calculators in her classroom. When asked about this she said that she would "love to have a set in the room but I don't have one". She said she would use them more frequently if they were readily available as calculators are "another learning tool".

Rose indicated on the questionnaire that she uses all four of the question types that were described. This included exercises, application problems, open-ended problems and unfamiliar problems. She believes that the best problems engage students in searching for appropriate strategies and provide hands-on experiences. Rose stated that children should be "exposed to a variety of questions. In this way I hope to appeal to each child's preferred learning method". The only qualifying comment was that when using unfamiliar questions, it is important "to be aware of the kids because you can't have too big a gap between what you're asking of them and what ... they can do".

In response to the request to record a recent problem that she had used in the classroom, Rose wrote:

> I gave each child a number between 5 and 19. They had to write as many questions as they could with that answer.

This example was a favourite open-ended task that supported Rose's belief that these questions were good teaching and learning vehicles. She indicated that open-ended tasks allow the more able students to be challenged while less able students can still attempt the question at their own level.

Interview Analysis

Interviews were conducted so that questionnaire responses could be elaborated, interpretation of questionnaire items could be clarified and additional information could be collected about Rose's beliefs and practices. These interviews were conducted before and after the classroom observations and involved conversations based on the questionnaire responses and classroom observations. Because of limited space, data from the first interview is summarised and will not be fully reported in this paper.

To analyse the transcript of Rose's first interview, points were made under key ideas that arose in the discussion and a diagram was used to represent these ideas (figure 2). Rose's interview indicated that there were several issues that she believes have had a profound influence on her beliefs and practices. These issues can be summarised under three broad categories of Rose as a "learner", a "teacher" and a "change agent". Rose's experiences as a teacher and her own learning of mathematics have influenced her teaching practices and reactions to students. She believes in life-long learning and continues to study. She has strong views about children experiencing success while learning but also that they must be taught how to think. Approaches to teaching mathematics include hands-on, meaningful experiences that can be achieved by using a problem solving focus. She also believes that the children enjoy this approach. As a change agent, Rose believes that she can influence other teachers, parents and the future development of the curriculum.

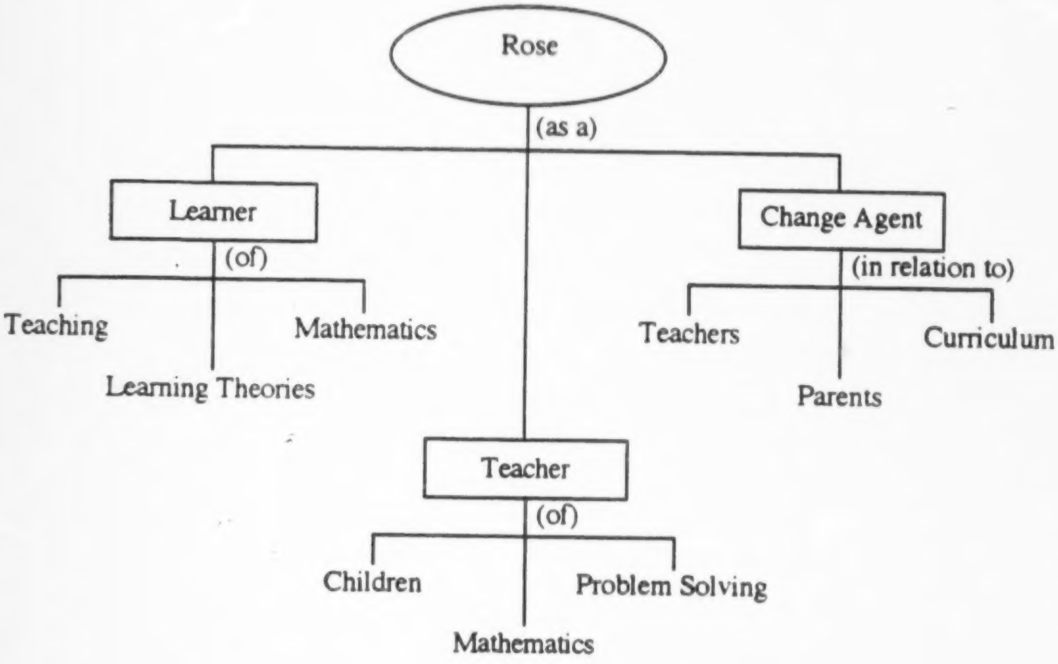

Figure 2. A summary of the key ideas from Rose's interview

Rose as a Learner

Rose's teaching career has spaned thirty years although she has only taught for sixteen of those years. She began teaching in the mid-sixties and taught for seven years before a long break for child rearing. In describing her history as a teacher, she spoke at length about the level of change she experienced when she returned to teaching as well as the amount of change that continues in education. A new *Mathematics Syllabus K-6* (NSW Department of Education, 1989) was about to be introduced when she re-entered the teaching profession. This document included Number from the previous syllabus but introduced Space and Measurement as two additional strands of study; two strands Rose described as "only ever incidental before". She also mentioned change in other curriculum areas including the "process writing approach" to teaching English.

Rose reflected on the level of change she had confronted and described herself as feeling like "a beginning teacher". Not only did she have to cope with new content and new teaching approaches but she also had to cope with teaching in difficult schools. She described this period as "an enormous learning curve" although it was apparent that she is still concerned about her knowledge of teaching. She stated "I always worry about my own knowledge and confidence and understanding".

Since returning to teaching, Rose has completed two degrees at university. Further study, and in particular, her knowledge of learning theories has offered Rose supporting reasons for her classroom efforts. She mentioned Vygotsky's notions of scaffolding and the zone of proximal development. In discussing her belief of challenging the children to think about new ideas she suggested that it was important to extend them but cautioned that the challenge must not be beyond their reach.

Rose's own learning of mathematics has played an important role in her views about teaching mathematics. She described herself as "a person who failed maths" and that because of this she is a different kind of teacher of mathematics than those she had experienced. She stated:

> Maths is a bogey in my life and, but I maintain that actually because of that I'm probably a more sensitive teacher in the area because I remember we had a teacher who couldn't understand why we had problems when I went to school and I hated maths and she used to just dismiss our problems as being stupid therefore you know I had a problem and I do think that maths is taught fairly poorly so, in the school system.

She claimed that she learnt things in mathematics but she did not understand them. She believes that this approach to teaching mathematics is poor as she believes that students up to year ten should have hands-on experiences instead of just sets of rules to learn.

Rose as a Teacher

Rose believes that her own negative experiences as a learner of mathematics has enabled her to develop a more sympathetic approach to teaching mathematics. She is more aware of how the children feel when they cannot do the work. She described her concerns about their individual needs and said "I do try to look at every child as if they are mine and say well now what do you need, what is it that I can give to you to solve this problem for you". The Year Two class that Rose currently teaches has been described as a challenge because of the diverse needs of the pupils. Her class is a middle set in a streamed year two with 20 boys and five girls. Five are quite bright and about five need much extra support and individual help. She spends considerable time with these students who experience difficulties although she expressed frustration because she feels that other children are being ignored while this is occurring.

Rose is concerned about providing children with meaningful yet challenging experiences and believes that problem solving is a good way to do this. She likes to teach the children how to think. Rose described a recent lesson and felt that it represented a good problem-solving lesson because the children were "buzzing" and helping each other. She said "they got really enthusiastic about it and they loved it". Not all children found the number patterns involved in the lesson easy to find. She observed that some of the children found the patterns difficult to work out and she suggested that this would help to inform the focus of future lessons. She would provide a "scaffold of some counters and things" in the next lesson for the students she identified as needing this support.

Rose does not like to "tell" the children how to do problems but describes her approach as "the guide on the side" rather than "the sage on the stage". She described another problem solving lesson she had embarked upon recently. The children were required to consider how they might construct three-dimensional shapes. She gave no instructions but left them to think about the idea. She reported that some children tried to work this out whereas others did not bother. She listened to their suggestions and guided their ideas over the next ten lessons until all of them had constructed reasonable solids. She commented that this was a good lesson because she did not tell them how to do the task. In the survey, Rose indicated that she "hardly ever" explains "in detail what the students have to do to solve problems".

Rose was concerned about the level of understanding of teachers in relation to problem solving. When asked about her overall views on problem solving and the professional development needs of teachers at her school, Rose wrote in the questionnaire "the teachers - me included - need a lot of PD re problem solving. We are all still locked into old style concepts of maths". When this was discussed in the interview, she said "I still think as teachers we're really only at the beginning of this I don't think we have enough, um understanding of it".

Rose as a Change Agent

Rose also saw herself as a change agent and wanted to support other teachers in their efforts to confront change. Her view of herself as still being a learner was not viewed as a bad thing. She suggested that "because I feel I'm still learning ... I can show other teachers that learning continues". She revealed that she has ambitions to be a principal so that she can have a greater influence on teachers and their approach to change. She believes that teachers can be afraid of change and so they often use excuses of being stressed or too busy.

Further evidence of Rose's beliefs about change was identified in the questionnaire. One of the open-ended questions stated:

> The following statement was made recently at a teacher inservice course:
> "People who push problem solving in mathematics obviously don't work in classrooms. It is a waste of time."
> What is your reaction to this statement?

In response to this Rose wrote:

> My reaction is that this teacher is afraid of change, afraid that his/her knowledge of mathematics learning will be exposed - and he/she is expressing and reflecting community views about maths. He/she is probably into control in the classroom.

When questioned about this response, Rose indicated that she believes that many teachers in the local area are older and use community views as an excuse not to change. She acknowledged that other teachers are not confident about teaching mathematics as well.

There were many signs displayed around the school in prominent positions. The one facing the teachers' car park was "change is a conscious choice". These signs were organised by Rose and she explained that teachers, parents and students had voted on the twenty best signs to display. She said:

> And the one right in the back paddock I actually fudged and I had that put up even though it wasn't in the top 20 signs that went up and that is "change is a conscious choice" and I deliberately put it so that every teacher who walks into this school every morning has to see "change is a conscious choice".

She indicated that she can understand why teachers find it difficult to change but she would like "to drag teachers kicking and screaming into this next level of understanding".

Rose was also prepared to take a role in influencing parents' views as well as actively participating in curriculum development. She indicated that she always informs parents about her approach to teaching mathematics and that she has not experienced problems with parents views. She believes that this is partly because of her communication to parents but also because she teaches in the infants department and so hands-on experiences are acceptable. Rose recently reviewed a draft of the new *Outcome Statements* distributed to schools by the Board of Studies in NSW. She was quite concerned about the document because she believes that there is a gap between the syllabus and teachers' knowledge and understanding. She feels quite passionate about it and stated "I found myself really reacting to that".

In summary, from the questionnaire and interviews, detailed information was collected about one teacher's problem solving beliefs and practices. Rose is an experienced, dedicated teacher of a Year 2 class who believes that children need to experience challenge and success in mathematics learning. She believes that the best way to achieve this is to use a problem solving approach in the classroom. This is achieved by using a variety of problem types on a regular basis. Problems provide opportunities for students to think about mathematical ideas, to share their ideas with each other, to explore concepts using concrete materials, and to learn about mathematics in a meaningful way. Rose's espoused beliefs and reported practices were confirmed in classroom observations.

Implications

In addition to collecting information about beliefs and practices, this research explored the sources of influence on those beliefs for one teacher. Rose's beliefs about learning in general and learning mathematics in particular have been influenced by several factors. Her experiences of learning mathematics were fraught with difficulties and poor teaching. Her life-long learning has revealed alternative approaches to teaching and learning and has made her question the way she was taught. Returning to the classroom after a period of extended leave provided her with the experience of feeling like a beginning teacher and further changed her views about teaching and learning as well as the role of professional development. She has responded to advice given in curriculum documents, and in undergraduate and postgraduate courses. Her experiences as a teacher of mathematics have reinforced her belief that this is the best way for children to learn. She employs teaching strategies that support these approaches even though there are constraints operating in her school that support more traditional teaching approaches.

A model was proposed (see Figure 1) that connects espoused beliefs and reported practices with anticipated sources of influence. For the teacher in this study, the main sources of influence on beliefs and practices were her experiences, the advice from postgraduate courses and the impact of a new mathematics curriculum. Classroom observations confirmed Rose's reported practices that were adapted to accommodate constraints operating within the school context. These results clearly support the proposed model for this teacher. Raymond (1997) developed a model of relationships between mathematics beliefs and practices that included the additional factors of students' lives, teacher's life and personality traits. It is possible that extra sources of influence would need to be considered for a larger group of teachers. Even though Raymond included other factors, she reported that the strongest influences were past school experiences, teacher education, and the impact of actions and the immediate classroom situation. Further factors will be explored in the overall research.

Another purpose of this study was to explore the level of coherence of beliefs and practices across a variety of data collection methods. Rose consistently and coherently reported the same beliefs and practices on the questionnaire and during the interviews. Because she has strongly held views about mathematics teaching and learning, she has been able to resist constraints on using this approach. Rose believes that her position as a member of the school executive as well as her increased knowledge through further study have empowered her to resist the constraints that are operating in her school. This suggests that strategies need to be considered in preservice and inservice courses that empower teachers to similarly resist constraints.

Focusing on teachers' beliefs and practices and encouraging them to reflect and describe what they believe may be the first stage in changing teachers' beliefs and practices. Identifying the sources of influence on beliefs provides a starting point for discussions in preservice and inservice education. As many teachers have not responded to advice to incorporate problem solving into their teaching approaches, it is possible that prevailing beliefs held by many of these teachers mitigate against the implementation of contemporary approaches in classrooms. Confronting these beliefs seems to be a necessary step if change is to be embraced.

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The Use of Action Research To Assist the Transition into Teaching

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This paper discusses the learning's from an action research project conducted in collaboration between beginning teachers and a group of university researchers in mathematics and science education. Participants formed action research cells based on their common interests. This paper presents the overall structure of the project and discusses the resulting benefits to the participants and the benefits and limitations of the use of action research to assist transition into teaching.

First year teachers of mathematics enter the teaching profession with varying levels of skills, content and pedagogical knowledge, and confidence. Because of the lack of employment opportunities, many apply for, or are posted to isolated schools or schools with students who are culturally unfamiliar to the teachers. While first year teachers are attempting to overcome difficulties faced in the new school environment, Veenman (1984) suggested that these teachers "need both pedagogical assistance and psychological support." Katz (1972) described four stages of teacher development survival, consolidation, renewal and maturity. It was suggested that the first two stages characterise the first two or three years of teaching. The survival stage is distinguished by self interest and self concern, for instance, getting through the day and planning for a short period of time. In the consolidation stage, concerns move beyond self, and towards children.

Fuller (1969) described three major phases in teacher development pre-teaching, characterised by non concerns; early teaching phase, characterised by concerns for self; and a late teaching phase, characterised by concerns for pupils. Fuller and Bown revised this model (1975) to three stages of concerns of an inservice teacher's development The stages were characterised by concerns for survival, the teaching situation (e.g., content, methods, materials), and pupils (e.g., students' learning and emotional needs). Other models have been reported in the literature, for example, Vonk (1983) and Burden (1980). Common to all of these models is the initial survival stage.

The Board of Teacher Registration (BTR), Queensland (1991) noted that many beginning teachers started "out with idealistic attitudes towards democratic classroom management, but ..(have) to abandon these in favour of the more custodial approach." Sullivan and Leder (1992) also reported beginning teachers tending towards authoritarian classroom control, and over directing classroom activities, including drill and practice activities. Further, there were many factors influencing beginning teachers' beliefs and behaviour. These included their own schooling and family background, preservice education, the transition process from preservice to inservice, the school and its induction programs, and the classroom itself. In their study of the influences affecting novice teachers' instruction, Sullivan and Leder found that a very significant factor was the students, themselves. Among recommendations for change in preservice courses to address this issue, one proposal for future research, suggested by Sullivan and Leder, was to look at the ability of teachers to reflect on their practice, with the view that reflection may influence the direction of their teaching.

Induction and support programs

Common induction programs for beginning teachers reported by Veenman (1984), include provision of printed materials about employment conditions and school regulations; orientation visits to schools before taking up duty; release time; group meetings among beginning teachers for emotional support; consultations with experienced teachers; and team teaching. However, in Australia in the 1970s, and even the 1980s (Davis, 1988) few beginning teachers (fewer than half) had participated in induction programs and many of the recommended forms of assistance were not being offered (Board of Teacher Education, 1981).

The James Report in England (1972) recommended an induction program of *teacher-tutors* where both beginning teacher and the more experienced teacher-tutor would be given reduced teaching loads. The teacher-tutor would arrange inservice and provide professional help for the beginning teachers. Costs for such a program were found to be exorbitant. Also on the negative side, beginning teachers felt strongly about their perceived reduced professional status under such arrangements.

The teacher-tutor model is a similar program that has been initiated in many countries, for instance, *Indiana Mentor Internship Program*, *California Mentor Teacher Program*, *Mentor Teacher Internship Program*, *Kansas Internship Program* (BTR, 1991, pp. 21-22). In the mentor model, beginning teachers are assigned to "expert" teachers who provide support and advice (professionally or emotionally, or both), and organise professional development

A related mentor model, a loosely structured "Buddy System" was trialled in Idaho (Klug, 1988), where mentors provided assistance on request. This model usually did not include set times for meetings, no set arrangements for observations and did not have formal guidelines. Participants found that the lack of structure to be confusing, and argued that the roles and goals should have been better defined. This was compared with another model, "Induction Team", where a team, comprising an administrator, a staff member from a higher education institution, and a mentor worked with a beginning teacher. Beginning teachers under this model were allocated some release time and underwent regular observations and attended set meetings. It was found that the participants preferred the more structured approach although they did find the observation requirements to be rather demanding.

Some induction programs have included university input, where university staff provided expertise, support and advice, and ran inservice courses (e.g., Cheney, Krajewski, & Combs, 1992; Dianda & Quartz, 1995; Reiman, McNair, McGee, & Hines, 1988). Some of these partnership programs associated with universities have incorporated reflective practice or action research at both preservice (e.g., McLaughlin & Hanifin, 1994) and inservice levels. Bartell (1990) asserted that "beginning teachers need to develop not only the capacity for seeking out ideas and resources but a framework for making decisions about what is or what is not useful or effective in their own practice." The advantage of such practice is that the participants seek knowledge and make decisions for their own settings, thus empowering them

The EMSTAR project

The *Enhancing the immersion of beginning women teachers into Mathematics and Science Teaching through participatory Action Research networks* (EMSTAR) was a collaborative participatory action research project among nine first-year women teachers and university researchers. For the participating university staff, one of the main aims of the project was to investigate the support needed to enhance the transition of teachers from their university course into the profession, and the use of action research for facilitating transition. For the participating teachers, the project allowed them to collaborate with each others and the university staff to deal with specific aspects of their teaching of mathematics and science in their schools. The analysis in this paper deals with the use of action research in the facilitation of the transition of teachers into the teaching profession.

Action research in Education

The methodology adopted in this project was participatory action research (PAR). Kemmis and Wilkinson (1998) discussed the following characteristics of action research. First it is a *social* activity in that "it deliberately explores the relationship between the realms of the individual and the social." It recognises that "no individuation is possible without socialization, and no socialization is possible without individuation" (Habermas, 1992, p. 26). PAR is also *participatory* in that "it engages people in examining their knowledge (understandings, skills and values) and interpretive categories (the ways they interpret themselves and their action in the social and material world)." It is also participatory in the sense that people can only do action research "on" themselves - individually or collectively. It is *not* research done "on" others. PAR is also *collaborative* in that "[a]ction researchers aim to work together in reconstructing their social interactions by reconstructing the acts that constitute them. It is a research done "with" others. PAR is *emancipatory* in that "it aims to help people recover, and unshackle themselves from the constraints of irrational, unproductive, unjust, and unsatisfying social *structures* which limit their self-development and self-determination." PAR is also *critical* in that "[i]t is a process in which people deliberately set out to contest and to reconstitute irrational, unproductive (or inefficient), unjust, and/or unsatisfying (alienating) ways of interpreting and describing their world (language/discourses), ways of working (work), and ways of relating to others (power)." Finally PAR is *recursive (reflexive, dialectical)* in that "it aims to help people to investigate reality in order to change it (Fals Borda, 1979), and to change reality in order to investigate it ... It is a process of learning by doing - and learning with others by changing the ways they interact in a shared social world."

Participants

The participants in the study were eight beginning women primary teachers and one participant still in her final semester at university. All teachers came from a four year BEd course at the Queensland University of Technology. During their final year in their course, these teachers had participated in the Women Trainee Teachers in Mathematics study (Atweh, Kyle, & Burnett, 1996; Atweh & Burnett, 1997) or worked as mentors in the Peer Assisted Study Sessions (PASS) program for students enrolled in a core unit called Science Foundations (Watters & Ginns, 1997). The participants represented a range of abilities in mathematics and science. They joined the project in the belief that it might benefit their teaching and that the findings would also help with the planning and implementation of the preservice primary teacher education program. The participants were invited to commit themselves to collaborative work with each other and with staff from the university in action research projects within their schools. Five of the beginning teachers were located in Queensland schools and three worked interstate - two in the Northern Territory and one in New South Wales. The teachers were joined by three university lectures, two in science education and one from mathematics education, and one research assistant. From time to time the expertise of more experienced people was called upon to assist in the discussions of the project.

Procedures

The fundamental aspect of this study was the formation of a *network* among the beginning teachers, staff from University and some more experienced teachers. Various *action research cells* emerged from the network. The university staff acted as facilitators for connecting teachers with similar needs and interests. During the third network meeting three groups of common interest were established. Three teachers decided that they were interested in working on catering for the needs of the gifted and talented; another three teachers were interested in aspects of assessment; the last three teachers shared an interest in making mathematics more relevant and inclusive to a wide cross section of students. This paper deals with the overall learning from the project. Other papers consider the findings from the action research cells (Atweh, Harris, Garrett, Pitman, & Sitton 1997; Suhrbier, Moman, Fitzgerald, & Ginns, 1997; Watters, Andrews, Henderson, & Everett, 1997).

Being an action research study, the exact procedures used in the study emerged from the various discussions at network and action research cell meetings, and classroom and school issues that teachers faced. The first network meeting allowed participants to identify their personal aims and negotiate the general operation of the study, and acquire some experience in using email. Principles of action research were elaborated upon at the first network and subsequent action research cell meetings. The second network meeting provided teachers with an opportunity to discuss and share their early experiences in their new schools as well as consider and reflect on their initial plans for action research in their own classrooms. Areas of common interest continued to emerge. The principles of action research were reiterated at this meeting.

Findings

Benefits to participating teachers

The other publications from this project present the voices of the women teachers enabling them to elaborate on the benefits to themselves. However, from our observations of the meetings we can identify some aspects of benefit to the teachers themselves.

There were times when the project did not seem to us to be going as we envisaged. Teachers were not sending information or fulfilling the agreed action from the action research cell meetings. We often wondered if they saw this project as a not-so-useful activity that they were engaged in for our purposes only. However, all nine teachers who remained in the project after the end of the first term of school, remained in the project for the rest of the year. They all had the opportunity of leaving the project as some of their colleagues had done so early on. They had not done so. Obviously they were receiving some benefit from the project.

From our observations, the discussions at the network as well as action research cell meetings provided a feeling of mutual care and support. At the early stages of the project the participants were asking each other about their placements and plans. Later, some of them volunteered suggestions and ideas for each others' considerations. The project developed a sense of community among the teachers. Two of the teachers were placed in isolated communities in the Northern Territory and this sense of community was particularly important for them. We believe that teachers had a chance to reflect on the major problems that they encountered in their teaching, and many found this aspect useful to increase their self awareness about their practice. Similarly the sharing of their stories with each other assisted them to compare their practice with others. More importantly, they were confident that they could share their main concern in a supportive and non-judgmental atmosphere. In one of the action research cells the participants had a chance to share with one another a situational analysis of their school and in a special meeting they had an opportunity to comment and make suggestions on issues arising from the situational analyses. Many of these comments were accepted gracefully by the participants.

Naturally the question whether the practice of those teachers and the understanding of their practice has actually changed as a result of their involvement in this project, we can not answer in this context. The papers by the teachers themselves address some of these points.

Benefits to the university researchers

As university researchers we had to negotiate our roles with the participants. We were careful not to let our agenda and interests dictate the action research conducted by the teachers. Among us we had a wide interest in topics and areas that cover mathematics, science education, and gifted and talented children. As participants in the first two meetings of the network, we declared our interests just like any other member of the group. We also indicated several areas in which each one of us had some interest. Each participating teacher did the same as well. The three action research cells that were formed using this process were based on the patterns of interest demonstrated by each participant. The degree that our interests have determined the outcome of the groups is not clear. However, we are confident that each member of the action research cells was pleased with her selection. What was pleasing to us was that the action research cells have been selected in areas that we were very interested in as well.

As university researchers we have a range of interests, commitment to, and understanding of action research. In our planning meetings we debated our practices and plans for our groups. Ultimately we worked independently within our action research cells. Our actions within these cells were determined as much by our values and beliefs as by the needs of the participating teachers. How the particular groups functioned is once again illustrated in the in other publications from the project. One thing we are confident of, is that we are more committed to the process of action research as a means of professional development of teachers at the end of this process than we were at the beginning. None of us has been involved in an action research project where the participants were at a distance from us, and where we were unable to meet them face to face on a regular basis. This was a learning experience for all of us.

This project increased our understanding of the isolation and the problems that first year teachers face. This knowledge, also documented in the individual papers mentioned above, was more real to us than a widely circulated survey or review of the literature because it arose from talking face to face with teachers as they experienced these problems.

Difficulties Encountered

By the nature and scope of the project some learnings evolved out of dilemmas and difficulties we encountered which resulted in several compromises. The first type of difficulty encountered arose due to the geographical distance separating the participants. In one action research cell all the members of the group were interstate. Hence, the only contact possible was electronically. Other groups also had difficulty arranging face to face meetings. Undoubtedly this created serious difficulties for the usual meetings of action research cells. Although it did not take participants very long to get used to teleconferences, the nature of such meetings prohibited important aspects of communications. It had a tendency of making the meetings more structured and formal. This may increase the efficiency of meetings, but also it places artificial constraints on the interactions among participants. There are protocols of politeness in talking on the phone and these may prevent some participants from debating issues. Further, teleconferencing is an expensive way to meet. Even though the project received a grant for that aspect, we were very careful not to exceed the time limits in our meetings. Communication problems with some teachers were not restricted to telephone. One teacher in an isolated area received mail only once a week. Another teacher did not like receiving faxes from the other participants because of lack of privacy at the school. At the planning stages of the project we expected that every participant would be connected through email. In spite of accounting for that service in our budget, this aspect of the project did not work at all. Those who had email facilities often had to share them with many other teachers in the schools. Others did not have the software or hardware to connect to the internet. Access to email is only part of the problem. An email culture needs to develop before people use email confidently for regular communication and sharing of learnings and problem solving. The culture of the school did not incorporate email communication as a normal means of communication. Lastly, being at a distance created limitations on how much we as facilitators can really understand the context of the teachers without having visited it. This difficulty was shared by teachers from the various schools as well. This is an inherent limitation of action research groups such as these.

The second main difficulty we faced related to the competing demands on teachers' time. First year teachers are always under pressure to meet commitments and satisfy the demands of their classes and schools. In certain ways, a decision to be part of an action research project is a commitment to carry the burden of additional responsibilities and activities. These additional responsibilities may take teachers attention away from more immediate and urgent tasks. The literature on beginning teachers indicate that the first year is a survival year. Are we being unfair to add to those responsibilities concerns about the gifted and talented, varied assessment and inclusive curriculum? Is the involvement in this project justified? Is it good value for money? Once again, the persistence of the teachers in the project is a partial answer to these questions..

The last type of dilemma we faced was related to the different understandings of the nature of the project by the various participants. On several occasions the participants would ask the university researchers about what was the next stage in the process. Even though it appears some of the teachers took responsibility for reflecting on their own practice, none of them took charge of the process. Guidance was left to the university staff. In one sense the teachers did not own completely the process of collaboration as a means of improving practice. The project, even though it might have been seen as useful and enjoyable, was also peripheral to the main concerns of those teachers. Similarly, where our concerns may have been on emancipatory aspects of action research and a critical understanding of practice, it seemed to us at times that the needs of the teachers were more technical and practical.

Perhaps in our naivety we underestimated the pressure for survival that these teachers are under. Perhaps that is all they need as new teachers.

Discussion and Conclusions

Kemmis argues that PAR is participatory in that the people affected by the research findings should participate in its design, conduct and analysis. This project was planned to allow the teachers the greatest input in determining the issues pursued and the methods adopted. However, the participation of the different players was not necessary equal. The university researchers, as holders of the grant to conduct the research and as more experienced action researchers, outlined the general structure of the project at the early stages of its implementation. Although the plans were not adhered to rigorously, they determined to a large extent the structure of the project. At the early stages of the project the teachers felt some unease because they did not know clearly what the project was supposed to investigate and what was the expectation of them. However, by the time the action research cells were formed, the teachers did take a little more responsibility for the planning of the project. In discussing the problems of participation in research with the profession, Grundy (1998) discusses the issue of "parity of esteem" where the different expertise of the different participants is brought to bear in the design and conduct of the project. All participants should be aware of these limitations to equal participation and should negotiate the roles and expectations early in the project. Distinction should be made as to the variants and constraints of the project and to the roles of the different partners and their expectations of each other.

Kemmis also agues that PAR is collaborative in that different players are involved in the practice, and work together to develop individual and collective understanding and improvement of the practice. Naturally, the problems associated with the transition and their solutions are dependent on other people in addition to the beginning teachers themselves. For example, the crucial roles of the school administration, other staff and other members of the school community can not be overemphasised. Should they be participants in the project as well? It is conceivable that the action research cells could have been formed around the individual schools with the participation of the school administrators and other more experienced teachers. We are confident that such an organisation would have been useful as well. However, in this project, we decided to work with the teachers from the same cohort and leave the nurturing of the contact with other players to the teachers themselves. In the organisation adopted here, the beginning teachers have been able to develop a sense of rapport with each other being from the same cohort of a teacher education course. Further, the formation of support groups from outside the individual schools implied the teacher could be open about the problems experienced without fear of reprisal. Lastly, talking to teachers from other schools and knowing what is available or possible in other contexts implied that the teachers could be more critical about what was happening at their own schools. Working with teachers from other contexts gave them the opportunity to explain their own context to others.

The further characteristic of action research as identified by Kemmis is that it is a social activity. This project allowed the teachers a deeper awareness of the social context of their teaching. At least in one action research cells, the participating teachers decided to write a situational analysis of their respective context to share with each other. Their writings showed a deep insight and knowledge of the social background of their students and the ethos of the school, and other limitations to their practice. Further, through the discussion with each other, they learnt how to identify the practical difficulties that could more easily be changed.

The fourth and fifth characteristics of PAR as identified by Kemmis are critical and emancipatory, in that the practice in which the participants are involved is seen as a part of a system that, at times, acts and is structured contrary to the interests of the beginning teachers, and it attempts to empower teachers to improve their own practice. We felt that, in general, these aspects of action research were not highly successful in the three action research cells. The teachers tended to be more concerned with day to day problems of how to conduct and manage classes, how to conduct appropriate and manageable assessment, and so on. Although not unexpected or unreasonable at this stage of teaching, technical needs have a priority over practical and emancipatory needs.

Finally, Kemmis argues that PAR is reflexive and recursive. As argued above, the project was designed to maximise the input of the teachers themselves in understanding and improving their practice. The teachers adapted to this change in philosophy from their teacher training days, where the lecturers were often considered as authority, to the more participatory action research project. This project was very much researching *with* the teachers rather than *on* the teachers. Whether the practices of action and reflection would be entrenched in the professional life of the teachers in future years remains to me seen. Yet we believe that the teachers have taken a step in that direction.

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Towards the Study of the Processes and Effects of Internationalisation in Mathematics Education

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This proposed study is designed to investigate a rapidly expanding area of activity in mathematics education that has not been researched and theorised. Arguably, mathematics education is one of the most internationalised areas of the curriculum. This paper outlines a longitudinal study to investigate issues in the internationalisation and globalisation of mathematics education in 3 regions with ties to Australia. It is a part of a workshop that aims to obtain feedback from peers and trial the instruments to be used.

Robitaille and Travers (1992) claim that mathematics education is one of the most internationalised areas of higher education. This is evidenced in similarity of the curriculum reforms happening around the world and the number of international conferences and research activities that have propagated during the past few decades. Yet, very few research studies have problematised the processes and outcomes of such internationalisation. This proposed study investigates issues related to internationalisation and globalisation of mathematics education with special emphasis on the professional development of educators.

Although the terms internationalisation and globalisation are relatively recent in academic discourse (Waters, 1995), they are playing an increasingly significant role in higher education policy and practice. Waters claims that while postmodernism was *the* concept for the eighties, globalisation may well be *the* concept for the nineties. Several national (Internationalisation of Higher Education in the Asia-Pacific Region, 1996) and international (de Wit, 1995) conferences have been held on issues of internationalisation of education. Back, Davis and Olson (1996) have shown that almost every Australian university has developed policies and strategies to deal with this trend. Research is essential for further developing Australia's interests in this competitive market, as well as enabling it to provide programs that meet the needs of its international clients.

Conceptualisation of internationalisation and globalisation

Since the concepts of *internationalisation* and *globalisation* sometimes are given different meanings by different authors and other times are used interchangeably, we need to conceptualise their usage in this study. A widely used definition of internationalisation is that of Knight in a report commissioned by DEETYA (Back, Davis, & Olsen, 1996, p. 1): "Internationalisation of higher education is the process of integrating an international/inter-cultural dimension into the teaching, research and service of the institution". Waters (1995) defines globalisation as "a social process in which the constraints of geography on social and cultural arrangements recede and in which people become increasingly aware that they are receding" (p. 3). Globalisation is often associated with "forces [that] are impersonal and beyond the control and intentions of any individual or groups of individuals" (p. 2). McGinn (1995) presents a similar view by asserting that "efforts at internationalisation ... [are] seen by ... authors as activities that improve higher education institutions without diminishing their autonomy. ... There is some evidence, however, that the processes of globalisation are compelling rather than invitational, and therefore require careful scrutiny" (p. 78).

In accordance with the definitions presented above, *internationalisation of mathematics education* is taken here to mean the integration of an international dimension in the curriculum, research and professional development including activities that promote inter-country collaborations. The term ***globalisation of mathematics education*** is taken here to refer to the phenomenon of knowledge, values, principles and curricula developed in some context gaining a global adherence.

Internationalisation and globalisation issues in mathematics education

Robitaille and Travers (1992) discussed the relative similarity of content and importance of mathematics curricula around the world and concluded that the "language of mathematics is, in many ways, truly international" (p. 687). Silver and Kilpatrick (1994) and Kilpatrick (1992) discussed the increasingly crucial role in the development of mathematics education in different countries of international conferences (e.g., ICME and PME), publications (e.g., JRME, FLM, ESM and MERJ), international studies of achievement in mathematics (e.g., IAEEA and the more recent TIMSS study), and international collaborative groups (e.g., the BACOMENT project). Although these forums allow researchers from around the world to share results of their studies, they have often been criticised in that commonly they have greater representation of American and European researchers and views (Bishop, 1992). Further, they often consist of brief reports of research projects that do not allow the discussion of the complexity of issues affecting mathematics education (Silver & Kilpatrick, 1994). Similarly, international studies on achievement in mathematics have come under scrutiny on their failure to consider the interaction of the different countries' contexts with achievement levels of students (Robitaille & Travers, 1992). The scant literature that considers internationalisation of mathematics education seems to concentrate on research activities. Bishop (1992) has argued how the "growing mutual international influences of ideas, methods, practices, and expectations" (p. 710) has lead to the difficulties in identifying national perspectives of mathematics education in the different countries. Watson and Atweh (1992) have discussed the role of Australian research in the international community and called for an increase in the Asian co-operation on research activities.

In a recent chapter, Jacobsen (1996) outlined the effect of international organisations such as UNESCO, the World Bank and ICMI in changing mathematics education world-wide. He argued that the increasing gap between the rich and poor countries and the curtailing of funds from these international agencies make it "more difficult to look for governments for improved international co-operation in mathematics education" (p. 1253). He joined Miguel de Guzman, the past President of ICMI, to call for an increasing role of co-operation between professional mathematics educators and their associations to work to improve mathematics education world-wide. This proposed study in an investigation of some conditions under which such collaboration is effective.

Within the context of internationalisation, questions arise as to whether mathematics education, or parts of it, are becoming globalised. Commenting on the 7th ICME conference in Canada, Usiskin (1992) summarised the feeling of many participants by expressing his impression about "the extent to which countries have become close in how they think about their problems and, as a consequence, what they are doing in mathematics education" (p. 19). Yet, he goes on to express his hope "that the new world order does not result in a common world-wide curriculum; our differences provide the best situation for curriculum development and implementation" (p. 20). This concern about uncritical globalisation of issues is shared by Rogers (1992) stating that "the assumptions that mathematics is a universal language, and is therefore universally the same in all cultures cannot be justified. Likewise, the assumptions that our solutions to local problems in pupil's learning and our own teaching of school mathematics will have universal applications is even further from the truth" (p. 23).

One area that has received considerable debate in the mathematics education literature is the relationship of mathematics learning to its cultural, language and social context (Bishop, 1988; Ellerton & Clarkson, 1996; D'Ambrosio, 1985; Lave, 1988). In reviewing the literature on ethnomathematics, Nunes (1992) refers to the debate in the literature between those who assert that different contexts give rise to different types of mathematics and those who argue for some invariant features underneath surface differences. Clarkson (1992) has noted how curriculum decisions can change depending on which cultural group is dominant within a country, and the political decisions of the ruling group. Nebres (1995) argued that early international collaborations consisted of developing countries modelling the curricula of the developed countries that colonised them while current collaborations are more based on the formations of common interest networks across national boundaries. He argued for what he calls an axiom of collaboration: "The more global and multicultural we seek to become, the deeper must be our local cultural roots" (p. 39). This proposed study is an investigation of how local context can be accounted for within international collaborative activities.

The area of particular interest to this proposal is the professional development both of school teachers and university educators in mathematics education. Countries in the West, in particular the United States, United Kingdom and Australia, are attracting an increasing number of international students and consultancies in Asian and other countries and literature on issues of internationalisation of professional development is almost non existent.

This proposed longitudinal study aims to

1. investigate theoretical issues in globalisation and internationalisation of mathematics education with particular emphasis on cross-country collaboration in professional development;
2. investigate the experiences, perceptions and beliefs of mathematics educators from four regions about internationalisation and globalisation of mathematics education;
3. develop case studies of programs and activities that can be regarded as best practice in cross-country collaboration in professional development in mathematics education; and
4. develop model(s) for inter-country collaboration in professional development; and
5. design, implement and evaluate a collaborative program based on the model(s) developed in three postgraduate courses in Australian universities and in selected overseas universities.

Research Design

Project Reference Group

In this study it is essential that the design of the project reflects its theoretical underpinnings, in particular, that the project be conducted in an international cooperative way. Hence, the international Project Reference Group (PRG) is a crucial component of the design. This reference group will consist of leading representative educators each of the three international regions and will be consulted at each stage in the design and implementation of this study. These people represent leading educators for the region and would provide contact with mathematics educators from the respective regions not only from their specific countries.

Study Overview

This study consists of three main phases. This section presents an overview of the phases which are detailed in the following sections. The study phases follow a contingent design where the results obtained from one phase effect the final design of the following phase. The project commences with a ...

1. *theoretical study and a survey study*. This phase, continues with the development of a theoretical model for discussing internationalisation and globalisation of mathematics education. It uses focus groups that aim to identify issues, benefits and concerns by leading mathematics educators about internationalisation and globalisation of mathematics education (Aim 1). The major issues identified will inform the *survey study* to a large group of mathematics educators in the three international regions and in Australia to identify the extent of international contact by lectures in these regions and their perceptions of acceptable practice in international collaborations (Aim 2). Both the focus groups and the survey will be used to identify international programs/activities in existence in the identified regions, especially those that are

widely regarded as cases of best practice. A diverse selection of activities/ program will be identified for further investigation as ...

2. *case studies.* At least 3 international projects/activities will be identified within each region (total 16) and investigated in depth. This phase of the study aims at identifying features of these programs/activities that make them successful in meeting the needs of the various collaborators (Aim 3). Larger activities would be given more emphasis. The successful design and implementation features of these programs will assist in developing model(s) (Aim 4) for international collaboration and will be used to conduct an ...
3. *action research study.* This will involve the design, implementation and evaluation of courses/programs at the three participating Australian universities and at least one co-operating university from each of the three international regions. These will form the basis for the action research component of the study (Aim 5). Activities conducted in the first four phases of the project will provide knowledge that would lead into ...

Phase 1 (theoretical study and survey study)

Sample: Two samples are needed in this year. For the focus groups we will be inviting at least 6 leading mathematics educators from each of the participating regions who are recognised as experts in mathematics education by their leading role in their countries and by their contribution to international conferences selected in conjunction with the PRG. We will be ensuring a broad representation of views on and experiences in international collaboration. The second sample will be selected to undertake the survey. Since the population is not very large, all full time university mathematics education lecturers in the participating countries, identified with assistance from the PRG, will be surveyed.

Instruments: Focus groups: Focus group are efficient and powerful means for raising issues that the participants see as important (Stewart & Shamdasani, 1990). The model used here will proceed as follows: A synopsis paper, developed by the researchers, highlighting the main issues for discussion (e.g., the scope and types of international activities in their countries, the different models used in international co-operation, available funds and sources, how they meet the needs of their country and major hindrances) would be distributed to participants before the commencement of interview;. The group facilitator (one of the researchers) would introduce the general topic for discussion and requesting comments on theoretical issues as well as specific experiences that the participants may have had. Focus groups tend to be self directing, hence there will be no further interaction from the facilitator.

Survey: The survey will seek information from the participants about the extent of the influence of international activities in the development of their knowledge in mathematics education. The survey will seek their attitudes about the relevance of the various activities (e.g. conference attendance, overseas training, reading journals, joint research) to their local context and personal needs. Similarly, the survey will investigate the participants' attitude and beliefs on the theoretical matters stemming from the focus group. Participants will have a chance to nominate best practice activities that they are familiar with and are requested to provide documentary samples of such.

Procedures: The focus groups will be conducted during the second half of the year. Visits to the three international regions for the conduct of the focus groups will be arranged where possible to coincide with an international conference. Focus groups will be audio or video taped. Some of these focus groups will be conducted in languages other than English. The transcripts will be translated into English. The development of the survey will commence in August. The instrument will be developed in consultation with the reference group. The original form will be developed in English. Where needed, it will be translated into other languages using the back translation method (Brislin, 1986). The survey will be administered in early October by electronic mail if available or by post otherwise.

Data Analysis: Both qualitative and quantitative data would be generated from this stage of the research. Closed ended survey items would be analysed as frequencies and percentages. Responses will be analysed as function of country of origin and level of exposure to international activities of the respondents. Open ended questions would be coded into meaningful codes based on grounded research techniques (Strauss & Corbin, 1990). The codes are simple descriptions attached to discrete occurrences of events or phenomena. The codes will then be categorised to provide more abstract concepts called categories. Some of these categories would arise from the data while others may arise from the literature reviewed. Since it is expected that a large amount of data would be generated from this phase, the computer program NUD*IST would be utilised to generate and manipulate codes.

Phase 2 (case study)

Sample: The case studies will represent a purposeful sample of a wide range of activities developed for cross-country professional development in the overseas countries or in Australia. These programs will be identified in conjunction with the PRG and based on the focus group and the survey results. This method of selection will assure that programs undergoing in-depth analysis are regarded by the community as examples of best practice and cover a range of types of activities (e.g. formal degree courses with large number of international students, formal courses designed specifically for international students, international teacher and educators development projects, activities that depend on the use of telecommunication technology, and internationally funded consultancies).

Instruments: Case study: A variety of instruments will be used to develop the case studies in this investigation. The model used in this study is that presented by Stake (1995). Documents outlining the principles behind the design of the program/activity will be examined. Interviews with the key players (e.g. designers, delivers and selected participants) would be sought. Non participant observation of the program activities will be conducted whenever possible. Where appropriate, questionnaires would be sent to participants in these program to seek their attitudes and experiences in the program. Aspects of the activity that deal with internationalisation and globalisation issues and their adaptation to local context will be of special interest in these case studies.

Procedures: The data from the focus groups and survey will be analysed. The results will be used to generate criteria for the selection and conduct of the case studies. Contact would be made with various activity leaders for possible in-depth study of their activity as a case study. The case studies will be conducted in the second half of the year. Visits to the three regions will be conducted. Each case study would involve at lease one week visit by one of the researchers. Work would commence in August on summary of data from the case study leading to development of models of best practice. Plans and preparations will commence in September with participating instututions for collaborative action research in the following year.

Data Analysis: Two types of analysis would he conducted on data generated from this phase of the study. Case study descriptions based on Stake would be developed for each of the cases studied. Member check with each case would be conducted to assure authenticity of the data. Further, grounded research techniques discussed above will be used to identify the features of the activities that make them successful/unsuccessful. These features would be used to generate the model(s) for best practice. Continuous hermeneutic cycles would be followed to assure that the models obtained would identify the factors that are effective toward the success of these programs. Special emphasis will be made on issues arising from the focus groups and the literature review. It is expected that the model will include features in the activity's design, implementation and evaluation outlining the role of the different participants and features that make them appropriate to the local context of the participants.

Phase 3 (action research study)

Sample: The subjects selected for this part of the project depend on the specific projects that emerge from the previous year's negotiation with the collaborating countries. Students enrolled at three M Ed subjects at the three participating Australian universities, many of whom are international students, and staff teaching these subjects would form the nucleus of the sample used for the action research. Similar courses will be selected from some of the participating countries that are willing to participate and that can guarantee the resources for such participation. Naturally, participants who refuse to participate in this study may still participate in the activity without providing data to this study.

Instruments: A variety of instruments will be used for action research component of this project: Email correspondences, reflective diaries, regular feedback from subjects, final assessment items in the subjects and questionnaires as formal evaluation by the participants.

Procedures: As is the case with action research designs, the form of these activities is contingent on results from the previous phases of the study. The model of Action Research adopted in this study is that of Kemmis and McTaggart (1988). It includes cycles of planning, action and reflection. The design of the subjects would include participants from different countries working together on assessment tasks using electronic telecommunication. Further, all participants would be involved in some common readings and lectures about mathematics education in the different countries. Other features of the subjects would be developed based on the results case studies developed in phase 3 of the project. Regular electronic contact will be maintained between the designers of the activity toward its conduct and evaluation.

Data Analysis: Similar techniques for analysis of data used in the case studies will be employed in this part of the research. In particular, attention will be given to categories related to participants' experiences, language of discourse, and relationships that develop between the participants (Kemmis & McTaggart).

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THE EFFECTIVENESS OF A NO-GOAL APPROACH IN SOLVING PYTHAGOREAN 2-STAGE PROBLEMS

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Four groups of Year-8 students completed a set of 2-stage Pythagorean problems under differing conditions. One group completed the set with specific goals (find x) without instruction; the second and third groups were encouraged to either work forwards or backwards from the goal (x); whereas, the fourth group completed the set with unspecified goals (no x's). On a subsequent set of test problems, the no-goal group demonstrated a superior performance than the other groups in applying Pythagoras' Theorem.

Introduction

In the early stages of learning about the theorems and rules of geometry and trigonometry, many students follow a traditional sequence of instruction. In the introduction of formal geometry for example, students often begin their study by learning about the parallel line theorems (alternate, corresponding and co-interior). At first, the learners will apply these theorems to a number of elementary problems which only require the application of one theorem. Following this phase, students will be required to solve problems which need several applications of the theorems. Many of these types of problems can be classified as transformation problems (see Greeno, 1978), where an initial state is transformed to a specific goal state. In order to solve a transformation problem, a subgoal or a number of subgoals need to be calculated before the goal state can be found. For many learners, these elementary geometry problems may be their first experience of mathematical transformation problems.

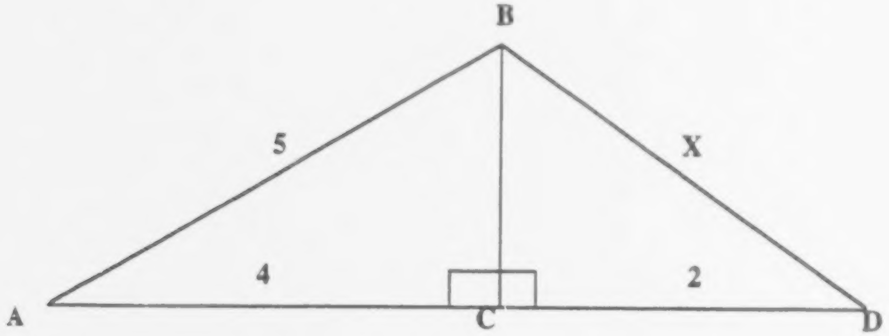
To solve transformation problems, novice problem solvers rely heavily on backward-working strategies such as means-ends analysis (see Mayer, 1983). Although means-ends analysis can be an effective problem solving strategy, its use creates a high cognitive load, as problem solvers constantly need to refer back to the goal or subgoals regardless of what position they occupy in the problem. Research conducted by Sweller and his collaborators (see Sweller, 1988, 1994; Ayres & Sweller, 1990) has shown that instructional phases which require problem solvers to use means-ends analysis are not particularly effective. Although problems may get solved, the strategy imposes a heavy cognitive load which may interfere with schema acquisition.

Much of the research conducted into the effects of using means-ends analysis and its relationship to learning was conducted in the 1980's, and the results of these studies were influential in the development of cognitive load theory (see Sweller, 1994). A main goal of cognitive load theory is to design instructional techniques which facilitate schema acquisition by creating learning environments which reduce the demands made on working memory. Use of no-goal or goal-free problems is one instructional design which has been investigated by researchers in a number of different domains such as dynamics (Sweller, Mawer & Ward, 1983) trigonometry (Owen & Sweller, 1995), geometry (Ayres, 1993) and biology (Vollmeyer, Burns & Holyoak, 1996). A goal-free environment is created by removing specific goals in a problem set and asking problem solvers to find all unknowns rather than specific goals. By removing specific goals, the use of means-ends analysis is minimised (if not eliminated), which in turn reduces cognitive load.

Typically in goal-free studies, groups of subjects who completed no-goal problems were found to perform significantly better than groups who completed traditional goal-specific problem. In a study by Ayres (1993) in the domain of geometry, the no-goal group made fewer errors than the specific-goal group. Furthermore, no-goal subjects were more able to navigate a path through the problem space. On these two-stage transformation problems, requiring the calculation of a subgoal and a goal, many subjects in the specific-goal group could not progress as far as finding the subgoal. Successful navigation through the problem space is a fundamental requirement for the development of problem solving skills in geometry and trigonometry. The Ayres (1993) study found that no-goal problems aided this development, whereas undirected problem solving strategies, employing heuristics such as means-ends analysis, did not. Earlier research into expert-novice differences found that experts in a particular field tend to work forward from the givens, whereas novices work backwards (see Larkin, McDermott, Simon & Simon, 1980) from the goal, using strategies such as means-ends analysis. Instructional phases which promote the use of backwards-working strategies appear to retard the development of expertise. However, it is worth considering whether learning would be enhanced if novices could be instructed with forward-working strategies.

Consider the problem shown in Figure 1. This is a two-stage transformation problem requiring two applications of Pythagoras' Theorem with the subgoal (side BC) to be calculated before the goal (X) can be found. To work forward, a problem solver would calculate the subgoal (BC) first without consideration of the goal (BD). The problem is very restricted, as there are no other unknown sides to calculate, and therefore the solution path is fairly simple to find. The rule "find the side which joins the two small triangles together first" will identify the subgoal for problems of this particular configuration. As a consequence of having a non-complex solution path, instructional techniques may be able to induce a forward working strategy more readily in this domain. It is also conceivable that backwards-working strategies may also be effective. A simplified solution-path with few feasible alternatives may reduce the level of cognitive load sufficiently to enable backwards-working strategies to be effective. Evidence to support this argument was found by Ayres & Sweller (1990) and Ayres (1993) in a two-stage geometry domain. Problem solvers were more likely to find the solution-path if the problem configurations involved were familiar or simplistic in nature.

Figure 1: Example of a 2-stage Pythagorean Problem



This study was designed to not only investigate the effectiveness of no-goal problems in the domain of Pythagoras' Theorem, but to also explore the effectiveness of forward and backward-directed strategies. Four groups of subjects completed different acquisition phases. One group of subjects was encouraged to work forward, another backwards; a third group completed a set of no-goal problems, while the last completed a traditional problem solving exercise with no instruction phase other than undirected problem solving.

Method

Subjects

Fifty six Grade Eight girls of average mathematical ability from a Sydney High School participated in this study. They had previously been taught the Theorem of Pythagoras and had completed problems requiring single applications of the theorem only. The students had little experience in solving transformation problems other than in an elementary geometry domain.

Materials and Procedure

Two sets of twelve problems were designed in a similar fashion to the two-stage transformation example depicted in Figure 1. The first set of problems was used as an acquisition phase, while the second set was employed as a test set. Both sets were presented on paper. Each problem consisted of a diagram which contained two unknown sides. To calculate these unknown sides two applications of Pythagoras' Theorem were necessary. Furthermore, one side (the subgoal) had to be calculated before the second side (the goal) could be found. In each case the subgoal was common to both triangles. Using Pythagoras' Theorem to calculate an unknown side in a right-angled triangle leads to two possible algebraic outcomes. If the unknown side is the hypotenuse then an equation will be formed with an expression which contains a plus: $x^2 = a^2 + b^2$ for example; otherwise the unknown side leads to an expression which contains a minus; for example: $x^2 = a^2 - b^2$. Both problem sets were generated so that each of these two expression types occurred an equal number of times at both the subgoal and goal positions. A common error made by students working in this domain is to apply one of these expressions to the wrong situation; for example, adopting the formula which has a plus when the negative is required. By counterbalancing the problem-sets, results would not be biased by a student who routinely favoured one of the two formulae only.

The four instructional methods were designed as follows. To encourage a forward working strategy, a group of subjects was asked at the beginning of each problem in the acquisition phase to, "Calculate the first unknown side you can". It was thought that by drawing attention to the subgoal initially, rather than the goal, subjects may be encouraged to work forwards. After finding this side, subjects were then told, "Now calculate x". In contrast, to encourage a backward-working strategy, a group of subjects were given the following instructions on each problem, "To find x we must first calculate an unknown side. Calculate this side". By focusing on the "x" initially it was thought that students may be directed backwards. After finding this side, subjects were then told- "Now calculate x". For the conventional group, subjects were told, "Find x" in all problems", and therefore had a specific goal task. For the no-goal group subjects were given the directions, "In each problem calculate as many unknown sides as possible". No mention of a goal (x) was made.

The subjects were randomly assigned to one of the four groups: *the Conventional Group*, the *No-goal Group*, the *Forward Directed Group* or the *Backward Directed Group*. Each group contained fourteen subjects. It should be noted that in other no-goal experiments in elemetary geometry and trigonometry (see Ayres, 1993; Owen & Sweller, 1985) significant effects were found following short acquisition periods. Subjects were given the acquisition set of problems and allowed twenty minutes to complete as many of twelve problems as they could in the time. Following this phase, subjects were given the second set of problems which were presented in the conventional goal-specific format with the instructions, "Find x in all of the following problems". Subjects were given thirty minutes to complete as many of these problems as they could. No feedback about correct solutions was given during the experiment.

Results and Discussion

For the acquisition phase, the mean number of problems attempted by each group was calculated (see Table 1). A one-factor ANOVA revealed that there was no significant difference between groups on this measure; F(3,52) = 0.27, p =0.85. To observe how accurately subjects applied Pythagoras' Theorem in this phase, success rates were calculated. Success rates were found for each subject by dividing the total number of sides correctly found by the total number of attempts at calculating sides (there are two sides which can be found in each problem). For example, if a subject calculated ten sides in five problems but made one error in applying Pythagoras' Theorem, then this subject would have a success rate of 0.90. It should be noted that errors which resulted from basic slips of arithmetic, rather than fundamental misunderstandings of Pythagoras Theorem, were not included in this analysis, as the focus of this experiment was to examine the students' understanding of the theorem. The most frequent error (61%) was to use an algebraic expression with a plus rather than a minus (see above). The reverse error of using a minus when a plus was required accounted for 20% of all errors, while the remaining 19% were varied and could not be easily classified.

Table 1: Number of problems attempted and success rates for each group in the Acquisition Phase

	Number Attempted		*Success Rates*	
	Mean	*SD*	*Mean*	*SD*
Conventional	5.14	2.71	0.51	0.31
No-goal	4.86	2.14	0.75	0.29
Forward-directed	4.86	1.88	0.54	0.34
Backward-directed	4.43	1.65	0.54	0.34

A one-factor ANOVA revealed that there was no significant difference between groups on success rates; F(3,52) = 1.51, p =0.22.

For the Test phase (see Table 2), both the number of problems attempted by each subject and the success rates were recorded in an identical method to those in the Acquisition Phase (see Table 1). Most errors (76.5%) were made by using a plus in the expression derived from the theorem rather than a minus. The reverse error of using a minus when a plus was required accounted for 11.2% of all errors, whereas the remaining 12.3% could not be classified. A one-factor ANOVA revealed that there was no significant difference between groups on the number of problems attempted; F(3,52) = 0.49, p =0.69. However, there was a significant difference between groups on the success rate measure; F(3,52) = 3.03, $p < 0.05$. Post-hoc Fisher PLSD tests revealed that the no-goal group had significantly higher success rates (at the 95% level) than both the conventional and forward-directed groups. Comparisons between other pairs failed to produce significant results. Clearly, the subjects who experienced a no-goal acquisition period were more able to identify separate triangles within the problem configurations and use Pythagoras' Theorem appropriately in the Test Phase.

Table 2: Number of problems attempted and success rates for each group in the test phase

	Number Attempted		*Success Rates*	
	Mean	*SD*	*Mean*	*SD*
Conventional	7.79	2.22	0.60	0.31
No-goal	6.64	2.62	0.85	0.20
Forward-directed	7.50	3.03	0.60	0.26
Backward-directed	7.43	2.50	0.63	0.28

To investigate the improvement in success rates between the instructional phase and the test phase , one-tailed t-tests (paired) were conducted between the two phases for each group. One-tailed tests were used as it was expected that students would show some improvement as a result of the acquisition phase. The no-goal group significantly improved from 75% in the acquisition phase to 85% during the test phase ($t(13) = 2.37$, $p < 0.05$). As the test phase consisted of conventional "x-format" problems, there is a strong indication that the acquisition phase was good preparation and allowed subjects to transfer easily to the test format. For the three other groups all showed an improvement in performance but not significant: the backward-directed group increased from 54% to 63% ($t(13) = 1.18$, $p= 0.13$); the conventional group increased from 51% to 60% ($t(13) = 1.54$, $p = 0.08$); and the forward-directed group increased from 54% to 60% ($t(13) = 0.81$, $p = 0.43$). Clearly, the forward-directed group did not improve significantly from the acquisition phase to the test phase, indicating that the forward-directing strategy was not very effective. In contrast, the p-values of the conventional and backward-directed groups may be approaching significance. However, even if real effects were present for these two groups, their effectiveness compared with the no-goal group was poor.

Conclusions

Consistent with previous research (Ayres, 1993; Owen and Sweller, 1985), a no-goal approach has been shown to be a very effective method for learning in domains which require solutions to simple transformation problems. Removal of the goal appears to simplify the problem-space searches and perhaps reduce cognitive load, which in turn, may enhance learning. Furthermore, Silver (1990) argues that goal-free problems may encourage conjecturing which is fundamental to learning. Attempts to induce either a forward or backward working strategy only succeeded in producing results at the same level of performance as a conventional approach. Although, these attempts lacked sophistication, they nevertheless provided a further contrast to the standard no-goal and goal designs of other studies. However, it should be noted, that in both cases, little data emerged which indicated whether the simple instructional formats achieved their objectives of stimulating forward or backwards-working strategies. A further study replicating this experiment could collect qualitative data to discover how students react to such instructions. Such a study might lead to more sophisticated ways to induce a forward-working strategy. In summary, even though the experimental design of this study has not found any evidence to support the argument that learning might be enhanced by instructing novices in forward-working strategies, more data has emerged to demonstrate the impact of a no-goal approach.

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FACTORS INFLUENCING PREDICTIONS ABOUT RANDOMLY GENERATED SEQUENCES

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The detailed analysis of four probability experiments conducted with Grade 5 and 6 students revealed trends and patterns in both the group and individual data. These results suggested that certain variables in the experiments, such as particular sequences of outcomes and the confirmation/refutation of student predictions, influenced the students' decision making strategies. The use of video recordings of deliberately controlled probability experiments offers the potential to systematically explore these influential factors with large samples of students.

Introduction

Several research studies exploring probabilistic reasoning have highlighted the influence of particular sequences of randomly generated outcomes on decision making strategies (for example: Peard, 1995; Shaughnessy, 1981; Tversky & Kahnemann, 1982). One such strategy is *representativeness* (Kahneman & Tversky, 1972), which is the expectation that a random set of outcomes should be representative of the composition of the known sample space. Related to representativeness is the type of thinking known as *negative recency* or *gambler's fallacy* , where there exists the expectation that as the frequency of a particular outcome increases the probability of that outcome ocurring decreases. For example; when repeatedly flipping a coin, a run of heads would lead to the expectation of the next flip being a tail. The opposite strategy, in this case of predicting a head because that's the trend, is referred to as *positive recency*.

Much of the research into misconceptions and inappropriate reasoning strategies has been with adults, using written tasks or 'tests', in which preconceived sets of outcomes have been presented to the subjects. However, most researchers working with children prefer to use real random generators to accommodate children's need for concrete experiences (for example: Carlson, 1970; Fischbein, 1975; Hoemann & Ross, 1971; Truran, 1992; Way, 1996). Random generators naturally produce sequences which vary from trial to trial. Consequently the ability to test large groups of students with controlled variables is restricted. The use of video-recorded probability experiments can overcome these research difficulties by providing both a realistic medium for children, and controlled sequences of outcomes (see Ayres & Way, 1998).

Truran (1996) analysed the nature of primary and secondary students's responses to a task in which they were required to state the most likely outcome of each of nine draws (with replacement) from a sample space of three green and one blue. The use of the negative recency heuristic in such tasks has been quite well documented, but more interestingly, Truran examined the effect that either confirmation or refutation of the predicted outcome had on the following prediction. In other words, he analysed the changes in prediction in regards to the next outcome. One finding was that when green (the more likely outcome in this experiement) was predicted, it didn't really matter whether the next draw confirmed or refuted that prediction. However, if the less likely colour (blue) was predicted, the subject was highly likely to change the prediction, particularly if the following outcome refuted the blue prediction. The strong influence that knowledge of the sample space had on the students' predictions was obvious, so this raises the question of what effect the absence of this knowledge might have on prediction patterns.

Probability experiments with hidden sample spaces force the subjects to rely on the information provided by the experimental outcomes (relative frequencies) when considering the likelihood of various outcomes. Fischbein (1975) established that even young children possess the *intuition of relative frequencies* and can therefore estimate the composition of an unknown sample space given adequate frequency data. But how much data is adequate, and how representative of the actual sample space does this frequency data have to be to stimulate reasonably accurate estimates?

The two main research questions currently of interest to the authors are as follows: a) What is the relationship between the degree of representativeness of frequency data produced from an unknown sample space and students' probability judgements? and; b) What is the relationship between the confirmation or refutation of student predictions and students' decision making strategies?

As part of an on-going project to investigate the development of probabilistic reasoning the researchers have completed two preliminary studies. Although each study had a different specific purpose, the design of the experiment and the type of data collected in both studies were similar. This particular paper reports the patterns and insights which have emerged by bringing the findings of the two studies together and applying for the first time, common data-analysis techniques. Firstly, brief summaries of the studies are given.

Study 1

The first study (Ayres, 1996) was exploratory and investigated Grade 6 responses to a sequence of coloured balls drawn at random from a bag containing six orange, three white and one yellow, with the actual contents of the box unknown to the students. (Note that these colours have been altered from the original study to standardise the reporting of both studies). Students were required to specify the colour they thought most likely to be drawn after every five selections. An introductory activity and discussion was used to assure the students that the researchers did not expect them to actually *know* the outcome of the next draw. A record of the outcome of each selection was displayed for the class. Students made six predictions in total and their individual choices are shown in the Appendix. Two classes (referred as 1A and 1B) participated in this study and observed radically different sequences (consistent with naturally occurring random outcomes), and consequently made differing predictions from a class perspective. The colour sequences that groups 1A and 1B observed contained a total of 73% and 53% oranges respectively. Although both sequences produced experimental data consistent with orange being the most likely colour to occur, the sequence observed by 1A contained a greater number of orange than the expected value (60%). The students in the group 1A constantly predicted the colour orange (93% of the time). It was therefore concluded that the students in this group were guided by experimental outcomes and were able to link ratio with likelihood. In contrast, the students in 1B only choose orange 43% of the time, suggesting that the different frequency data influenced their predictions.

Study 2

Following this study the researchers concluded that to test large groups of students in this domain, and understand the reasons for their decision-making, it was desirable to control the variables. As a result the second study (Ayres & Way, 1998) tested the effectiveness of using a video-recording as a method of controlling variables and reproducing random sequences. In this study, a group of Grade 5/6 students were shown a video-recording of balls being drawn from a box and asked to specify the most likely result of a draw in a similar fashion to the first study. However, the video-recording was a reproduction of a sequence of random colours which a parallel class of Grade 5/6 students had observed *live* and made predictions on. Quantitative and qualitative data revealed that the group of students who observed the *video* responded to the prediction tasks in a similar fashion to the live group and were not concerned about its legitimacy: students believed that the sequence was random, and not artificial.

Two Study Analysis

Overall, the two studies produced three sets of frequency data which are recorded in Table 1. Given the nature of the prediction tasks in these studies, it can be assumed that students who were guided by experimental probability were overall more likely to predict the most frequently occurring colour. Over the six trials the mean number of oranges predicted was 5.6 (SD = 0.9) for Group 1A and 2.6 (SD = 1.4) for Group 1B. Similarly the mean number of oranges predicted was 2.9 (SD = 1.33) for the Live Group and 2.5 (SD = 1.4) for the Video Group. Clearly, only Group 1A adopted a consistent policy of predicting the most frequently occurring colour. In contrast, the other three groups all predicted the most frequent occurring colour less than 50% of the time. Although, there appears to be little difference in overall means for these three groups, substantial differences can be found in the prediction patterns.

Table 1: The random colour sequences produced in the two studies

Groups	*Colour Patterns*
1A	OWOOO WOOOO OYOOO OOWOO OOOWW OWOOW W
1B	WOOYW WYOOO YOOOO WWOOO OOOYW WOWYW W
Live and Video	OYWYO WOWOO WOWOW OOOOO YOOOO WOOOO

Notes: a)The underlined colours indicate the outcome immediately following each request for a prediction; b) O = orange, W = white, Y = yellow.

Prediction profiles for each group were found by calculating the frequency of each colour selected at every prediction point (see Table 2). Whereas, Group 1A was very consistent at each prediction, the other groups differed considerably, especially on their first three selections. Although these differences could be due to random prediction factors between groups, there are also intrinsic differences contained within the sequences themselves which may have been influential.

Table 2: Colour selections (%) at each prediction point

	Group 1A			Group 1B			Group Live			Group Video		
Predictions	O	W	Y	O	W	Y	O	W	Y	O	W	Y
First	95	5	0	45	10	40*	17	47	37	61	26	13
Second	90	10	0	30	40	30	47	23	30	4	30	65
Third	85	0	15	55	35	10	43	33	23	26	39	35
Fourth	95	5	0	60	5	35	57	20	23	57	22	22
Fifth	95	5	0	40	25	35	67	3	30	30	52	17
Sixth	95	5	0	30	50	20	60	13	27	65	17	17
Means	93	5	3	43	28	28	49	23	28	42	31	28

* Note: other colour selected

Group Trends

By calculating the unfolding experimental probabilities (see Table 3) the differences between sequences can be observed. For example, for the Live and Video groups the first four colour outcomes produced a high (0.5) experimental probability of yellow occurring (2 yellows out of four). Consequently the selection of orange was not necessarily an informed choice by these groups at this stage. However, by the 20th selection the experimental colour proportions were more representative of the actual theoretical probabilities for both the Live & Video groups; and similarly for Group 1B.

Table 3: Cumulative experimental probabilities (%) per cluster for each random sequence.

	Group 1A			Group 1B			Groups Live & Video		
Clusters	O	W	Y	O	W	Y	O	W	Y
First	80	20	0	40	40	20	25	25	50
Second	80	20	0	50	30	20	44	33	22
Third	80	13	7	60	20	20	50	36	14
Fourth	80	15	5	60	25	15	58	32	11
Fifth	76	20	4	60	24	16	63	25	13
Sixth	73	23	3	53	30	17	66	24	10

(Note: Each experimental probability percentage has been calculated by combining the results of each prediction cluster. For example, the first experimental probabilities for groups 1A and 1B are calculated after five selections; the second after ten selections and so on).

In the Ayres & Way (1998) study the slow development of representative experimental probabilities were taken into account by comparing the first three predictions with the last three. It was argued that this measure would be a stronger indicator of probabilistic reasoning as the experimental probabilities were converging more towards the theoretical in the latter half of the sequence. By calculating the mean number of oranges (most likely colour) for the first and last three predictions it was possible to assess whether the groups had refined their strategies for the last three predictions. This data is reproduced in Table 4, along with a similar analysis for Groups 1A and 1B. It should be noted that this analysis was not previously applied for Groups 1A and 1B. One tailed (improvement expected) Wilcoxon signed-rank tests for matched pairs were conducted. As recorded in the Ayres & Way (1998) study both the Live & Video groups demonstrated a significant increase in the number of oranges selected, as the frequency data became more representative of the theoretical probabilities, and hence the sample space.

Table 4: Mean number of predictions made of the most frequently occurring colours.

	1A	1B	Live	Video
Mean number of "most likely" colour chosen on first 3 predictions	2.7	1.3	1.1	0.9
Mean number of "most likely" colour chosen on last 3 predictions	2.9	1.3	1.8	1.6
One-tailed Wilcoxon signed-rank tests for matched pairs	T (3) = 0	T(16) = 68.5	T(21) = 36*	T(19) = 37.5*

* Significant at the 99% level

Significance was not reached by Group 1A because n=3 is too small a number to evaluate under the Wilcoxon test where n=5 is usually required; although group means have increased. It should be noted that as 75% of this group selected all oranges anyway, there is little capacity for more oranges to be chosen. Only Group 1B did not show any indication of an overall group movement towards favouring the most likely colour. Individual analysis indicates that 8 students in Group 1B increased their choice of orange; whereas eight decreased. Only one student (5% of the group) completed a sequence of three or more oranges to finish the prediction sequence. This contrasts with five (21%) students in the Video group and 10 (33%) in the Live group who finished with 3 or more oranges. Further more, four students in Group 1B predicted three or more oranges to begin with, but then changed to another colour.

Factors which may have influenced Group 1B on this measure can be found by analysing the colour sequences further. Table 3 indicates that the experimental frequencies quickly converge towards the expected values by the third prediction. Although the experimental probability of orange occurring does decline from 60% to 53% by the last prediction, it is still the most frequently occurring colour, and arguably does not explain the low means prediction rate of 1.3 for the last three predictions. Ayres (1996) suggested that the lack of success in their predictions by this group influenced their strategies. More evidence can be found to support this argument by examining the colours which did occur immediately after the prediction points. (These colour outcomes form a subset of the frequency data and are underlined in Table 1). For Group 1A and the Live & Video groups the most frequently occurring colour (orange) was drawn four times out of five immediately after each of the predictions. (For this analysis the sixth prediction need not be considered as students were not required to make further predictions following its selection). In contrast, for Group 1B, an orange only occurred once after each of the five predictions. This may well be a significant factor. Students in Groups 1A, Live and Video applying a positive recency strategy (in regards to the subset of post-prediction outcomes) had their predictions confirmed, but students in Group 1B applying this strategy did not. Analysis of prediction success rates indicated that Group 1B had the lowest success rate of 1.5 (SD = 1.1) compared with 3.7 (SD = 0.64) for Group 1A, 2.2 (SD = 1.08) for the Live group and 1.9 (SD = 1.1) for the Video group. This finding suggests another variable which may need to controlled in future experiments.

Individual Analysis

Further insights can be gained by examining individual responses in more detail. In group 1A, orange was so dominant in the frequency data that very little additional information can be extracted, except at the third prediction, where three students chose yellow. This was the only time that yellow was chosen and followed a yellow occurring in the previous cluster which may have influenced these students' choices. There is some evidence to suggest that some students may have been utilising a positive recency strategy in some predictions. For example, following a white occurring three times in the sixth cluster, 50% of the students in Group 1B selected a white at the next prediction point. In addition, following two whites occurring in the third cluster, nine students (39%, the highest number) in the Video Group chose white, and ten students (33%) in the Live group. In contrast, there were examples of colours chosen where students may have adopted a negative recency strategy. Fifteen students (65%) chose yellow for their second prediction following the non-selection of a yellow in the previous cluster. Similarly 12 students (52%) chose white for their fifth prediction in the Video Group, following the non-occurrence of a white for nine selections. It is interesting to note that five students (two in Group 1B, two in the Live group and one in the Video Group) did not chose the most frequently occurring colour at all. In addition, it can be seen from the means in Table 2 that yellow was chosen as frequently as white, despite having the least likelihood of occurring (see Table 3).

Changes in Prediction

Further analysis (conducted for the first time on the data from both studies) of individual responses revealed that many students changed the choice of their colours frequently. The extent of these changes was calculated by counting each change of colour from one prediction to the next, thus giving a maximum of five possible changes. For example, Student 1 in Group 1B had a prediction sequence of O W W O Y W giving a total of four changes. The mean number of changes by group are shown in Table 5.

To investigate the extent to which changes are influenced by successful predictions, changes were categorised according to whether they followed confirmation or refutation of predictions. For each student the number of changes made following either outcome was calculated as a proportion. For example, if a student made four correct predictions which were followed by two changes in colour following each success, then the student had a ratio of 0.5 changes per successful predictions. Group means for this data are also recorded in Table 5.

Table 5: Changes in colour predictions

	1A	1B	Live	Video
Mean Number of Colour Changes	0.8 (SD = 1.4)	3.8 (SD = 1.3)	2.7 (SD = 1.4)	3.3 (SD = 1.0)
Mean number of proportional changes following a *successful* prediction	0.12 (SD = 0.3)	0.7 (SD = 0.4)	0.4 (SD = 0.3)	0.7 (SD = 0.3)
Mean number of proportional changes following *unsuccessful* prediction	0.23 (SD = 0.4)	0.8 (SD = 0.3)	0.7 (SD = 0.3)	0.6 (SD = 0.4)

As might be expected from a group who constantly favoured one colour, Group 1A demonstrated the least number of changes with a mean rate of 0.8 changes per five selections. Furthermore, it can be seen that this measure appears to form an inverse relationship with the prediction success rates of the groups. As reported above, the mean number of correct predictions made over the first selections were 3.7, 2.2, 1.9 and 1.5 respectively for groups 1A, Live, Video and 1B, which may indicate a negative correlation with the corresponding number of changes made (0.8, 2.7, 3.3 and 3.8). In other words, students who successfully predict a colour are less likely to change that prediction at the next selection point. Further evidence can be found to support this possibility by examining the changes made following confirmed or refuted predictions. The two most successful groups (1A & Live) at predicting, had the lowest rates of changes (see Table 5) following successful predictions. But these rates increased considerably (1A: from 0.12 to 0.23; Live: from 0.4 to 0.7) following an unsuccessful prediction. Although Group 1B followed this trend to a lesser degree (from 0.7 to 0.8), the Video group did not (from 0.7 to 0.6); a point which requires further investigation.

Conclusions

Although, a certain amount of caution should be shown as a result of examining the combined data from two independent studies, more patterns and trends emerged than could be identified from the studies individually. In any domain involving random generators, different sets of outcomes emerge which make the analysis of the data complex. However, by combining the data from four different groups, and searching for similarities, contrasts and patterns, the researchers were able to identify factors that should be considered when designing further investigations. The video-recording technique allows control over the frequency data, which can then be presented to many groups of children. The relationships between variables in the frequency data and the students' predictions which have been tentatively identified, can now be used to inform the ways in which the researchers manipulate these variables in future studies. As suggested by other researchers (for example: Truran, 1996; Way, 1996), the strategies employed by children in probabilistic situations are more complex than can be explained by currently identified heuristics. Although still in its early stages of development, the overall methodology outlined in this paper, has the potential to unravel some of these complexities, and more clearly identify some aspects of the development of probabilistic reasoning.

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Appendix: Predictions made by each student in both Studies

	Study 1			*Study 2*	
	Group 1A	*Group 1B*		*Live Group*	*Video Group*
1	OOYOOO	OWWOYW		OOWOYW	YWOYOO
2	OOOOOO	WYOYWY		OWYOYY	WYWOOO
3	OOOOOO	YWOYWY		YOOOOO	WWYOOW
4	WOYOOW	WYWYOW		OYYOYY	OYOOOO
5	OOOOOO	OOOYOO		YOOWOO	YYWOWO
6	OOOOOO	OYWOWY		YOOOOO	YYOWWY
7	OOOOOO	OOOOYW		YOWOOO	OWWOWO
8	OWOOOO	OYOWYO		WWWWOO	OYWYWW
9	OOOOOO	OOOOWO		YWOYOO	OYYWWY
10	OOOOOO	OWWOWO		WYWOYO	WYWWWO
11	OOOOOO	YYWYYY		YYYYYY	OYYYWO
12	OOOOOO	OWOOWW		WYOYOW	OWWOYO
13	OOOOOO	YWOOOW		WWOOOO	WYYYWY
14	OOOOOO	YOWOYO		OYYWOY	OYWOYO
15	OOYOOO	YOYOOW		WYOWOW	OYOOOO
16	OOOOOO	YYWYOW		YYYOOO	WOYWWW
17	OWOWOO	YYWYOW		YYWWOO	OYOOOO
18	OOYOWO	OOOOOW		WOWYYY	OYWOYW
19	OOOOOO	YWYOYO		YOOWOO	OYYYYO
20	OOOOOO	GWOOOO		WOOYWY	OWYOWO
21				YOOOOO	OWWOOO
22				OWWOOW	WWOOWO
23				WYYYYY	OYYWWY
24				WWOOYO	
25				WOOOOO	
26				WOYOOO	
27				WOOYOY	
28				WOWOYO	
29				WWWOOO	
30				YOWOOO	

Towards a framework for analysing power relationships in small group discussions

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Based on an approach to power derived from the work of Michel Foucault, this paper describes the development of a framework for analysing power relationships in small groups of students working on collaborative activities. Student-student interactions were observed, videotaped and transcribed. Key features that emerged were techniques used to control the flow of the discourse in the group and behaviours which influenced the mathematical knowledge constructed. Other factors include body language and the control of resources.

Introduction

This paper describes the development of a framework for analysing power relations in small groups of students working on collaborative activities independently of a teacher.

Collaborative learning—students working together, usually in small groups, on a shared activity and with a common goal—has been widely recommended in recent years as a strategy to enhance mathematics learning for all students, especially girls (e.g., Cordeau, 1995; Jacobs, 1994; Morrow & Morrow, 1996; Solar, 1995). I am beginning a study which will investigate the effectiveness of this strategy, with the aim of describing optimal conditions for its use. My principal research question asks how students experience learning mathematics in collaborative settings and how the gender of the student impacts on, and is affected by, this experience. In particular, I am seeking to find out how students working in collaborative groups interact with one another, and how they construct themselves as learners of mathematics. The study is focused on senior students (mainly in Year 11) at the stage where they are beginning calculus, because it is around this time that they make key decisions on course selection affecting their post-school options, and their future relationship to mathematics. These key decisions are predicated on students' evolving constructions of themselves as learners of mathematics.

In this paper, I analyse data from an ethnographic case study of a single classroom using collaborative learning activities, undertaken as a preliminary to the main study. Naturalistic inquiry methods were used in order to disturb normal classroom processes as little as possible. Data included videotapes of all lessons for a three-week period, field notes from classroom observations, and interviews with the teacher and target students. The aim was to assess the adequacy of the data-gathering techniques for answering the central questions of the main study and to explore possible frameworks for data analysis.

Feminist theory, and my own experience, suggest that a study exploring gender effects in collaborative learning needs to take into account the exercise of power within student groups, and to investigate the potential of these to influence learning outcomes.

Gender, power and knowledge

What is power and how can it be investigated?

My starting point is an understanding of the nature of power, and an approach to analysing power relations, proposed by Michel Foucault. Foucault defined power as "a mode of action which does not act directly and immediately on others. Instead it acts upon their actions … on existing actions or on those which may arise in the present or in the future" (Foucault, 1982 p. 220). Foucault stressed that power is exercised through individuals and must be studied at the individual level: "What characterizes the power we are analyzing is that it brings into play relations between individuals (or between groups)." (Foucault, 1982 p. 217).

The questions most often asked about power deal with its nature and sources, (the "What?" and the "Why?" of power). Foucault, on the other hand, chose to ask about the "How?" of power. The shift in focus, from theorising about the sources of power to asking how it is exercised, opens up the possibility of empirical investigation. This does not mean that questions of the nature and sources of power are ignored. On the contrary, empirical evidence of the exercise of power may provide data for investigating these questions and so help us understand how power functions in different contexts.

Foucault claimed that power in modern society is not a commodity, which some possess and others do not. Rather, it is jointly constructed, a structure of relationships which shapes people's actions. "Power exists only when put into action." (Foucault, 1982 p. 219). Furthermore, the effects of power are not all negative: "it induces pleasure, forms knowledge, produces discourse. It needs to be considered as a productive network which runs through the whole social body, much more than as a negative instance whose function is repression." (Foucault, 1980 p. 119). This point is of particular importance when we are studying the classroom, where the formation or construction of knowledge is the object of the enterprise.

Power relationships and the construction of knowledge

Systematic observation is needed to clarify the operation of power in pedagogy (Gore, 1997). It is particularly important to investigate in this way an innovative teaching practice such as collaborative learning, which involves a shift in traditional classroom power relationships. By relinquishing some control over classroom interactions, the teacher is sharing power with the students. I claim that the ways in which power is exercised among a group of students working together on a mathematical activity can influence the construction of knowledge within the group, both the personal understandings of mathematics constructed by each member of the group, and the knowledge which becomes "taken-as-shared" within the group.

For example, a student who is able to control the direction of the discourse within a group has great potential to influence the group's progress. Watson and Chick (1997) give examples of the "charismatic intellectual", with both cognitive and leadership abilities, who is able to lift the performance of an entire group, but point out that leadership skills without cognitive capability could have the effect of lowering the performance. If a student is able to command the attention of the rest of the group to his or her ideas, they may be taken up by the group in preference to more profitable ideas. Here again, Watson and Chick give examples in which one student's persistence with a wrong idea pulls down their whole group's performance on a task.

The extent of a student's influence on a group's discussions has the potential to affect their self-perception of mathematical competence and feeling of ownership of the mathematics constructed; and also how their capabilities are perceived by others. Thus the exercise of power within small groups is potentially important in a study of how students construct themselves as learners of mathematics.

Gender and power

Much of the research on gender, in mathematics education and in other fields, has focused on investigating gender differences, and developing strategies to minimise or eliminate them. But as Crawford (1995) points out, concentrating on difference may result in a failure to confront underlying issues of power, status and domination. Analysis of male-female power relations has been a major emphasis of feminist studies, revealing the many ways in which women's interests are subordinated to those of men at both the systemic and the individual level (see e.g., Eisenstein, 1984; Grosz, 1988).

Observation of young children has shown how the social construction of gender is shaped by the discourse of powerful and dominant masculinity (Clark, 1989; Davies, 1989). Gendered patterns in conversational interaction have also been documented. Malz and Borker (1982) observed that in cross-sex conversations males are more likely to interrupt; to challenge or dispute another person's utterances; to ignore what the other said; and to try to control the topic of conversation. They explain these differences as due to cultural differences in communication patterns produced in distinct male and female subcultures. This view has been widely criticised as representing a failure to recognise the influence of power differentials (Crawford, 1995). "The two-cultures approach fails to theorise how power relations at the structural level are recreated and maintained at the interactional level" (Crawford, 1995 p. 96) Crawford suggests that "those who want to understand how the gender system is recreated and maintained are wise to focus on language use. Attention to language practices can be a crucial way of unmasking the politics of everyday life" (Crawford, 1995 p. 180).

Perspectives on power in the classroom

Studies of teacher-student power

A Foucauldian view of classroom power sees it as a relationship between the participants, claiming that there can be no power relations without the possibility of resistance. Manke (1997) has adopted an interactive conception of power, taking into account actions of the students as well as the teacher. Her focus, however, is on the struggle for power between teacher and students, and the strategies adopted by teachers to achieve their objectives in the classroom. In a current project, Gore (1995; 1997) is using categories derived from Foucault's writings to analyse the practice of power in a variety of educational settings, including some described as radical. But so far no data related to male-female interactions within collaborative groups has been reported.

Studies of interaction in mathematics classrooms, including those dealing with gender issues (Koehler, 1990; Leder, 1990), have not had power relations as an explicit focus, although some of their findings could be interpreted as indicating the exercise of power by male students. Again, most of these have dealt with teacher-student interactions, finding fairly consistent differences in teachers' interactions with male and female students, favouring the learning of males. Leder noted "the pervasiveness of males' domination of teacher attention" (Leder, 1990 p. 165). Jungwirth (1991) found gender-related modifications of "typical" teacher-student interaction patterns, and argued that the effect of these was the interactive constitution of mathematical competence on the part of boys and of mathematical incompetence on the part of girls.

Power relationships within collaborative groups

In the studies discussed above, the style of classroom teaching observed did not permit an analysis of interactions between male and female students. Forgasz (1995a; 1995b), in a study of group work in two Year 7 classrooms, did study groups as they worked together. Her observations included disruptive behaviour by boys, occasional abusive behaviour by boys towards girls, and boys adopting work-avoidance tactics, leaving the girls in their group to do most of the work for which all group members would receive credit. All of these could be interpreted as ways of exercising power. Forgasz, however, did not explicitly address power issues, choosing to focus instead on autonomous learning behaviours and attributions for success and failure.

Procedures and results

Sources of data

The study was conducted in a government high school in Sydney, and involved a Year 11 class doing introductory work on calculus. A series of lessons was videotaped, with the camera trained on small groups when appropriate. Field notes and student worksheets supplemented the videotape record. "Rich" transcripts of the videotapes were prepared, including descriptions of actions, gestures, facial expressions or voice intonations which were judged to be relevant. To validate my judgements about what is relevant, a colleague was asked to view a sample of the taped lessons and comment on the information included in, or omitted from, the transcripts.

Indicators of the exercise of power

In seeking to develop a framework for analysis grounded in the data, my starting point was to study and reflect on the transcripts and to re-view the videotapes, to get a feeling for the power relations involved. There is always a risk in research of finding what one is looking for—simply confirming one's personal biases. As the researcher who collects and transcribes the data, I cannot avoid being aware of both the genders and the personalities of the students, and knew that I needed to guard against the risk of prejudging some important issues. I therefore aimed to develop a set of criteria for identifying the exercise of power which could be applied to rich transcripts by someone with no information about the gender of the protagonists. This would make it possible for a colleague to check the reliability of my analysis.

I was able to classify significant influences under three main headings: control of the flow of the discourse, construction of knowledge and use of resources. Other headings which I considered included the allocation and use of space and time, but few instances of students' use of these have been observed in the data analysed so far.

Control of the flow of discourse

The main way in which a student can control the discourse in a group is by influencing the topic to be discussed, including the timing of transitions from one topic to another. I found that the study of the resolution of uncertainty by Clarke and Helme (1997), provided a useful approach to this analysis. Clarke and Helme proposed that a mathematics lesson may be considered as consisting of many episodes, each defined by a consistent purpose such as the solving of a particular problem. Each episode is made up of one or more negotiative events associated with identification of, and attempts to resolve, a sub-goal. Negotiative events are usually initiated by an expression of uncertainty in some form, such as the asking of a question.

In practice, the transcripts I was analysing did not divide neatly into sequences of negotiative events, each satisfactorily resolved before the group moved on to the next. The negotiation was often inconclusive, or interrupted by off-task talk. Nevertheless, negotiative events appear to be a key unit for analysis, because transitions from one to the next mark the progress of a group's work on an activity. A student who enacts closure of one negotiative event and initiates the next to a very large extent controls the discourse.

A study of the transcripts revealed the following occurrences, representing ways in which students act to control or influence the direction and flow of group discourse:

- Initiates a negotiative event.
- Initiates off-task talk.
- Rejects or ignores off-task talk (usually by continuing the negotiative event).
- Claims the attention of the group by gesture, emphatic speech or other means.

Note that terminating a negotiative event is not a very powerful move. It is never clear that an event has ended until the next event, or discussion of some other topic, is established. Until then, although the group may appear to have reached agreement, it is always possible for one member to have a change of mind and resume negotiation. The sequence of negotiative events is thus interactively constituted by all participants in the discussion. One member can only control the proceedings if the others permit it.

Construction of knowledge

The sequence of negotiative events tells only part of the story. In a mathematics lesson, what is important is the mathematical ideas. We need to look at the influence of different students on the knowledge that is constructed or negotiated. For this, we must turn attention to individual turns within negotiative events. A study of the transcripts revealed the following types of moves which could be significant:

- Introduces a new idea, or makes a suggestion about solving the problem.
- Rejects an idea or suggestion.
- Accepts an idea or suggestion.
- Asks for an explanation or justification.
- Gives an explanation or justification.

The objective of a negotiative event is the resolution of uncertainty. The key to this may be contained in an idea suggested by one group member, but what really counts is its reception by the rest of the group. No matter how good the idea, if the group reject it, it will not advance their endeavour. This means that successful rejection moves are powerful. Ignoring is a form of rejection—collective rejection by the rest of the group. Being ignored can signify an individual's lack of power within the group.

Acceptance moves need to be closely examined. There is a big difference between positive support or endorsement of a suggestion, which causes the group to take it up and is clearly an exercise of power, and uncritical acquiescence to the ideas of an influential peer (like the "charismatic intellectual" described by Watson and Chick (1997)). A good idea may be accepted because of the status of its originator, without the rest of the group understanding its significance or being able to use it independently. Equally, a false or misleading idea may be accepted, and cause a time-wasting digression, or even failure of the group to complete the task satisfactorily. Such occurrences are exercises of power by the student who originated the idea. Discrimination between weak and powerful acceptance moves requires careful interpretation of the transcript and possibly a return to the original tape. The manner of saying and doing things can be as important as what is said or done: confident and tentative behaviours can have very different effects.

Asking for an explanation or justification can be an important and powerful move—almost equivalent to initiating a new negotiative event—but again this depends on the manner of asking, whether it is challenging or simply seeking help or clarification.

Finally, giving an explanation or justification is powerful if it convinces the hearers. An additional coding category is needed for the nature of the authority which is found to be convincing: appeal to the teacher, the textbook, or an established formula or rule (reliance on external authority), or the use of a rational argument constructed by the student (reliance on internal authority).

Use of resources

Resources available to a group to help them complete a mathematical task may include worksheets, textbooks, models, calculators, drawing implements or graph paper. Some of these may need to be shared. Observation shows that resources are used in a variety of ways to support control of discourse and construction of knowledge.

If there is a single copy of a worksheet or other resource, the student who has it is at an advantage in controlling the discourse. Conversely, passing a worksheet to another student can be a way of handing over control. Occasionally, one student hands over a worksheet to another but then dictates to the other what to write, in this way maintaining control, and indeed dominating the other. A gesture such as pointing to a model, diagram or graph can be a way of drawing attention to what one is saying, that is, a way of exercising power. But it may also be a device to help explain one's ideas, or an aid to clarifying one's thoughts, and so can support the construction of knowledge without the exercise of power over others.

Illustrative examples

Excerpt 1

R and N were working together on a card-sorting task: matching a set of functions with their derivatives. R had begun by arranging the cards in two columns on the table between them, the (numbered) function cards in numerical order and the derivative cards in random order. The second function, $3x^{-2}$, caused difficulty. They had never before differentiated a negative power, and initially selected the wrong card for this place. At the beginning of the excerpt, R had just terminated a negotiative event by moving the derivative card originally put in second place, into a position lower down.

Explanation of symbols used in the transcripts:

→	points to initiation of a negotiative event or off-task talk
(…)	indicates an indecipherable utterance
(what)	indicates the best guess for an indistinct utterance
[ ]	observations from videotape or field notes

→ 1 R: So which one is this? [Points to place opposite second function card, now vacant.]
2 N: Negative six. [Moves a card hesitantly towards the blank position, but does not put it in place]
3 R: Negative six. [Looks at the card N has suggested.] No it should have only x.
4 N: This one? [indicates a derivative card]
5 R: (What about) this one? [points to a different card.]
6 [Both study the cards for several seconds. R scratches his brow.]
7 R: Minus n uh minus one [emphasis] which is minus three, oh yeah. [Moves a card from near the bottom of the derivatives column into second place.]
8 R: Yeah, that one goes here.
→ 9 R: These two, and these two, these two, these two, these two, and this one is there.[Checks the pairs already in place]

Control of discourse: This excerpt consists of a single negotiative event, initiated by R in turn 1, and terminated by R with the initiation of a new event in turn 9. During the whole of the pair's work on the card-sorting task, which lasted about seven minutes, twelve negotiative events and two instances of off-task talk were counted, all initiated by R. This is a fairly extreme example of one partner completely dominating the discourse.
Construction of knowledge: In turns 2 and 4, N made tentative suggestions, which were immediately rejected by R. In turn 7, R realised that they had been making an error in applying rule they had just learned. R gave a brief explanation, drew attention to it by speaking with emphasis, added a confirming "oh yeah", moved the correct card into place and confirmed acceptance of the idea (turn 8) without checking with N. Note that the manner of speaking and acting is as important as what is said and done. N behaved tentatively, whereas R generally behaved with confidence. These differences in manner contributed to R's domination of the proceedings.
Use of resources: R took control from the start of the activity, by opening the envelope and laying out the cards. R also moved the cards into place. N did no more than point to cards or move one hesitantly towards a position, but never actually put one in place.

Excerpt 2

The class were working in pairs, with two pairs facing one another at each table. B and L had just resolved a disagreement. The shared worksheet was in front of B.

→ 1 B: Okay, then we've got (...) [B mutters mainly to self. L watches.]
2 B: (...) and nine x minus sixteen x plus six. There you go. [passes worksheet to L. Glances towards researcher (off camera) — looking for approval?]
3 [The worksheet now lies on the table between B and L. L looks at B's working and nods, then leans back, putting both hands to head.]
→ 4 L: I'm so tired, man.
→ 5 B: Okay, what's the rest of the question?
6 [L leans forward again and points to the worksheet.] What's the gradient of the curve at one.
7 [B pulls worksheet closer again and picks up pen.]
8 L: How do you do that?
9 B: Sub one in as x.
10 L: I thought it was something to do with x.
→ 11 [B leans forward to see what the pair sitting opposite them are doing.]
... (some off-task talk with the students opposite, sustained by L.)
20 [B is staring at the board.]
→ 21 L: Substitute x is one yeah. Wouldn't that still be the same thing?
22 B: I don't know. I think I'll go check my notes.
23 [B gets out notes and flips through the pages. L sits back, yawns, looks bored]

Control of discourse: In turn 1, B initiated a negotiative event—differentiating the polynomial they had just simplified. L terminated the event (turn 3) by nodding and disengaging from the discussion. L then tried to introduce off-task talk (turn 4). B rejected this invitation by initiating a new negotiative event in turn 5. This appeared to end at turn 10, when L again gave minimal assent to B's answer, and they began to talk about other things. B initiated this off-task talk, but L joined in and sustained it. B tried to terminate the off-task talk by returning to work (turn 20). It seems that both L and B felt uncertain about the conclusion agreed on earlier, as L reintroduced the question, beginning a new negotiative event (turn 21), and B expressed uncertainty (turn 22). They reached no conclusion. B searched through notes and L "switched off". Note L's use of body language to signal engagement or disengagement from the activity (turns 3, 6, 23).

Control of the discourse passed from B to L and back again. L showed little interest in the task and made little effort to understand the details. L tried to introduce and sustain off-task talk, but moments later switched to thinking about the task again. B, on the other hand, did appear to want to engage with the task, but lacked confidence that the idea proposed was correct, and resorted to searching through notes for confirmation. Neither L nor the students opposite responded in any way to B's expression of doubt.

Construction of knowledge: Very little knowledge was constructed in this excerpt. It is also notable that there were few powerful moves. No explanations or justifications were given and no statements were rejected. L acquiesced uncritically to B's ideas (turns 3 and 10) and asked how to do it (turn 8). B answered with apparent confidence (turn 9), but seemed to find L's question in turn 21 challenging, and responded by expressing doubt. B then sought confirmation from an external authority (the notes).

Use of resources: The paper on which the problem was written was passed to and fro, as each tried to push the responsibility onto the other.

Discussion and conclusion

The most important indicators of the exercise of power in small group discussions appear to be the initiation of a new topic, either a negotiative event or off-task talk; and the endorsement, rejection or challenge of a statement by another student. Other indicators, such as gestures, body language, and use of resources have a modulating effect on these.

An interesting insight that has emerged from my analysis is an apparent parallel between powerful student moves and the traditional teacher's role. In teacher-centred classrooms, discourse has been observed to follow the tripartite initiation-response-feedback (I-R-F) pattern: the teacher asks a question, a student answers, and the teacher gives feedback by evaluating the correctness of the answer (Stubbs, 1983). A key characteristic of this pattern is that the teacher already knows the answer to the question.

We do not expect to find the I-R-F pattern in student-student interaction, except perhaps in the case of peer tutoring, where one student is recognised as more capable and has been explicitly assigned a "teacher" role. However, it is noteworthy that some indicators of student power or control carry echoes of the teacher role: the initiation of a new negotiative event has much in common with the initiation turn in the tripartite pattern, and the act of endorsing, rejecting or asking for an explanation is very similar to the feedback turn. Thus the student exercising power in either of these ways is, to some extent, behaving like a teacher.

In the first excerpt, R assumed control from the start of the activity. The first three turns of the excerpt (186-188) are a classic I-R-F sequence, even though R did not at that point know the correct answer to the question. In the other excerpt, control of the discourse was contested, and the parallel with the I-R-F pattern was not so clear.

The ideas presented here constitute the starting point for a framework for analysing power relations in collaborative groups. This now needs to be tested by applying it to more data, including transcripts of collaborative interactions from classrooms where the types of activities differ from those in the class described here. This will allow the descriptions of the indicators of power to be refined, and more added, as necessary.

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Year 5 students' available and accessible knowledge of decimal-number numeration

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This paper reports on a study of the decimal-number numeration knowledge held by 112 Year 5 students who had completed formal instruction in tenths and hundredths. It explores the interaction between available and accessible place-value and regrouping knowledge. Available knowledge was elicited through diagnostic test items; accessible knowledge was elicited through an error analysis of the students' numeration procedures used in addition and subtraction algorithms. The results showed that performance varied markedly between classes (indicating an instructional effect), that regrouping was more difficult than place value, and that there was generally a direct relationship between available and accessible knowledge (although there were many instances of high-available and low-accessible knowledge).

According to Greeno and Riley (1987), the fundamental question of understanding a cognitive procedure is whether a person performs with some understanding or whether the performance is rote or mechanical. Correct responses may be based on inappropriate knowledge (e.g., claiming 4.52 is larger than 4.3 because it has more digits), and incorrect responses may be based on appropriate knowledge (e.g., knowing that 43 –17 is 4 less 43 –13 but counting back incorrectly to get 27 instead of 26). Therefore, performance alone is an insufficient indicator of the presence or absence of understanding. Moreover, because "one can never be certain whether impaired performance is due to faulty or absent use of well-articulated (memory) knowledge or 'efficient' use of inadequate knowledge" (Cavanaugh & Perlmutter, 1982, p. 15), it is difficult to distinguish between *unavailability* (a knowledge limitation) and *inaccessibility* (a performance limitation).

Bransford, Sherwood, Vye and Reiser (1986) argued that, whilst acquisition (availability) of relevant knowledge provides no guarantee of access, poor performance can often be attributed to access failure (not unavailability). Schoenfeld (1985) concurred: "The issue for students is often not how efficiently they will use the relevant resources potentially at their disposal. It is whether they will allow themselves access to those resources at all" (p. 13). Therefore, as Prawat (1989) claimed, teaching for accessibility is a much more complex process than teaching for availability (i.e., knowledge acquisition).

Decimal-number numeration knowledge. In her study of students' acquisition of, and access to, the cognitions required to function competently with decimal numbers, Baturo (1998) tested 173 Year 6 students from two schools (different socioeconomic backgrounds) with a pencil-and-paper diagnostic instrument to determine the students' available knowledge of the numeration processes, namely, number identification, place value, counting, regrouping, comparing, ordering, approximating and estimating for tenths and hundredths. As a result of analyses of the students' performances and of the cognitions embedded in decimal-number numeration processes, Baturo developed a numeration model (see Figure 1) to show these cognitions and how they may be connected.

The model depicts decimal-number numeration as having three levels of knowledge that are hierarchical in nature and therefore represent a sequence of cognitive complexity. *Level 1 knowledge* is the baseline knowledge associated with *position, base* and *order*, without which students cannot function with understanding in numeration tasks. Baseline knowledge is unary in nature comprising static memory-objects (Derry, 1996) from which all decimal-number numeration knowledge is derived. *Level 2 knowledge* is the "linking" knowledge associated with *unitisation* (Behr, Harel, Post & Lesh, 1994; Lamon, 1996) and *equivalence*, both of which are derived from the notion of base. It is binary in nature and therefore represents relational mappings (Halford, 1993). *Level 3 knowledge* is the structural knowledge that provides the superstructure for integrating all levels and is associated with *reunitisation, additive structure* and *multiplicative structure*. It incorporates ternary relations that are the basis of system mappings (Halford, 1993).

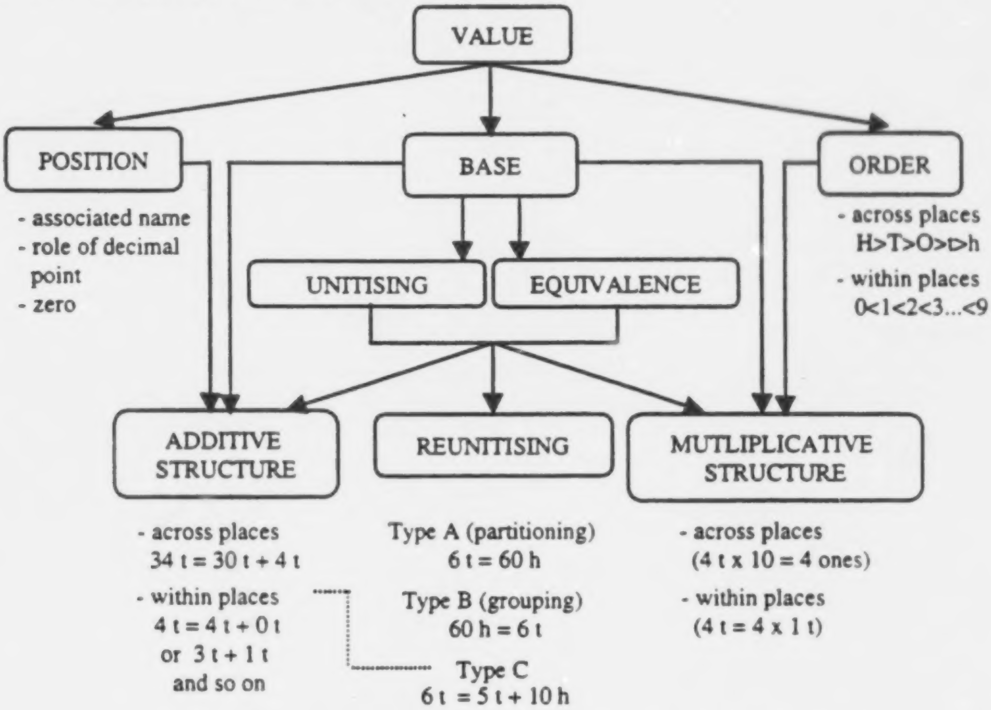
VALUE
POSITION
BASE
ORDER
- associated name
- role of decimal point
- zero
- across places
H>T>O>t>h
- within places
0<1<2<3...<9
UNITISING
EQUIVALENCE
ADDITIVE STRUCTURE
REUNITISING
MUTLIPLICATIVE STRUCTURE
- across places
34 t = 30 t + 4 t
- within places
4 t = 4 t + 0 t
or 3 t + 1 t
and so on
Type A (partitioning)
6 t = 60 h
Type B (grouping)
60 h = 6 t
Type C
6 t = 5 t + 10 h
- across places
(4 t x 10 = 4 ones)
- within places
(4 t = 4 x 1 t)

Figure 1. Cognitions and their connections embedded in the decimal number system (Baturo, 1997).

Within the model, *multiplicative structure* relates *position* and *base* into an exponential system (Behr, Harel, Post, & Lesh, 1994; Smith & Confrey, 1994) to give *value* and *order*. It is continuous and bi-directional and, for binary relationships, relates all adjacent places to the left through multiplication by 10 and to the right through division by 10. (For ternary relationships, it relates all adjacent-but-one places to the left through multiplication by 100 and to the right through division by 100.) It is the knowledge structure that underlies the concept of *place value*, the development of which is a major teaching focus in the primary school. This structure conflicts with the pattern of the place names which is syntactic in nature (about the ones), a conflict which is compounded if the decimal point is seen as having a position similar to ones (Baturo, 1997).

Also within the model, *unitisation, equivalence* and *reunitisation* underlie the decimal-number numeration processes of *renaming (*reunitisation Types A and B*)* and *regrouping (*reunitisation Type C*)*. These processes are particularly important in understanding decimal-number numeration beyond tenths (i.e., hundredths and thousandths). Decimal numbers can be named in standard form (e.g., 2.43 as 2 ones 4 tenths 3 hundredths), renamed (e.g., 2.43 as 2 ones 43 hundredths), and regrouped (e.g., 2.43 as 2 ones 3 tenths and 13 hundredths).

Available knowledge of decimal-number numeration can be directly probed, but accessible knowledge is mainly evident from situations where knowledge is applied. This paper reports on one study which compared Year 5 students' available knowledge of place value and regrouping for decimal numbers (limited to hundredths) as measured by diagnostic items with accessible knowledge as measured by error analysis of applications of this knowledge within addition and subtraction situations.

Method

Subjects: The study involved 112 Year 5 students from two schools each of which had three classes of Year 5 students. School A's clientele (Classes A, B, C) was drawn from a high socioeconomic background while School B's clientele (Classes D, E, F) was drawn from a low-to-middle socioeconomic background.

Instruments: The first instrument was a diagnostic test developed to cover all the numeration processes prescribed in the Queensland mathematics syllabus. These were categorised as *number identification* (identifying decimal numbers represented in pictorial, word and symbolic forms), *place value*, *regrouping*, *counting*, *comparing*, *ordering*, *approximating* and *estimating*. Because the numeration items were designed to probe available semantic knowledge, several different items were developed to assess each numeration component. The nonprototypic (Herhskowitz, 1989) items designed to create "expectation violations" (Schank, 1986) yielded insight into the robustness of the students' available knowledge of place value and regrouping (see Figure 2).

PLACE VALUE		REGROUPING	
Write the number that has:	______	Write the numbers.	
2 tenths, 5 hundredths, 4 ones	______	86 tenths 3 hundredths	______
19 ones, 2 hundredths	______	7 tenths 14 hundredths	______
7 ones, 4 tenths	______		
3 hundredths, 6 tenths	______		
8 tenths	______		
5 hundredths	______		

Figure 2. Test items to probe decimal-number place value and regrouping.

The second instrument comprised two computation tasks. The focus of these tasks was to determine the accessibility of the students' place value and regrouping knowledge, the availability of which was determined from the diagnostic tasks (see Figure 2). The addition task (23.5 + 3.67) was designed to probe the application of decimal-number place value knowledge. For example, it was anticipated that students with low available place value knowledge would merely align the digits from left to right or from right to left without any regard to the values of the digits. This task was thus seen as a means of determining whether students with high available place value knowledge would transfer this knowledge to a different task (and therefore indicating a metacognitive awareness as well as a procedural awareness of place value). Whilst there was also a regrouping component, this was not considered to be as cognitively demanding as the regroupings required in the subtraction task.

The subtraction task (8.2 - 1.76) was designed to probe accessible decimal-number regrouping knowledge with respect to knowing **when** to regroup (e.g., when having to subtract the 6 hundredths) and **how** to regroup prototypic instances (e.g., when having to subtract the 7 tenths). Whilst there was also a place value component to this task, it was anticipated that students with low available place value knowledge would either write the "larger number" (i.e., 1.76) on top (and therefore have no chance of a correct solution) or align the places from left to right, a syntactic procedure that would produce the correct alignment without any regard to knowing that like places should be aligned.

Procedure: The diagnostic test was administered, at the same time, to all students within each school with the tasks being read to students to overcome possible reading problems. The computation tasks were administered to the students following the test. Students were required to record their working for the computation tasks. Most students completed the test and the computation tasks in about 20 minutes.

Analysis: Individual and class performances (available knowledge from the diagnostic test) were collated with respect to place value and regrouping and then individual performance was categorised as *low, medium, or high*. The classification criteria for the 6 place value items and the 2 regrouping items were: *low* (place value – ≤2; regrouping – 0); *medium* (place value – 3 or 4; regrouping – 1); *high* (place value – ≥5; regrouping – 2). For the computation items, performance was analysed in terms of flow charts that showed all options with respect to place value and regrouping in the solution procedures. Comparisons were then made between each student's available and accessible place value and regrouping knowledge.

Results

Table 1 provides the class means for the place value and regrouping components of the test and the computation tasks. As shown in this table, there were major class and school differences within and between all components. (Independent Sample T-tests revealed that many differences were significant at $p<0.001$.) In particular, performances were more impoverished for subtraction than for addition.

Table 1
Class Means (%) for Place Value, Regrouping and Computation.

	School A			School B		
Items	Class A	Class B	Class C	Class D	Class E	Class F
Place value	58.9	44.9	78.9	31.0	28.9	54.7
Regrouping	40.8	37.5	43.6	28.1	42.6	40.9
23.5+3.67	73.7	68.8	62.5	62.5	35.3	81.8
8.2–1.76	15.8	37.5	31.3	04.2	00.0	36.4

Available knowledge: From the test results, students were classified as high, medium and low with respect to available place value and regrouping knowledge. Table 2 shows the number and percentage of students within each categorisation. All but one high regrouping student were also high place value students. Approximately one-third (23) of the high place value students revealed low regrouping knowledge.

Table 2 reveals a markedly different category distribution for available place value knowledge than for available regrouping knowledge. The number of high available regrouping students is significantly less than the number of high available place value students. This indicates the inherent complexity in regrouping – having high available place value knowledge appears to be an essential but not sufficient prerequisite for high regrouping knowledge.

Table 2
Number and Percent of Students in Each Available Knowledge Category with Respect to Place Value and Regrouping Knowledge.

	Available knowledge categories		
Knowledge	Low	Medium	High
Place value	37 (33.0%)	17 (15.2%)	58 (51.8%)
Regrouping	59 (52.7%)	42 (37.5%)	11 (09.8%)

Accessible knowledge: Details of the students' performances on the two computation tasks are shown on the flow charts in Figures 3 and 4.

Figure 3 reveals that, as expected, the most critical point in the solution of this addition task which involved decimal numbers with different whole-number "lengths" was in the alignment of the numbers. Only one third of the students knew to align and add like digits. Of these 77 students, 49 (representing 84.5% of their category) had exhibited high available place value knowledge, 12 (representing 70.6% of their category) had exhibited medium available place value knowledge, and 15 (representing 40.5% of their category) had exhibited low available place value knowledge. These results indicate that having high available place value knowledge is not a sufficient indicator of having accessible knowledge

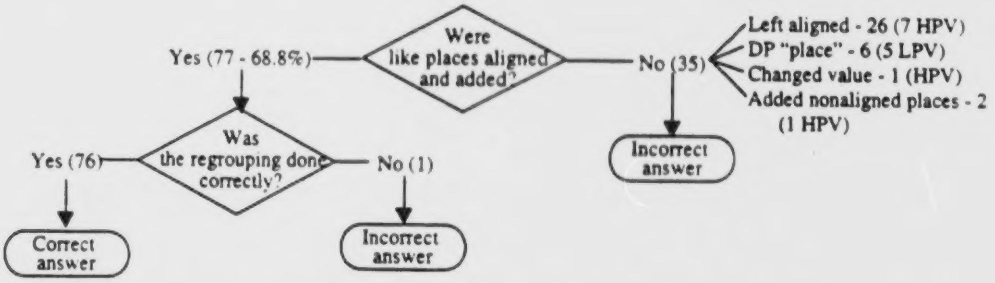
Yes (77 - 68.8%)
Were like places aligned and added?
No (35)
Left aligned - 26 (7 HPV)
DP "place" - 6 (5 LPV)
Changed value - 1 (HPV)
Added nonaligned places - 2 (1 HPV)
Incorrect answer
Yes (76)
Was the regrouping done correctly?
No (1)
Correct answer
Incorrect answer

Figure 3: Students' performances on the addition task (23.5 + 3.67).
(HPV = high available place value knowledge; LPV = low available place value knowledge.)

Figure 4 reveals that 12 students who had exhibited high available place value knowledge (HPV) were unable to access this knowledge when preparing the decimal numbers for subtraction. The HPV student who had aligned the subtraction numbers incorrectly had also aligned the addition numbers incorrectly. Figure 4 also reveals the complexity of regrouping (in knowing when and how to regroup) with only 21 of the 73 students (28.8%) who had prepared the numbers correctly being able to complete the regrouping component correctly. Of these 21 students, 4 (representing 36.4% of their category) had exhibited high available place value knowledge, 13 (representing 31.0% of their category) had exhibited medium regrouping knowledge, and 4 (representing 06.8%) of their category) had exhibited low available regrouping knowledge.

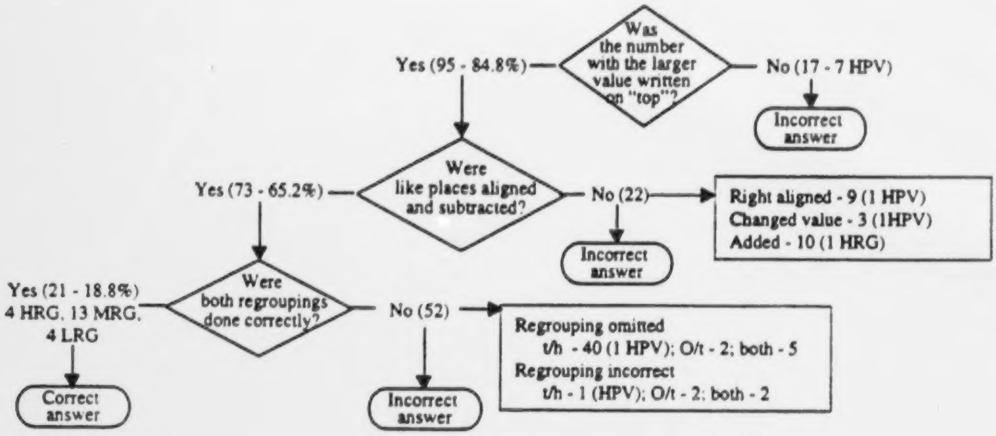
Was the number with the larger value written on "top"?
Yes (95 - 84.8%)
No (17 - 7 HPV)
Incorrect answer
Were like places aligned and subtracted?
Yes (73 - 65.2%)
No (22)
Right aligned - 9 (1 HPV)
Changed value - 3 (1HPV)
Added - 10 (1 HRG)
Incorrect answer
Were both regroupings done correctly?
Yes (21 - 18.8%) 4 HRG, 13 MRG, 4 LRG
Correct answer
No (52)
Incorrect answer
Regrouping omitted
t/h - 40 (1 HPV); O/t - 2; both - 5
Regrouping incorrect
t/h - 1 (HPV); O/t - 2; both - 2

Figure 4: Students' performances on the subtraction task (8.2 – 1.76).
(HPV = high available place value knowledge; HRG/MRG/LRG = high/medium/low available regrouping knowledge.)

Comparing available and accessible knowledge: Table 3 provides a cross-tabulation of students' available and accessible place value and regrouping knowledge with respect to the performance categories. It presents accessible regrouping data as Regrouping P (referring to the prototypic, that is, standard/"simple" regrouping) and Regrouping NP (referring to the nonprototypic, that is, involving an "understood" zero regrouping). Table 3 also contains reductions in the response totals for regrouping because incorrect setting up of the subtraction task resulted in removing the need for regrouping. It also includes the results of the 29 students (20 low, 8 medium, 1 high) who had not prepared the subtraction correctly but had encountered an instance where regrouping (mainly prototypic) was required. Of these 29 students, 15 (11 low, 4 medium) indicated that they knew when and how to regroup.

Table 3 shows a strong direct relationship between available and accessible place value knowledge, a modest relationship for accessible nonprototypic regrouping, but no relationship for accessible prototypic regrouping. For both regroupings, the relationship was not supported for high available knowledge. This may have been a consequence of the distribution difference between available place value and regrouping knowledge (see Table 2).

Table 3
Percent and Number of Students' Accessible Place Value and Regrouping Knowledge With Respect to the Available Knowledge Categories.

	Available knowledge categories		
Accessible knowledge	Low	Medium	High
Place value	45.9% (17/37)	70.6% (12/17)	87.9% (51/58)
Regrouping			
– RGP	62.7% (37/59)	78.6% (33/42)	63.6% (7/11)
–RGNP	10.2% (6/59)	42.9% (18/42)	45.5% (5/11)

Note. RGP = prototypic regrouping; RGNP = nonprototypic regrouping.

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Discussion and conclusion

Students' performance on place-value, regrouping and computation items varied markedly between classes and schools (see Table 1), but overall, performance was better on place-value items than regrouping (see Table 2). It was also evident that regrouping was much more difficult for subtraction than for addition (see Figures 3 and 4). The significant differences within and between classes and schools could be attributed to instructional experiences, particularly to certain surface forms that, for some classes, appeared to have been over-practised (e.g., inserting a "rightmost zero"). The test revealed that the prerequisite whole-number and fraction concepts and processes (e.g., Resnick et al., 1989) were impoverished in some classes. Effective instruction needs to focus on establishing the prerequisite knowledge and by ensuring that prior learning is connected to new learning. Special attention needs to be given to place value, particularly for decimal numbers of unequal length, and for regrouping, particularly for subtraction.

The results for regrouping warrant an investigation into the mathematical structure of both types of regrouping and into how students acquire and access both types, particularly when internal and rightmost zeros are involved. Figure 1 shows that regrouping is a Type C reunitising construct that has been classified as Level 3 knowledge (structural knowledge). Level 3 knowledge encompasses ternary relationships (Halford, 1993) which are more cognitively complex than the binary or unary relationships of Levels 2 and 1 respectively. Type C reunitisation is more difficult than Types A and B because the decomposition within the given unit links it to additive structure (another Level 3 knowledge type). The complexity of regrouping and the current curriculum trend promoting mental and calculator computation raise pedagogical issues. For example, should time-consuming pencil-and-paper computation be eliminated from modern curricula?

This paper takes the stance that there is a place for all types of computation. Calculators and/or mental computation should be employed when speedy and accurate calculations are required, and pencil-and-paper computation should be employed as a vehicle for promoting understanding of mathematical principles rather than procedural proficiency. For instance, underlying pencil-and-paper computation are mathematical principles (e.g., set inclusion which develops the understanding that like things only can be added and subtracted; the commutative and distributive laws) that apply across whole-number, fraction and algebra domains. The structural knowledge embedded in computational knowledge suggests that a study of computation (not merely computational procedures for "getting an answer") should be undertaken at the upper primary and lower secondary levels.

The analysis of the students' computations indicated the existence of rules that appeared not to have been identified by the literature. For example, several students appeared to align numbers from the left, irrespective of place value. As well, many students seemed to employ an "equalising" procedure to ensure that the given numbers had the same "length". Whilst most students inserted a rightmost zero (which did not change the value of the given number), some children inserted a zero internally (i.e., immediately after the decimal point) so that the value was changed. These latter students may employ a rule "zero has no value" no matter where it is placed.

Whilst the data indicated that there was generally a direct relationship between available and accessible knowledge (see Table 3), it also indicated what is evident to any teacher – that correct individual performance is possible without understanding and that availability of knowledge does not mean it will be used (accessed). Students who failed to access their available knowledge may have lacked awareness (a key metastrategy in Prawat's, 1989, theory of accessibility) of the role of place value and regrouping in computation.

The low-available knowledge students who "accessed" unavailable appeared to be following procedures routinised from overpractised whole-number procedures (Hiebert & Wearne, 1985), thus bypassing the need to access available semantic knowledge by replacing it with now available syntactic knowledge. It seems doubtful that these students demonstrated a semantic understanding of the role of place value in the addition operation and it seems more likely that they had applied the rule "line up the decimals". As well, there were medium- and high-available place-value students who did not correctly align the decimal numbers in either computation. Similarly, there were correct low-available and incorrect high-available regrouping students. In several cases, errors resulted from failure to activate a procedure rather than from incorrect execution of the procedure.

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Mathematics curriculum and development

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This paper explores curriculum development issues in nine regions in five federal countries. The focus is on the curriculum contents, the development process, and the role of teachers in development. The study explores alternatives that others may consider when reviewing their curriculum.

Introduction

In 1997 I compared curriculum in five federal countries—Australia, Canada, Germany, Switzerland, and the United States of America, where I looked at nine regional case studies. In these five countries curriculum development is a regional responsibility and the regions are comparable to New Zealand with respect to population and area. In three of the countries—Canada, Switzerland and the USA, the curriculum decisions are expected to be modified further at sub-regional levels.

This studies was motivated by my concern about the curriculum development process in New Zealand. In 1989 this process was restructured but consideration needs to be given to further changes within the new system. My focus is to provide alternatives for regions to consider when they are planning development priorities or processes.

I had a limited time in each of the countries and was concerned with official policies and the intentions of curriculum developers. I had informal interviews with curriculum developers in each of the nine regions using an open-ended interview schedule then I followed this up by talking with colleagues who were involved either the curriculum process or in teacher education that took cognizance of the emerging curricula.

Some details about the five countries and nine regions are in table 1.

Table 1: Details of nine regional case studies.

Confederation	(Notes)	Regional Case Study	Population	Area (km^2)
Australia (6 States and 2 Territories)		Queensland	3.3 m	1 730 000
		Victoria	4.5 m	230 000
Canada (10 Provinces & 2 Territories)	1	Manitoba	1.1 m	550 000
	1	Ontario	11.0 m	920 000
Germany (16 Länder or States)		Berlin	3.5 m	890
		Nordrhein-Westfalia	17.8 m	34 075
Switzerland (24 Cantons)	2	Geneva	0.4 m	246
	2	Zürich	1.2 m	1 660
USA (50 States and Num. Territories)	3	Massachusetts	6.0 m	7 800
cf New Zealand (1 Nation)		(No regional units)	3.5 m	270 000

(Notes: 1. Decisions are made at school district and individual school level; 2. Decisions are made at commune level, (3029 communes); 3. Decisions are made at local school board level)

In talking about curriculum one needs to define one's terms. I was thinking of a regional curriculum as being a curriculum document together with associated teacher guide material, any compulsory assessment initiatives, and any regulations that influence what teachers do. I acknowledge that there are many other levels of curriculum such as the school curriculum, lesson plans and the implemented curriculum that all more directly rely on the teacher as curriculum developer.

National Coordination

Although my study was of nine regions in federal countries, most regions were influenced by some forms of national coordination.

In Australia state initiatives are partly coordinated by the Curriculum Corporation which was funded by the Commonwealth to assist in the coordination of curriculum development, and with the preparation of curriculum materials. They published "A National Statement on Mathematics for Australian Schools" for the Australian Education Council (1991) which is equivalent to a national curriculum and although it has no legal status it influences what each state does. At Ministerial level the Australian Ministers of Education meet regularly and this also has a coordinating effect.

In the USA the 'Standards' (NCTM, 1989) are the equivalent to a non-official curriculum and have influenced all the states. Apart from the Standards which were partly funded by federal funds, the federal government also influences regional initiatives by providing funds for research and exerting an influence on education through regulations.

In Canada the responsibility for education rests with each province (or territory) except that the federal government provides funds for education of some groups including aboriginal people. In Canada too there is a national council of Ministers of Education and they organise some joint projects. In mathematics there has been a Western provinces consortium and an Atlantic provinces consortium working on 'guides' and these are influencing the work in the respective provinces.

In Germany education is the responsibility of each of the sixteen länder but the Ministers of Education meet regularly and decide on common thrusts for development. This is viewed by some as having a conservative influence on development because the smaller and less populated regions which tend to be less liberal dictate policies to some of the larger länder.

In Switzerland there is no Federal Minister of Education, there are 24 cantons (seven French-speaking, one Italian -speaking, and the others German-speaking) and each is responsible for their own educational system. Cantons that use the same language coordinate curriculum activities to some extent and two institutions for research and development have been founded, each serving more than one länder.

While broad cross-regional initiatives have been argued for in terms of consistency and cost-efficiency, they have been recognised, in Canada and Germany, as limiting experimentation, limiting teacher involvement in the early aspects of development, and limiting the development of the initial ownership of a project which is recognized as being a factor contributing to the success of a project.

Timing

During 1997 the nine regions were at different stages of mathematics curriculum development. In Berlin the primary school curriculum was being revised but the high school one which dates back to the new maths of the seventies was not seen as needing revision because it was not written in terms of detailed operational objectives, it was very general, and it was seen as a guide rather than as a restrictive statement. Westfalia aims to revise its curricula about every ten years or so although this is becoming more difficult and the upper secondary mathematics curriculum has not been revised since 1980. In Westfalia there are seven mathematics curricula written for the different levels of and types of school, and while these are theoretically all coherent, in practice each is revised as part of a response to an expressed need. Queensland and Geneva had revised their curricula in the 80s and had no immediate plans for reviews. In 1993 Zürich had brought out a new curriculum for all subjects, in 1995 Victoria had published its new mathematics curriculum while Massachusetts had been working from 1993 to 1997 on its first ever state math curriculum. Manitoba and Ontario had reviews in progress during 1997.

Influences

In most of the regions reviews were initiated by political pressures (back to basics, accountability, standards, and cost efficiencies) although where standing committees exist these were able to create political pressure. Curricula have been influenced by others in the same language and by neighbouring regional initiatives. The non-European regions were influenced by or expecting to be influenced by region-wide high-stakes assessment, while in the non-compulsory school years assessment for university entrance influenced most curricula (although it was often controlled by a different authority).

No regions suggested that curriculum development in mathematics was research driven although consideration was given to what occurs overseas, to comparative studies such as TIMSS, and to the informal research of reflective teachers who provide ideas of good practice and are asked to provide feedback on drafts. No regions had an evaluation of implementation although Massachusetts intended to evaluate its new curriculum to inform future changes—such evaluation was expected to include feedback from teachers and Universities. Some regions assumed that results from the planned testing of cohorts or samples of students at numerous grades may inform future reviews.

Another influence on curriculum is school systems and tracking. In Germany and Switzerland after four (or six) years of primary education, most students go to a three tiered high school system that focuses on academic, general, and practical programmes. In these regions there are a number of curricula written. The equivalent to this in other regions occurs in the last years of high school when students have a choice about the type of mathematics they study. In all countries multiple courses seem to be slowly being abandoned as the choice has been a school one rather than being made by the student.

Curriculum administration

The administration of curriculum and development varies, it is sometimes with the Ministry, and sometimes in a quasi-Government body (Curriculum Council or Board of Studies) or an institution. In some regions more than one authority shares responsibility and examples of this occur where the syllabus for University entrance is not determined by those who control the school curriculum. Most countries have independent schools and whether these have to fit with state curriculum depends on local decisions.

In Berlin, Manitoba, and Massachusetts an ‘umbrella’ document that outlines the aims of schooling and put mathematics into broad context alongside other subjects has been developed. Ontario is developing such a document and Queensland has recognised the need for one. In Zürich an alternative was used to coordinate subjects, they were grouped together into five learning areas and one school curriculum produced. The areas were: Man & the environment (Religion, Life skills or civics, Science, Home economics from yr 7), Language (German, Writing, French from yr 5, English or Italian from yr 8 or 9), Arts & music (Handicrafts, Art, Music, Textiles or Materials in yr 9), Mathematics (Mathematics, Technical drawing in yr 9), and Sport. This curriculum is intended to be taught in a cross-disciplinary way and give teachers more freedom.

To coordinate developments within mathematics some regions had a permanent subject committee with a rotating membership which was able to provide advice and guidelines for developments. In most regions the convenor of the development project was a full time curriculum officer or consultant/adviser who was a development expert, but in the other regions a seconded inspector, a university mathematics educator, or someone seconded as a curriculum officer headed the project.

All the regions had set up development committee(s) with a balance of teachers, consultants/advisers, university mathematicians, and mathematics educators participating and in all cases teachers were in the majority. The teachers in some regions applied to be on committees, in others they were nominated. The number of committees in the regions varied considerably from only one central committee in one region to a number so that different aspects would be considered and so that all school districts were represented.

The time involved from deciding to review a curriculum to complete implementation with resources in place seemed to vary up to ten years. Now the trend is towards months rather than years with three years being regarded as usual. This variation is influenced by:

- the number of people involved and what consultation occurs between groups,
- whether the teachers involved are released from their teaching during the project,
- the frequency of meetings for the committee(s),
- if draft curriculum are tested as they are in some regions,
- how much consultation occurs with schools,
- how many drafts are circulated to schools for feedback,
- whether curriculum development is separated from the teacher development process,
- whether resources (teachers guides or texts) are prepared and distributed.

Curriculum status varied from being directive to being a guide depending on the autonomy teachers, schools and school districts had. The trend was towards directive curricula—Massachusetts had not had a state document before, the present developments in the Canadian provinces were more directive than in the past, and curricula in Australia were becoming linked with initiatives that would become part of the de facto curriculum.

Some relevant issues were not addressed in the curriculum development process or by the mathematics steering committees, but seemed to be decided without debate for all curriculum by a small central group of officials or politicians, these were issues such as:

- what status should a curriculum have? should it be directive or a guide?
- should the curriculum be visionary or detailed? and how much detail?
- what philosophical or theoretical underpinnings does the curriculum have?
- what opportunities are provided for the appointed personnel on curriculum development committee to debate or consider some general curriculum and development issues?

Curriculum contents and structure

Emerging curriculum from the non-European regions emphasised the mathematical processes as well as content. These included logical reasoning, problem-solving, modelling, estimation, mental maths, communication, visualization, making connections, using contexts, applications, using technology. Where processes were emphasised it was usually in the introduction or in a strand of the curriculum, they were not embedded in the content—the integration of content and processes was left to teachers guides and texts.

Content varied across regions. In the English-language curricula there was an emphasis on patterns in algebra and on statistics and geometry in the lower grades; and on number sense and estimation (including topics such as number theory and discrete mathematics). The European curricula were more similar to English-language documents from the early 80s with even trigonometry and statistics omitted in the Zürich curriculum.

Curriculum strands (or topics) do not integrate mathematics and how the strands were linked to teaching sequences was seen as something that teachers should decide although teacher guides and textbooks often provided suggested sequences.

Ideas in the various curricula related to pedagogy included exploring, inquiring, connecting between topics and within maths, doing cooperative work, writing in maths, using open-ended and open response questions, and explaining and justifying answers. All these statements about pedagogy were only suggestions and the curricula all assumed that decisions about teaching should be made by the teachers.

Much of the curriculum content was listed in behavioural terms, but in Victoria the introductory section of the curriculum stated that the *outcomes are not to be considered as a checklist and an atomistic outcome-by-outcome approach should not dominate.*

A number of concerns about pedagogy arose in some regions, these included:

- content seemed to be based on logical rather than psychological considerations,
- teachers were seen as lacking knowledge of concept development and learning theories,
- using students' questions as a basis for teaching seemed not to be accepted,
- teachers did not seem to share experiences to improve their work, and
- discovery learning was given undue emphasis.

The way that curricula were organized varied from three-year topic groupings, yearly programs, and ability levels which were seen as spanning a number of classes. Planned teacher guides reflected these different groupings but I heard nothing to suggest that teachers had had any input into which organisational grouping they preferred. One respondent was quite critical and remarked how curriculum was defined in terms of the direct outcomes of formal education rather than recognising the complexity of schooling and the influence that school has on aspects of socialisation.

The contents of the curricula varied considerably, they all had introductions and then aims and/or purposes for studying mathematics. The introductions were followed by the general aims for each strand or section, a statement of the content for each section, and a guideline which provides some elaboration of this and includes links with other parts of the syllabi. Sometimes notes on learning activities and assessment activities were usually given. The detail in the curricula depended on whether they were seen as directive and controlling or guiding with the expectation that schools or districts would develop their own curriculum to suit their particular situations.

All the countries said that they had ethnic minorities and broad ability groupings to cope with. In Europe many of the students from these minorities went after four years schooling to the third (practical) tier of high school where it was up to teachers in those schools to cope with them, although at least in these schools the curricula did vary so the expectations of all concerned were significantly different. In some countries translations of curriculum documents were available for large language groups but not usually for smaller groups or for aboriginal people. In Canada the first nation people were seen as a federal not a provincial responsibility and in Victoria aboriginal people had been partially considered by the use of reference groups during the development of the curriculum. In general the needs of aboriginal people, of ethnic minorities, of refugees, and of the less-abled and the gifted were left for teachers to address.

Teachers' professional development

Related to curriculum implementation, catering for minority groups, new content, and new teaching strategies is professional development. My belief is that curriculum and teacher development should be integrated and be mutually dependent. This was not the situation in any of the nine regions and the traditional linear model with dissemination involving teacher development following development was the prevalent view. This was most obvious in the two Australian regions where the responsibility for the development of the curriculum and the implementation were controlled by different authorities.

In Germany teachers are regarded as being very well qualified. They spend from five to seven years in preparation, they are assumed to be professional, and therefore teacher development is their own responsibility—unfortunately, and in spite of the opportunity provided by teaching each day only between 8am and 1pm, most German teachers were not involved in professional development. However in Berlin the teachers from comprehensive schools were seen as adventurous, they had developed a database of teacher-prepared units of work which they freely share with each other as they face the problems of mathematics for all. These units reflect modern ideas of mathematics and learning and emphasise applications and contexts, they also integrate social values and students interests with the mathematics when possible. Gymnasia teachers have recently started to work with this group as in Berlin the Gymnasia now take not merely the elite 1% or 2% but closer to 40% and sometimes even 60% of the age cohort and teachers are facing the problems associated with teaching students from this wider range of abilities.
In Northrhein-Westfalia the institute that developed the curriculum also had responsibility for teacher inservice, but while there used to be some provision for teacher development this was being reduced due to the lack of resources for education.

Manitoba had teachers from all districts involved in curriculum development and came the nearest to the ideal with teacher and curriculum development being mutually dependent. In fact their development was based on the Western provinces protocol so the initial involvement of teachers had not been great. Their teachers expect to participate in 10 days of inservice or administration work each year while schools are closed and schools budget $450 per teacher for professional development. When possible inservice work related to the new curriculum involves modelling the way that teachers might work with it. The problem areas in Manitoba were seen as the new emphasis on mathematical processes and the lack of background in algebra and statistics of many elementary school teachers. Change was not expected within a year but was expected to take 4 or 5 years which did not fit well with political thinking which assumed that after the mathematics initiative there would be other subject reforms that would impact on generalist teachers. The initiative in Manitoba for professional development rests with superintendents and principals although resource people are available and the pressures of assessment and testing are making people aware of the need to make development opportunities available.

In Geneva the primary curriculum is supported by the "Didactic Services" who provide advice on teaching methods and on ways that the curriculum can be developed. For high school mathematics teachers the main foci for professional development are their professional societies and their in-school department meetings. Professional development is mainly during holidays as the expense of 'teacher relief days' is high.

In Massachusetts a recertification procedures requires all teachers to gain 120 points in each five years and these points (development hours) are divided further into 60 in general education and 60 in maths education for mathematics teachers. This pressures teachers to attend courses and workshops on the curriculum and provides a focus for courses which districts must ensure are available for teachers. The idea of compulsory professional development and recertification sounds fine but it is not all that successful in some other states, a number of educators are cynical about its effect, and concerns about the 'professional development industry' that is growing so that teachers can satisfy the requirements by attending courses but not changing their practice were voiced.

Increasingly the implementation of curriculum is seen as a school or school district responsibility because of a belief in local autonomy. In some cases this is a traditional belief, in others it is the rhetoric behind political moves which are reducing budgets and the responsibilities of regional authorities. Most regional offices were doing what they could to at least ensure people are identified as consultants and so that teachers were aware of holiday courses, university courses, subject association meetings, as well as district initiatives, and in Victoria professional development activities were also being delivered by satellite with simultaneous on-line links.

Resources

One way to assist the implementation of curriculum is by providing teacher guides and to ensure that suitable textbooks are available. Most of the English speaking regions were developing teachers guides (in print or CD form). Ontario had decided not to do so and in Westfalia it was felt that there was a conflict of interest with publishers. The teachers guides were not legally binding but gave detailed suggestions related to teaching methods, sample programmes, essential learning, links between topics and links across subjects, assessment, ability-based outcomes, and references to other suitable resources.

Textbooks were seen in many countries as the defacto curriculum. To make the most of this some countries have approval systems. In Australia schools are free to buy whatever texts they want and the situation is the same in Massachusetts even though half of the states in the USA have approval systems. Approval is required in Ontario while in Germany approval is also required although this is usually given unless the book is actually contradictory to the curriculum or it contains errors. In Geneva texts are paid for by the state and in Zürich texts are produced and issued by the canton's own publishing house and these are seen to provide an interpretation of the curriculum for teachers.

The provision of prepared texts and worksheets produced by experts was commented upon by one interviewee as deskilling teachers, he believed that each teacher should prepare their own resources if they are to really understand the curriculum.

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In contrast to these regional curriculum initiatives an alternative view was suggested for Canada by Taylor (1997) from Queen's University. He recognized the influence of textbooks in North America but sees the need for an extended problem-solving focus as used by some schools in Japan. His philosophy is summarized in his book by: *This is a collection of classroom problems; they are not meant to be simply inserted into the curriculum—they are the curriculum* (p 1).

Conclusion

Many regions begin curriculum revision without adequate planning. They allow politicians to suggest national coordination that means that local teachers are not heard and they accept unreasonable timescales instead of considering the change process.

While research did not play a significant part in curriculum development apart from teachers informal research, I was surprised that action research projects and exploratory studies as a form of developmental research were not being fostered.

Too many questions are not asked during curriculum development, questions about status, purpose, assessment, and so on. While accepting that these relate to all subjects, there was little evidence of teacher (or public) debate in the few regions where a document provided a framework structure for subject curricula.

The idea that some variation might be possible within a curriculum seemed not to be considered although in the Czech Republic (Kotásek & Svecová, 1995), with approved curricula a 10% to 40% variation is allowed in some subjects at school level and an individual teacher can make further changes of up to 33%.

The changing emphasis on mathematical content and on the processes that are emerging as school mathematics is redefined was to be expected, but the disregard for minority groups and ideas of ethnomathematics was a surprise for me.

Teachers value practical strategies more than learning theories, but most teachers in high schools have a good knowledge of their subject and less knowledge of pedagogy. Perhaps curriculum for them should emphasise how to teach rather than what to teach.

Each region I visited had some unique contribution to this study, my hope is that mathematics educators and curriculum developers are able to reconsider the development process and look towards making changes in the future.

Acknowledgements

I acknowledge the University of Waikato's help in this project, they enabled me to take study leave in 1997 and spend time in the nine regions talking with colleagues there.

I also acknowledge my data sources and thank my colleagues for their assistance. In some cases my interpretations of what was said may be erroneous—I have presented what I constructed from the data and accept total responsibility for misrepresentations.

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The Impact of an Early Number Project on the Professional Development of Teachers

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This paper reports the findings of an investigation that evaluated the impact of the Early Number Project conducted by the NSW Department of School Education in 1996. It includes information obtained from a questionnaire distributed to all participating classroom teachers. Generally teachers benefited from their involvement in the project both professionally and personally. The classroom-based model of professional development was determined to be a major factor in the success of the project.

An ultimate objective of any professional development program is the enhancement of student learning (Schools Council, 1990). However, a re-occurring criticism of projects intent on change and the professional development of teachers is the lack of concern for such outcomes (Hopkins, 1994). Recent reports concerning the professional development of teachers in Australia, have emphasised the fact that teachers consider the most effective and useful learning occurs within their schools—from their students and other teachers. Until recently, the notion of the school as an educative workplace has been largely ignored by those responsible for the professional development of teachers. This paper reports the findings of an investigation that evaluated the impact on the professional development of teachers during their involvement in an early number project that was based largely in the classroom (Bobis, 1996).

Background to the Study

In 1996 the NSW Department of School Education trialed an early number project (Count Me In) in 13 schools throughout NSW. The aim of the project was to develop the knowledge of K-2 teachers in early number with the ultimate aim of improving young children's mathematical abilities.

The project employed a work-based model of professional development, with mathematics consultants working in classrooms alongside teachers. Exactly how consultants became involved varied from school to school, but basically their role was to assist teachers with the implementation of the learning framework espoused by the Count Me In (CMI) project. Generally, this was achieved by consultants helping teachers assess the mathematical development of children in their class, and by helping them plan and implement developmentally appropriate learning and teaching experiences.

Design and Methodology

Materials and Procedure

Questionnaires: Questionnaires were sent to all teachers participating in the project at the end of term 3. It consisted of two parts. The first part was designed to collect relevant demographic and biographical details about each teacher and their school. It contained questions pertaining to school district, student population, teacher age range, teacher qualifications, years of experience and the like. The second part contained open-ended questions intended to elicit teachers' perceptions about the program's effectiveness in relation to their professional development and its impact on the learning outcomes of the students in their class.

The questionnaires were distributed by the mathematics consultants, completed anonymously and returned to the researcher by mail to maintain confidentiality of the people involved. Quantitative data was collated and presented in tabular form. Qualitative responses were analysed for emerging themes or categories. Once an initial identification of themes was made, data was further analysed with the help of the qualitative analysis program NUDIST (1994).

Results

Reporting of all responses to open-ended items on the questionnaires will give prominence to the individual voices of the people in question. First, a summary of the demographic and biographical data of participating teachers who responded to the questionnaire will be reported. This will be followed by an analysis of their responses to the open-ended items.

The Questionnaire

Twenty-six questionnaires were returned, representing approximately 70 percent of the total number of teachers involved in the Count Me In Project. A summary of the demographic and biographical data of participating teachers is presented in Table 1.

Findings reported from Part B of the questionnaire were obtained by analysing open-ended responses for patterns and key themes. Four major categories of responses emerged: participation, outcomes, responses and needs. Each of these categories will be discussed briefly.

Participation: Fifteen percent of teachers who responded to the questionnaire indicated that the final decision to participate in CMI was not made entirely by themselves. For these teachers, the final decision was made by the "principal", "other teachers" or by the "entire school" at a staff meeting. Another 11% indicated that their final decision was influenced by the "respect" and admiration they held for the district consultant that introduced the school to the project. The desire for professional development was the most often cited reason for participating in CMI with 65% of teachers indicating that their involvement was an opportunity for "professional development and to strengthen maths teaching skills" in the early childhood years. Generally, teachers thought the project would benefit them professionally by helping their "understanding of maths concepts to increase", by providing them with "ways of extending children" in their classes and by informing them of "how children learn and acquire number skills".

Thirty percent of teachers referred to personal reasons for deciding to participate in CMI. For instance, a number of teachers commented that they found the whole CMI concept "exciting" and were personally "interested in learning about any new programs or teaching strategies". Another teacher considered herself to be "bored" with teaching maths and was hoping to "get my teeth into something" to relieve the boredom.

Table 1 Demographic and biographical details of respondents to teacher questionnaire (n=26)

Category	Details	Percentage
Gender	Female	85.0%
	Male	15.0%
School Size *	100-200 students	11.5%
	201-300 students	23.0%
	301 + students	62.0%
Nature of Population **	High % NESB	19.0%
	High % Aboriginal	12.0%
	Predominantly Anglo Saxon	54.0%
	Low socio-economic	35.0%
	Middle Socio-economic	58.0%
	High Socio-economic	3.8%
Age range (years)	20-30	23.0%
	31-40	38.0%
	41-50	31.0%
	50+	8.0%
Teaching Experience (years)	1-5	8.0%
	6-10	19.0%
	11-15	23.0%
	16-20	38.0%
	21+	12.0%
Current Year Level	Kinder	34.5%
	Year 1	31.0%
	K/1	19.0%
	Yr1/2	11.5%
	K/1/2	4.0%
Years Teaching Current Level	1-3	38.0%
	4-7	31.0%
	7+	31.0%
Highest Level of Teacher Ed *	Diploma of Teaching	38.0%
	Bachelor Degree	38.0%
	Teacher's Certificate	15.0%
	Masters Degree	4.0%
Undertaking Further Teacher Ed	Yes	12.0%
	No	88.0%

* Not all respondents completed this question. ** Some responded to more than one category.

Responses: Teachers recorded their affective responses to most aspects of the project. In particular they considered events, key people and documents. For instance, a number of events that occurred throughout the project triggered mixed reactions amongst the respondents. The initial inservice day was generally viewed negatively, including 20% of respondents who considered "the aim of the project was not clearly outlined nor what was expected of us" and the fact that many found the "new terminology confusing" and too much to absorb in one day. Three teachers indicated that they would "like to re-visit the theory side of the project" now that they more fully understand its aims and how it was to operate. The second session, however, was viewed much more positively:

> The second session with n...(lecturer's name) was *excellent* - where we viewed videos of children being taught and discussed what they were 'learning' and how they were being 'taught' - but it would have been more useful earlier-on in the program.

The initial and final video taping sessions of the children for assessment was considered "time consuming" and "difficult" by 19% of respondents. Despite this, many of the same respondents also considered the assessment process to be "extremely useful" and "helpful" as it gave teachers "a chance to focus on individuals and assess their growth" and taught them "to focus on the strategies not the answers".

Due to the perceived benefits teachers gained from "professional dialogue" with colleagues, consultants and teachers from other schools, a plea for "more opportunities for schools in the district to get together" with their consultants was a recurring request. Similarly, time spent "sharing knowledge, problems and anecdotes" with their colleagues and teachers from other schools was viewed positively by 35% of respondents, with many expressing the need to extend this contact by arranging for teachers to observe the "successful practices and activities" of others. One teacher suggested that:

> Now that it has been run, I feel the next group would benefit from a teacher explaining the project initially, ...to give a better overall view of how it is run in the classroom.

Other events that teachers found valuable were the opportunities to not only observe math consultants teach, but to observe their own students engaged in learning, thinking and reflection:

> I found it fascinating watching children engaged in lengthy times of thinking and reflection. We gave them the opportunity and time to do this and watched amazed at their concentration.

As an 'event', the Count Me In Project in its entirety, received overwhelming and unconditional support from 81% of respondents:

> It's a great way to inservice teachers. Inservicing is always more powerful when a number of teachers from the same school can benefit from it. Thanks - I've thoroughly enjoyed it and I'm no longer bored.
>
> An enormous beneficial experience for all involved, especially as an STLD as it was used well with primary children who need a strategy and to be directly taught.

Positive comments about aspects of the project were made by all teachers despite their reasons for participating in the project initially. However, teachers who participated "at the request of the principal" were more likely to have reservations about some elements than those who felt that they needed the "professional development".

The role of key people in the numeracy project was perceived by teachers to be a crucial element. In particular, the "supportive consultant" was viewed by at least 3 teachers as a major determining factor in deciding whether to participate in the project or not. Furthermore, reference to "helpful discussions", "helpful advice from our consultant", the expert knowledge of consultants, and respect for a consultant's "professional judgement" pervaded responses to almost every question for 31% of respondents. The overarching perception of teachers was that the success of the project depended to a great extent on the consultant - both the relationship they had with them and their expertise was considered important:

> I have a lot of respect for n....'s (consultant's name) professional judgement and I valued n....'s interest in the project. ...It has been a lot of work and involvement, but very worthwhile. I do not believe it could have been successful without the tremendous input we had from n.....

The only negative comment that was made about a consultant indicated that "too many ideas from a very helpful consultant" resulted in 'overload' given the short time frame of the project and that "more time to develop a program of lessons with other participants would have been helpful".

Other people that figured greatly for 35% of teachers responding to the questionnaire were "colleagues" and "teachers from other schools". To a lesser extent, contact with "university lecturers" was also considered invaluable:

> I've learned new strategies from working with n....(consultant's name), and discussing ideas/activities with the other project teachers at other schools and how these activities fit into the learning framework.
>
> The half day sessions with colleagues and consultants was invaluable when we shared knowledge, problems, interesting happenings in our classrooms and spent some time watching children being assessed on the video.

Thirty-one percent of responses from teachers referred to documents such as the *Mathematics Syllabus K-6* (1989) or to documentation supplied as part of CMI that outlined the project's theoretical perspective and classroom activities supporting the Learning Framework. For example, a number of teachers considered that they had benefited from the "additional reference to the current syllabus" with 23% of the opinion that the Learning Framework gave them "more detail ...than I've learnt from the curriculum":

> Learning about the 'Learning Framework' - I can see/understand where individuals fit in, and therefore what I need to teach them in more detail and more specifically than I've learnt from using the curriculum.

At least two respondents indicated a need for more documentation from the project coordinators in the form of "lessons ...providing a step by step approach from using concrete materials ...to the abstract" or "activities which could be used to support the SENA test".

Outcomes: The final category in which responses fell referred to *outcomes* of the project. This category was by far the largest of those emerging from the data. Teachers referred to outcomes in a number of different areas. For the purposes of analysis of this report it is helpful to subdivide this category into outcomes pertaining to: teacher professional knowledge; attitudinal changes; classroom practice; children's knowledge and skills; and children's affective responses.

Teacher professional knowledge related to comments about content knowledge, knowledge of strategies and of how children learn. Almost all teachers expressed the notion that they now felt they had a "greater understanding of content and strategies children need and use". The project allowed some teachers to go "beyond what has been seen as 'kindergarten maths'". For those teachers already familiar with the content and strategies presented "..it has refocussed my attention on how to most effectively teach these strategies."

Many teachers felt that their attitude to teaching mathematics had already been quite positive and therefore did not change significantly as a result of their involvement in the project. However, 42% of teachers commented that they were now more "enthusiastic", "confident about teaching mathematics" or no longer "bored" by their teaching of the subject.

All teachers acknowledged that they had made changes to their classroom practice. While some commented on the increased "use of games" or the adoption of a more "hands-on" approach, others considered that they now encouraged "higher levels of thinking" from their children, taught "specific strategies", teach more to the "individual levels" of their children and encourage more discussion in the classroom by allowing "the children to discuss their methods a lot more and to learn from each other".

It was the view of every respondent that the children in their class had benefited by their involvement in the project. Sixty-two percent of teachers felt that the final assessment of the children confirmed their observations that there was increased "understanding of math across all ability groups". With the lower ability children developing "strategies previously undeveloped and higher ability students extended". However, a small number indicated that the less able children did not progress as much as the higher ability children. Thirty percent of respondents commented on the children's increased confidence, enjoyment and enthusiasm to solve problems and use large numbers. Many teachers noted that their students were able to "think", "reflect" and "to talk about maths" as a result of their involvement in the project.

Needs: Almost three-quarters of the teachers responding to the questionnaire considered there were shortcomings of the project. These were often expressed in the form of perceived *needs*. Comments that fell into this category were further divided into the four sub-categories resources, guidance, knowledge and contact. For instance, under the sub-category of resources, the need for more *"time"* to be allocated to almost every aspect of the project was made by 46% of respondents. Some thought that the "program needed to be extended beyond two terms so that the work begun can be continued". In particular, the request was made for additional time to train themselves to conduct the assessment sessions more effectively and that professional assistance be given to help with video-taping the children.

Another perceived need emerged in the form of guidance. Thirty-one percent of respondents expressed the desire for more clearly defined "aims of the project" so that they might know exactly "what was expected" of them. To a lesser extent, teachers indicated a need for more guidance in the form of "specific activities for children at each stage in the Framework", especially at the start of the project. Guidance or clarification was sought on almost every aspect of the project by at least one respondent.

In relation to the need for knowledge, two respondents indicated that they would have liked the opportunity to "re-visit the theory" once the project was established. Others wanted more "expert knowledge sooner" from their consultant .

Finally, the overwhelming positive reaction to contact with colleagues, teachers from other schools, consultants and university lecturers saw respondents wanting more opportunities for "professional dialogue". One teacher suggested that "the half days for planning could be planned with other participating schools to discuss what they are doing and get new ideas".

Summary of Questionnaire Data Findings

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In short, a number of findings have emerged from an analysis of the questionnaire data:

1. Generally teachers were positive about the overall outcomes of the project regardless of initial reasons for participating.
2. All teachers acknowledged that they had changed their classroom practice as a result of their participation in the project.
3. All teachers considered that they had gained knowledge relating to content, strategies, how children learn mathematics or that their prior beliefs about these things had been reaffirmed by their involvement in the project.
4. The project was also seen to have a positive impact on other staff members not involved in the project.
5. Teachers found contact with consultants, colleagues and teachers from other schools invaluable for their professional development and requested that more opportunities for this type of professional dialogue be provided.
6. The role of the consultants and the relationships they established with individual teachers was considered crucial to the success of the project.
7. All teachers considered that the project had positive cognitive and/or attitudinal outcomes for the children in their classrooms.
8. Approximately one-third of teachers thought that the aims of the project were not clearly articulated during the initial inservice day and that there was some degree of information 'overload'. However, subsequent meetings were considered more helpful once the project was in progress and teachers were more aware of their roles.
9. Teachers shared similar concerns about time required for certain aspects of the project, namely the video-taping of assessment segments.
10. Teachers requested more guidance or clarification, particularly in the form of documentation, for almost every aspect of the project.

Final Summary and Conclusions

An initial aim of this investigation was to determine the impact of the Early Numeracy Project on teachers. It was evident from questionnaire findings that generally teachers benefited from their involvement in Count Me In, both professionally and personally. Teachers increased their knowledge and understanding of mathematical content, of strategies and of how children learn mathematics. They changed their classroom practices by allowing their students to take more responsibility for their own learning, by providing more discussion time in math lessons and by encouraging their children to be more reflective.

Findings indicate that the classroom-based model of professional development was a major factor in the success of the project. Similarly, the role of the consultants was vital—as mediators between schools, providers of expert knowledge and mentors with whom teachers might engage in professional dialogue. Finally, contact with colleagues, teachers from other schools, consultants and lecturers was seen to be invaluable for all those involved in the project. Hence, providing more collegial group gatherings could be beneficial for the ongoing success of the Count Me In Project.

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Arithmetic, pre-algebra, and algebra: a model of transition

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Learning to operate algebraically requires assimilation of new mathematical concepts and procedures. Current literature identified a gap between arithmetic and algebra and proposed a pre-algebra level. This paper reports on a longitudinal study that investigated students' readiness for algebra, from a cognitive perspective, to determine what constitutes a pre-algebraic level of understanding. Thirty-three students in grades 7, 8, and 9 participated. A two-path model depicts the transition from arithmetic to pre-algebra to algebra; students' understanding of relevant knowledge is discussed.

Pre-algebraic and Algebraic Knowledge

Secondary students often seem unable to apply basic algebraic concepts and skills and do not appear to understand many of the underlying structures. This becomes evident when a distinction is drawn between performance and understanding as outcomes of instruction (Rosnick & Clements, 1980). Recent research has focused on the transition from arithmetic to algebra and the difficulties in developing algebraic concepts caused by a cognitive gap (Herscovics & Linchevski, 1994) or didactic cut (Filloy & Rojano, 1989). It is suggested that the gap/cut is located between the knowledge required to solve arithmetic equations, by inverting or undoing, and the knowledge required to solve algebraic equations by operating on or with the unknown or variable. Linchevski and Herscovics (1996) found that students could not operate spontaneously on or with the unknown and that grouping algebraic terms is not a simple problem. They argued that students viewed algebraic expressions intuitively as computational processes (cf. Sfard & Linchevski, 1994) and suggested that in teaching, instead of moving from variable to expression to equation, arithmetical solution of linear equations might be more suitable initially for learning to operate on or with the unknown. Filloy and Rojano (1989) believe such concerns indicate the need for an operational level of 'pre-algebraic knowledge' between arithmetic and algebra.

However descriptions in the literature of what constitutes a pre-algebraic level of understanding are unclear. For example, Herscovics and Linchevski (1994) consider that the cognitive gap defines a level of pre-algebra and regard this as "involving those intuitive algebraic ideas stemming from the presence of an unknown in a first degree equation" (p. 75). Linchevski (1995) provided an explanation for pre-algebra as incorporating substitution of numbers for letters and allowing students to build cognitive schemes through reflective activity and spontaneous procedures. In contrast to this, Bell (1996) proposed six hypotheses about algebraic thought. These included: resolution of complex arithmetic problems by step-by-step methods working from given data to unknowns or by global perceptions and use of multiple arithmetic relations; recognition and use of general properties of the number system and its operations; and use of a manipulable symbolic language to aid this work. We believe that these hypotheses are concerned both with pre-algebraic and algebraic thought and that a sound arithmetic knowledge base is essential to developing pre-algebraic concepts. Research has shown however that students' understanding of arithmetic principles is often inconsistent and can be a source of cognitive difficulties in acquiring such concepts.

Cognitive Difficulties

The Arithmetic Knowledge Base

Algebra has been depicted as emerging from arithmetic and constructed in terms of students' prior knowledge of symbols, operations, and laws which are extended into higher order abstraction. For example, students need to form a more abstract view of addition in algebra as an object in Sfard's sense (see Linchevski, 1995). The function of operations must also be extended and new knowledge assimilated in an algebraic framework. Booker (1987) exemplified this by stating that in order to solve equations, multiplication processes must include factorisation. Gallardo and Rojano (1987) reported difficulties students had in understanding algebraic principles that stemmed from an inadequate arithmetic knowledge base. These included difficulty with inversion of operations, the need to accommodate a move from operating vertically in arithmetic to working with equations presented horizontally in algebra, and the nature of equality. Linchevski and Herscovics (1994) conducted a study with grade six students which also found inadequacies in students' arithmetic knowledge base. They reported that students over-generalised order of operations, failed to perceive cancellation thus resulting in sequential operation of equations, and displayed a static view of the use of brackets. They considered all of these obstacles to be of a pre-algebraic nature.

Booth (1989) stressed the importance of students' understanding various structural notions in arithmetic. Herscovics and Linchevski (1994) examined this contention by analysing the knowledge required to solve $4 + n - 2 + 5 = 11 + 3 - 5$. They stated that students need to be able to use commutativity to obtain $(n + 4) - 2 + 5$ and associativity to perform $[(4 - 2) + 5]$. However MacGregor (1996) suggested that students possess an unsure understanding of the commutative law while Booth (1988) reported students believe that division, like addition, is commutative. A sound understanding of the distributive law is also essential for algebraic functioning (Demana & Leitzel, 1988). Students' conception of equals has been documented (Herscovisc & Linchevski, 1994; Kieran, 1981; Linchevski, 1995) as indicating an operation to be performed on the left of the equal sign with the answer appearing to the right. Linchevski argued from a psychological point of view, that operating algebraically requires students to move from a unidirectional mode of reading an equation to multi-directional processing of information. Clearly, arithmetic principles are multifaceted; if they not well understood they may result in cognitive difficulties for students developing new algebraic skills.

Unknowns and Variables

The move from arithmetic to algebra also requires students' conceptions of operations performed on numbers to change so that the concept of operating on variables may develop (Filloy & Rojano, 1989). Conceptual obstacles in interpreting letters in algebra have included a lack of understanding concatenation. Herscovics and Linchevski (1994) found only 36% of 14 year old students gave a correct answer when asked to add 4 onto $3n$. They also reported that seventh graders made errors when substituting a number for n in $3n$ such as writing 32.

We consider that understanding of the unknown and solving to find the unknown in an equation constitute, in part, a pre-algebraic level of understanding. Panizza, Sadovsky, and Sessa (1997) suggest that the notion of unknown may become an epistemological obstacle when trying to conceptualise the notion of variable. However others, such as Graham and Thomas (1997), maintain that allowing students to gain an appreciation of letters as labelled stores will help develop an understanding that will improve assimilation of later concepts. This notion has significant implications in light of Ursini and Triguero's (1997) finding that college students had difficulty discriminating between variable as unknown and variable as generalised number. They proposed understanding of variable as unknown implies: recognising and identifying in a problem situation the presence of something unknown that can be determined by considering the restrictions of the problem; the ability to substitute for the variable, the value or values that make the equation true; and determining the unknown by performing the required arithmetic and/or algebraic operations.

A Sequence for Learning Complex Equations

Complex linear equations in algebra such as $2x+3=11$ include three crucial components: an equals sign, a series of more than one operation, and a variable 'x'. We are describing these equations as complex, because they include more than one operation, as opposed to binary operations such as $x+5=6$. We propose a two-path model for

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learning complex algebra where binary arithmetic (2+3=5), complex arithmetic (35÷7+8=13) and complex pre-algebraic operations [$3(x+7)=24$] are necessary components of one path and binary arithmetic (2+3=5), binary pre-algebraic ($x+7=16$), and binary algebraic ($x+y=12$) operations are necessary components of a second path. This means that understanding binary operations, such as $2x$ and $x+3$, should be a prerequisite to understanding $2x+3=11$ as should application of operational laws to series of operations. Additionally, we suggest that equations such as $x+7=16$ require solution procedures of a pre-algebraic nature which, at the lowest level, comprise use of inverse arithmetical procedures to find the unknown. The two-path model also assumes that learning linear algebraic equations will be facilitated by understanding isomorphic structures in complex arithmetic. The developmental literature (Collis, 1974; Sfard & Linchevski, 1994) supports this sequence in that it suggests acquisition of pre-algebraic and algebraic concepts in the following order: one occurrence of the unknown in binary operations, a series of operations on and with numbers and the unknown, multiples of the unknown, acceptance of lack of closure and immediate solution with a series of operations on the unknown, and finally relationships between two variables and operations on them.

The purpose of our study was to explore students' early understandings of algebraic concepts as they moved from arithmetic to algebraic. This was to determine: (a) the validity of the two-path model of sequential development of algebraic understanding and (b) what constituted a pre-algebraic level of understanding. Results from pilot work were published in Boulton-Lewis, et al., (in press), and for the pilot work and the first year of the study in Boulton-Lewis et al. (1997) and Cooper et al. (1997). This paper presents results for the three-years of the longitudinal study.

Method

Sample

The sample comprised 33 students who were tested in the first year, in grade 7, in four state primary schools in Brisbane. These were feeder schools for the high school where, in the second and third years of the study, students were in grade 8 then grade 9. Generally these schools were in a middle socio-economic area. Interviews were conducted with the grade 7 students before any formal algebra instruction took place and with grade 8 students after they had received instruction in operational laws, use of brackets, and solution of arithmetic word and number problems. Grade 9 students were interviewed after they had learned about an 'unknown' in a linear equation and solution of a linear equation using balance procedures.

Tasks and Procedure

Students were presented with expressions and equations written on cards and asked questions that investigated: commutative (x, -, +, +; 35 ? 76=76 ? 35) and distributive (6 x 13 = 60 + 18) laws; inverse operations (5x71=355, 355 ? 5 = 71; 64-29=35, 35 ? 29=64); order of operations [32+(12x8)+3]; meaning of equals in an incomplete equation (28÷7+20=) and a complete equation (28÷7+20=60-36); meaning of unknown (+5=9; $x+7=16$; $3x = 12$) and variable (+5; $3x$);); and solution of linear equations [$3x+7=22$; $3(x+7)=24$]. Students were interviewed individually and videotaped. They were encouraged to complete each task, however, if they could not the interviewer proceeded to the next task.

Analysis

Interviews were transcribed and analysed to identify key categories. The NUD*IST program (Richards & Richards, 1994) was used to classify response protocols under these categories and further sub-categories. Responses for laws, inverse operations and order of operations were categorised as *satisfactory* or *unsatisfactory* as a basis for learning algebra. Responses for the other tasks were categorised as *inappropriate*, indicating a lack of knowledge required for the task; *arithmetic*, focussing on arithmetical procedures and numerical answers; *pre-algebraic*, evidencing understanding between arithmetic procedures and intuitive algebraic ideas (Herscovics & Linchevski, 1994) and use of inverse procedures; and *algebraic*, evidencing recognition of relationships expressed in simplified form and use of general properties of the number system and its operations (Bell, 1996).

Results

The summary results below will be illustrated by Tables and examples of responses at the conference presentation.

Commutative and Distributive Laws, Inverse Operations and Order of Operations

In grades 7 and 8 the majority of students (19 and 17 respectively) could not explain *commutativity* of addition and multiplication satisfactorily. However by grade 9, 25 students gave a satisfactory explanation for commutativity. The majority of students in grades 7 (21) and 8 (17) could not give a satisfactory explanation for the *distributive* law and while a substantial number of students (14) still could not explain this law in grade 9, 19 students were able to provide a satisfactory explanation. *Inverse operations* were explained satisfactorily by the majority of students in each grade (26, 30, and 33 respectively) and by grades 8 and 9 most students explained *order of operations* satisfactorily (26 and 23 respectively) compared with only nine satisfactory explanations in grade 7.

Meaning of Equals

In each grade, the majority of students explained '=' in $28 \div 7 + 20 =$ as find the answer. Only one response in grade 8 and three responses in grade 9 evidenced knowledge that '=' denoted an equivalence relationship by stating that both sides had to be equal. For '=' in $28 \div 7 + 20 = 60 - 36$, the majority of responses moved from arithmetic in grade 7 when 19 students stated equals meant the answer, to arithmetic (12) or algebraic (12) in grade 8 as students explained equals as either the answer or denoting equivalence, to algebraic in grade 9 with most students (19) explaining equals as equivalence or showing a balanced equation.

Meaning of Unknown and Variable

The majority of students in each grade indicated that , in $+ 5 = 9$ (16, 22, and 21 respectively), and x in $x + 7 = 16$ (18, 24, and 26 respectively) represented an unknown number. However when x was presented in $3x = 12$ in grade 7 most students (18) did not know what this meant and gave an inappropriate explanation. In grade 8 most students explained concatenated x either arithmetically as a times sign (12) or pre-algebraically as an unknown number (12). In grade 9 most students' (25) explanations for x in $3x = 12$ were pre-algebraic stating that x was an unknown number.

For meaning of variable in grade 7 most students (18) stated pre-algebraically that in $+ 5$ represented an unknown number; another eight students stated algebraically it was any number. Five students gave inappropriate responses and two stated it was the answer. In grades 8 and 9 the majority of students stated pre-algebraically that was an unknown number (15 and 18 respectively) or algebraically that is was any number (14 each grade). In grade 9 there was only one arithmetic response and this indicated was the answer. For x in $3x$ most students in grades 7 (19) and 8 (15) responded arithmetically that it was a times (multiplication) sign. However by grade 9 the majority of students (17) stated pre-algebraically that it was an unknown number and a further 10 students responded algebraically that it represented any number.

Solution of Linear Equations

The majority of students in grades 7 (14) and 8 (13), as one would expect, did not know how to solve $3x + 7 = 22$. Eight students in grade 7 and 10 students in grade 8 used inverse arithmetic processes to find what they believed was missing after x because they interpreted x as a 'times' sign. Nine students in grade 7 and 10 students in grade 8 used inverse processes to solve for x which was categorised as pre-algebraic. By grade 9

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most students (23) solved $3x + 7 = 22$ pre-algebraically by using inverse processes. Two students did not know how to solve the equation and two used an incomplete pre-algebraic balance method which entailed balancing the equation by taking 7 from both sides, then dividing 15 by 3, rather than dividing each side by 3. Six students solved by using a complete balance procedure which was categorised as algebraic.

For $3(x + 7) = 24$ the majority of students (27) in grade 7 did not know how to solve the equation, while six students used a pre-algebraic inverse procedure. By grade 8 the majority of students still did not know how to solve the equation, however 12 students did use a pre-algebraic inverse procedure. Six students used arithmetic processes: two were inverse and depended on finding the space after the x and four were trial and error. By grade 9 most students (19) used pre-algebraic inverse processes to solve $3(x + 7) = 24$. Another four responses were pre-algebraic: three were incomplete balance and one was the incorrect balance method. Six responses were inappropriate, one student used a trial and error arithmetic process, and three students used a complete balance procedure which was categorised as algebraic.

Discussion

It was not until grade 9 that most students had sufficient understanding of the commutative and distributive laws to apply these to linear equations. These findings attest to MacGregor (1996) and Booth's (1988) contention that students have difficulty in understanding the commutative law. It also indicates a need for explicit instruction in these laws, in particular as Demana and Leitzel (1988) maintain that a sound understanding of the distributive property is essential for algebraic functioning. The majority of students displayed a satisfactory understanding of inverse procedures and correct order of operations by grade 8. Herscovics and Linchevski (1994) include understanding the order of operations as indicative of arithmetic functioning. Overall these results showed that it was not until grade 9 that most students displayed a satisfactory understanding of these arithmetic principles to apply them to algebra. Even then some students exhibited unsatisfactory understanding of each principle, except inverse operations, and would require instruction to facilitate understanding before they could apply them in an algebraic situation.

For equals in the incomplete equation, the majority of students each year indicated an arithmetic understanding by stating it meant find the answer. For the complete equation, understanding of equals moved from arithmetic in grade 7, to arithmetic or algebraic in grade 8, with most students in grade 9 stating algebraically that '=' denoted an equal or balanced relationship. However in all three grades almost one third of the students interpreted '=' pre algebraically. Kieran (1981) noted that students require an equivalence understanding of equals to operate algebraically. By grade 9, 19 students demonstrated an equivalence understanding, however 14 students were still operating at either a pre-algebraic or arithmetic level. This suggests that while students' knowledge of '=' had developed, there was still a substantial number of students who did not understand '=' in an algebraic sense and would need to learn the concept of equivalence. Providing explicit instruction of equals at a pre-algebraic level, that is each side is the same, may bridge the gap between arithmetic and algebraic understanding of equals.

Most students, over the three years, knew that in the expression and equation represented an unknown number. In the expression this is indicative of a pre-algebraic level of understanding as could be interpreted algebraically as representing 'any number'. In grades 8 and 9, 14 students did explain , in + 5, as any number and as understanding emerged in grade 9 some said it was a variable. These results indicate that understanding initially as an unknown number appears to be a suitable foundation from which to introduce the concept of any number or variable. Similarly by grade 9 most students explained x (not concatenated) in the equations as an unknown number.

Understanding of x in $3x$ was a more cognitively demanding task. Most students in grades 7 and 8 did not understand concatenated x and hence could not solve the linear equations. Allowing students opportunity to experience letters as labelled stores earlier in arithmetic instruction, as Graham and Thomas (1997) suggest, may foster intuitive understanding of concatenation and subsequent algebraic calculations. In grade 9, after students had been instructed in concatenation and use of inverse or balance procedures to solve an equation, most chose to use inverse pre-algebraic procedures to solve linear equations; a smaller number of students used balance procedures successfully. Sfard and Linchevski (1994) view the process of solving to find an unknown by reversing procedures or backtracking, as early algebraic thinking. We suggest that this solution process would be more appropriately placed at a pre-algebraic level of functioning.

Conclusion

Results for the three years of this study support the case for focussing on an explicit pre-algebraic level of understanding. This was particularly evident in students' unsatisfactory explanations of commutative and distributive laws, explanations for equals in the complete equation, and x in the expressions, and solution of the linear equations. The findings also support the sequence of instruction, as proposed in the model, that understanding of binary operations such as $3x$ is a prerequisite to solution of complex algebraic equations. We suggest that as arithmetic procedures are applied intuitively they should be taught explicitly in order to provide a sound basis for pre-algebraic instruction. Our findings indicated certain inadequacies in students' arithmetic knowledge base. In particular we feel there is a need for explicit and prolonged instruction in commutative and distributive laws and that letters could be introduced earlier, in arithmetic, to represent labelled stores. Finally we propose that pre-algebra should include instruction in: operational laws; equals as equality of sides leading to equivalence; solution of binary and complex equations using inverse procedures; use of letters to represent unknowns as distinct from variables; extensive use of 'x' as 'times' between an unknown and its coefficient; concatenation; and should be based on students' arithmetic as well as intuitive algebraic knowledge.

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The *invisible wall* project: problem solving processes of pupils who work on problems with a goal which can not be reached

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The *invisible wall project* analyses problem solving processes of children in grades 3-4 and 8-9. By now, we have transcribed more than 160 interviews; as a consequence, the qualitative interpretive methods were complemented by quantitative methods. Besides analysis of *difficulty of tasks* and *learning while being tested*, we describe the *process of problem solving* by *action profiles*. Using the method of *divergent coding*, we introduce interpretive methods to better understand action profiles.

The *invisible wall* project

Since 1992 M. Stein has been working - together with several groups of student teachers - on problem solving. The focus of the research is on such components of problem solving ability which are *not* subject of mathematics lessons. As a consequence, the project does not deal with word problems and other Aclassical problems@. The central idea of the research is to use sets of tasks which are all *unsolvable* which means they have a goal which can not be reached. The unsolvability, however, is of a kind which can be understood even by younger children (e.g.: try to find exactly 4 different numbers out of the set {1, 2, 3, 4, 5} which give the sum 9). Subsequently, we shall use the term *impossible task* as well. (See STEIN 1998, this volume, for more informations)

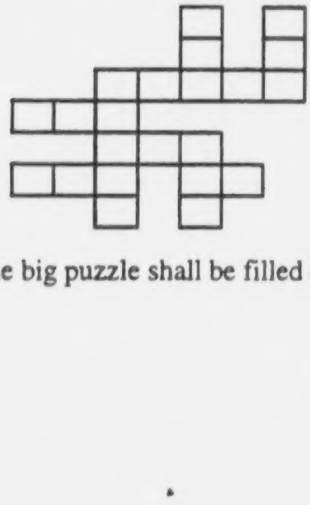
In the first part of the project elementary components of problem solving behaviour which are actually used by younger children were identified (the methods and some of the results are described in STEIN 1997). The search for those components was organized as search for noticeable patterns in the subject=s behaviour. The impossibility of a solution acts as an **Ainvisible wall@** for the pupils' actions: the pupil tries to solve the task, gets stuck ("bounces against the invisible wall"), and tries a different solution.

In the present stage of the project the authors try to describe profiles of problem solving processes. Funded with a grant from *Deutsche Forschungsgemeinschaft*, we have protocolled and fully transcribed app. 160 interviews with pupils of grades 3-4 and grades 7-8. This material is now open to analysis by *quantitative* methods.

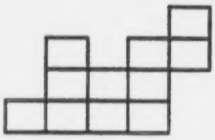
The tasks

We work with a range of different puzzles. In this paper, we concentrate on the following two puzzles:

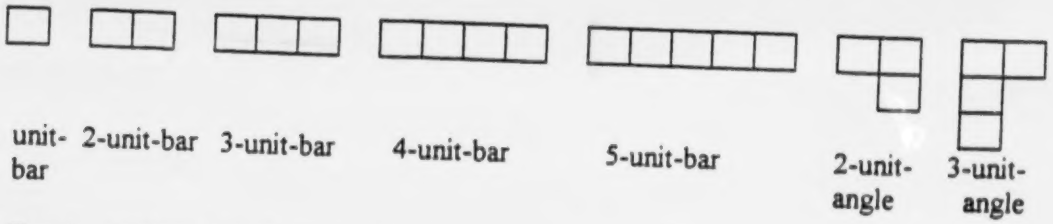
Puzzle 1 „big puzzle“



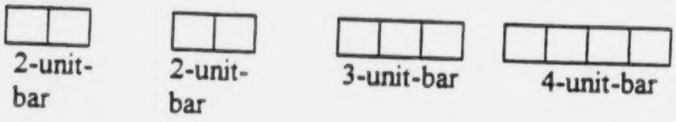
Puzzle 2 „small puzzle“



The big puzzle shall be filled exactly with the following parts (each is given once).



For the small puzzle, there are the following parts:



Design of the study

All problems are solved by groups of two pupils. The pupils are informed that some of the tasks can be solved, other tasks not, and that they have to find out what is the case and why.

As has been said, the search for components of problem solving behaviour is a search for noticeable patterns in the actions of the children. We do *not* assume, however, that such a pattern is guided by a consciously applied "strategical insight". If a teacher - or, in our case, the interviewer - starts asking questions about the behaviour, the pupils may discover the strategical implications of their behaviour and start talking about it as if they had planned their proceedings from the very beginning.

As a consequence, the interviewers *only watch the pupils work*. When the pupils *say* that the task can not be solved, or *show some "suspicion"* the interviewers ask whether there might be other ways to solve the task. This may be repeated once. Only at the end of the interview the interviewers are permitted to talk with the children about their understanding of the situation and about the process of solution.

The interviews are filmed with a video camera. Every action of the children is protocolled. The final transcription has the character of a script for a movie which allows to replay the interview.

The script is interpreted turn by turn (MAIER 1991) by a team of interpreters which includes the interviewer. The behavior of the children is analysed under a broad range of aspects. In many cases there will be more than one interpretation of the same behaviour. The same action may be interpreted, for instance, as a consequence of a social conflict between the two children, or may be seen as influenced by gestalt operations or be understood as guided by explicit use of heuristic strategies.

The tasks were presented to pupils in grades 3/4 and to pupils in grades 7/8. In grades 3/4 we tested 37 pairs of pupils of all abilites, from 8 different schools. In grades 7/8 we tested 46 pairs of pupils of all abilites, from 7 different schools.
Each pair of pupils got 4 tasks, the first two of them always were solvable. After that, each pair of pupils got one big and one small puzzle. The order (big puzzle first, or big puzzle as second task) was randomly assigned to the pairs of pupils.

Categorization of the final argumentation

At the end of each interview the pupils are encouraged to give reasons for their opinion that the task can not be solved. It may be assumed that the answers are an indicator for the pupils' understanding of the unsolvability of the task. We have developed categories for grading the tasks by the completeness and exactness of the answers. By the results, we can get informations about the difficulty of the tasks.

The answers are classified using three different categories:

Category I Systematic Reasoning or mention of the relevant fact.

It is said that the two big pieces are essential for the unsolvability of the task.

Examples: "The 5-unit-bar and the 4-unit-bar have fixed positions on the puzzle. But, if they are laid down, I have no place for the 3-unit-angle." - „It can not be done because the 5-unit-bar can be laid down in two different positions only."

Category II Modification of basic assumptions

The pupil makes comments about necessary modifications of the rules or the pieces of the puzzle "to make it solvable".

Example: "If I could cut the 4-unit-bar into two pieces, I could solve the task."

Category III Mention of isolated facts

The pupils mentions some facts which he observed as obstacles for the solution of the task.

Example: "There is always one piece left."

The classification of the pupils' answers by those categories was done by one of the authors. A choice of tasks and the coding rules were given to two independent coders. The results of the three coders were analysed using the Q-concordance-test by Gebert and Lienert (1971).

Difficulty of tasks

We assume that pupils have understood the unsolvability of the task if they are able to explain systematically why the task can not be solved (Category I). Pupils who gave answers in categories II or III *may* have understood why the puzzle is not solvable, but they seem not to be able to put their insight in good words. So, we take the numbers of answers in category I as an indicator for the difficulty of the tasks.

We ask:

- are "small" puzzles easier than "big" puzzles?
- do pupils in grades 3/4 have more difficulties with the tasks than pupils in grades 7 to 9?

The zero hypothesis is always, that there is no difference. The significance level is 5%.

We get the following results:

small puzzles

	grades 3/4	grades 7/8
I	23	30
II / III	14	16

Table 1

big puzzles

grades ¾	grades 7/8
15	28
22	18

Table 2

There are no significant differences between younger and older pupils in their respective ability to talk about the unsolvability the tasks (Altogether, even the older pupils do not show a convincing performance).

Learning in the test situation

While working on a task, the pupil considers different aspects of it. Though he/she may not be conscious of it, his/her considerations may lead to learning effects.

As was said in the first section, every pair of pupils got two unsolvable tasks, a small puzzle and a big one. The presentations

- first small puzzle, then big one
- first big puzzle, then small one

were (nearly) equally distributed.

We now analyse whether the second unsolvable puzzle was solved better than the first unsolvable puzzle.

In grades 3/4 we find the following situation:

	first task big puzzle	second task small puzzle
I	7	10
II / III	9	6

Table 3

first task small puzzle	second task big puzzle
13	8
8	13

Table 4

In grades 7/8 we find the following situation:

	first task big puzzle	second task small puzzle
I	14	20
II / III	11	5

Table 5

first task small puzzle	second task big puzzle
10	13
11	8

Table 6

Tables 3, 4 and 6 show no significant differences between the groups. Table 5 shows a high increase in quality from first task to secons task. However, it does not pass the χ^2 - test for significance. An analysis using the exact test of Fisher shows, however, that the probability to get by chance this or an even more extreme result, is not much higher than 5%.

Since table 3 shows an increase in quality from first task to second task, we can get some additional information by putting both tables together (though this was not part of the original hypothesis)

	First task big puzzle	2nd task small puzzle
I	21	30
II/III	20	11

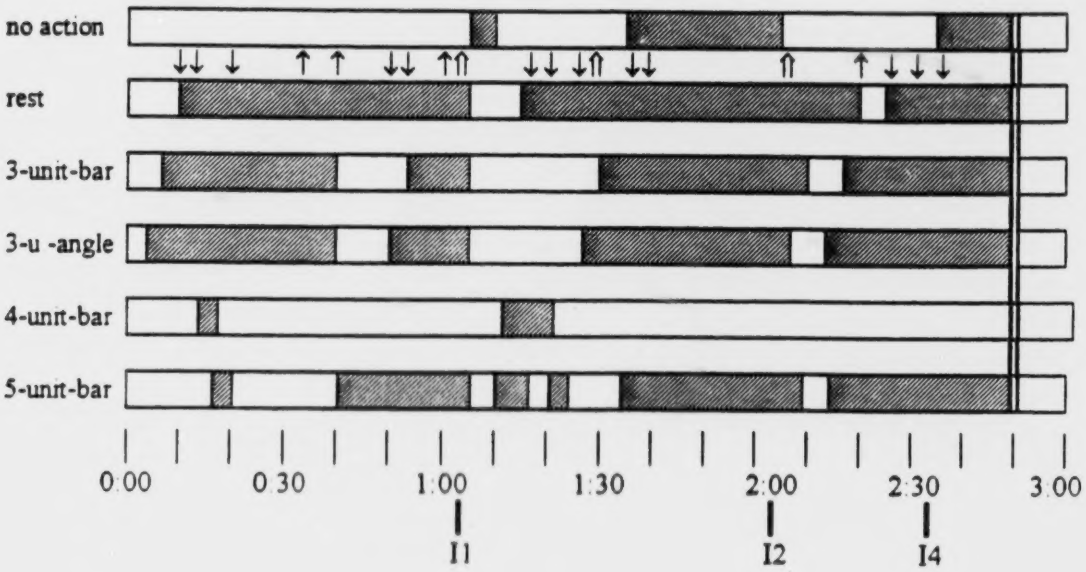
The increase in N leads to a table in which the inincrease in quality from 1st task to 2nd task is significant on the 5% - level.

Action profiles and divergent coding

Looking at the results above we have to take into account that we do not know whether a pupil who gave an answer in category III *really* did not understand the unsolvability of the task. It is possible that during his/her attempts in solving the task he/she acquired a good insight in the unsolvability, but found it so "self-evident" that he/she saw no necessity for giving a systematic argument. If we have pupils who gave "good" explanations in Category I, we are interested to find out where the insight in the reasons for the impossibility of the task happened.

To get more and better information about the connections between problem solving processes and the final "outspoken" answer, we analyse the action profiles of the problem solving processes by divergent coding (see STEIN 1998, this volume). Within the limits of this report, we can discuss three examples.

Example I

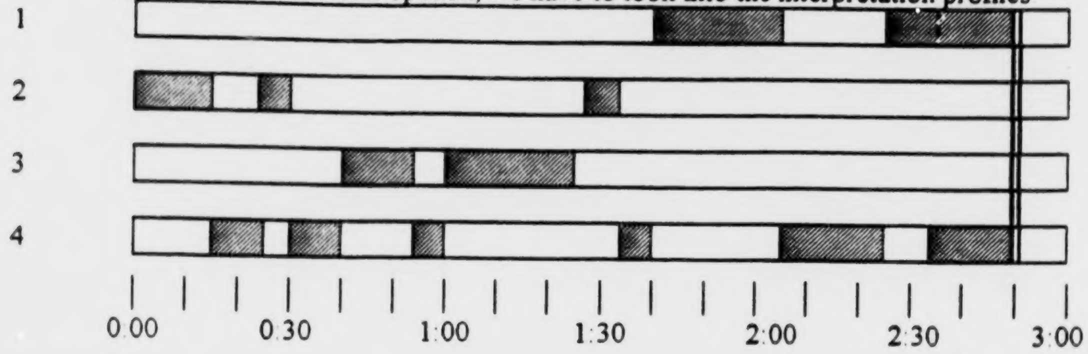


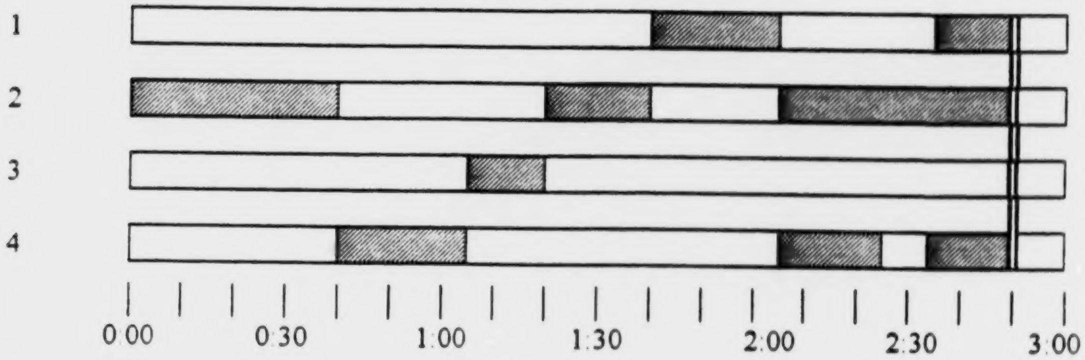
Two boys, both 13 years old and of average mathematical abilities, end up with the following pieces lying on the puzzle: 5-unit-bar, 3-unit-bar, 3-unit-angle. They say that the puzzle can not be solved : "The 4-unit-bar must be cut into two pieces, because there are not enough 2-unit-bars". This answer is categorized as Category II.

The action profile of the boys shows that the 5-unit-bar was nearly all the time lying on the puzzle. Additionally, nearly all the time we find the 3-unit-angle and/or the 4-unit-bar lying on the puzzle.

The combination of 5-unit-bar/4-unit-bar resp. 5-unit-bar/3-unit-angle is essential for seeing that the task is impossible: the 5-unit-bar/4-unit-bar - combination makes it impossible to put the 3-unit-angle down, the combination 5-unit-bar/3-unit-angle makes it impossible to put the 4-unit-bar down. (We call those combinations *blockade situations*). We may conclude that the boys "had the reasons for the impossibility of the task in mind" but found them after nearly 3 minutes of acting so self-evident that they did forget to formulate them. On the other hand, we find many actions with the small pieces. By this we may conclude that - though the big pieces were lying on the puzzle nearly all the time, the pupils did *not* recognize that this fact was the essential reason for the impossiblity of the task.

To get a better founded opinion, we have to look into the interpretation profiles





We see many differences in the interpretation profiles of both coders. There is not much unanimity whether actions belong to *gestalt operations* or to *other systematic behavior* (for instance, trying to fill up the puzzle, beginning at one end). This should not surprise us: following the *gestalt* of a figure can - with our puzzles - lead to a behavior which looks very systematical.

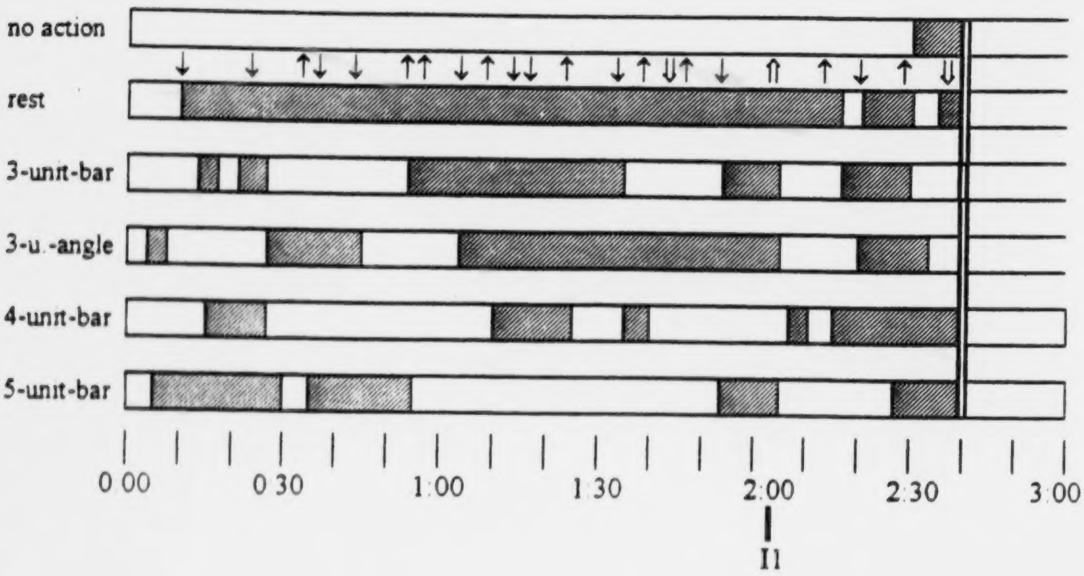
Both coders see, however, only a short period of *logically guided behavior* - coder 1 from 0:40 to 1:30, coder 2 from 1:05 to 1:20. We understand this better by having another glance on the action profile: only at 1:10, the two big pieces are laid down with no other pieces lying on the puzzle. *At all other times the 5-unit-bar resp. 3-unit-angle are laid down with some small pieces already on the puzzle.* Placing the big pieces on the puzzle seems to be part of a "filling - up - action", not part of logically guided sequence.

Conclusion In this case, we assume that the low quality of the outspoken answer reflects a problem solving process which did not use a basic analysis of the material. All attempts to fill the puzzle up ended in a situation in which the 4-unit-bar was left over or in which there were two free spaces with a 2-unit-length. This experience is summarized in the answer.

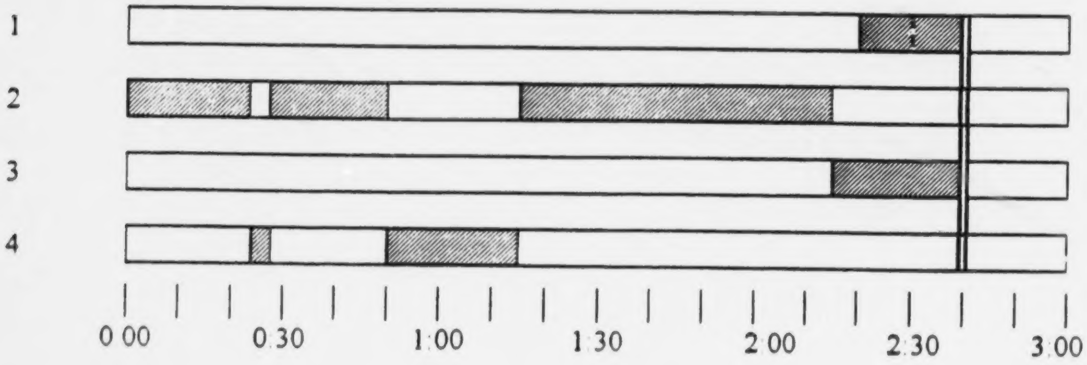
Example II

Two boys (12 y; 8m / 13 y; 5 m) of average mathematical ability, work for 2 minutes and 40 seconds. They end up with the 4-unit-bar and the 5-unit-bar in the right place. Their final argumentation is in Category I: "The 5-unit-bar and the 4-unit-bar have fixed positions on the puzzle. In any case there remain two free spaces of length 2, and there are not enough 2-unit-bars to fill those spaces up."

We now take a glance on the action profile. It shows many actions, involving pieces of all sizes. During the first 2:20 - period, there are only very short periods containing blockade situations.



Since both interpretive coders nearly fully agreed in their interpretation, we give only one interpretation profile. It shows that until 2:20 the pupils follow *gestalt* operations or show *other systematic behavior*.



Since the argumentation of the pupils was so good, we shall try to find out where this insight happened.

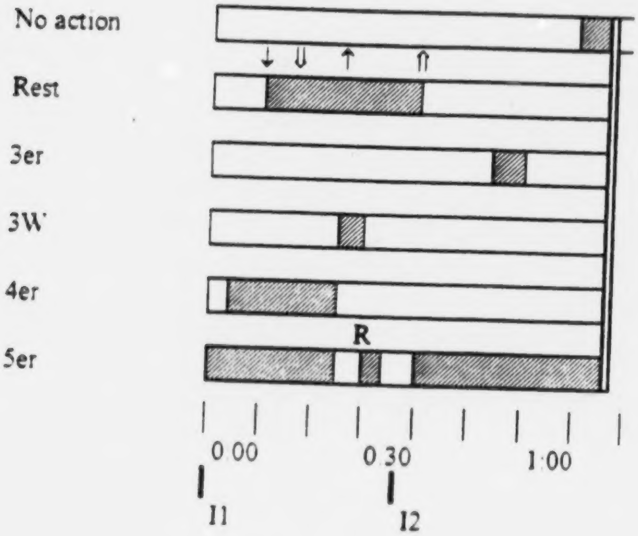
They started their work with the 5-unit-bar in it's place, and created some blockade positions. Since this was not successful, from 0:55 to 1:55 they worked without the 5-unit-bar. During this working phase they seem to have realised that the 5-unit-bar has only two possible positions, and has to be placed first. Some attempts follow to fill the puzzle up. Now the big pieces are always involved. At 2:35, the 3-unit-angle, the 4-unit-bar and the 5-unit-bar are lying on the puzzle. The 5-unit-bar is the last big piece laid down. The reader may convince himself that this *must* be against the rules.

Conclusion The end phase of the pupils' work seems to be the most interesting part of the interview. Having been unsuccessful for nearly two minutes, leads to increased number of actions involving big pieces. Those actions stabilize the insight in the reasons for the impossibility of the task. This insight is summarized in the answer.

Example III

Within the space of this paper, we can only summarize the results of the interpretive coding of the following transcript. The transcript shows the work of a 13 year old boy (medium ability level) and 13 year old girl (good in mathematics).

At the end of the interview, the pupils give the following explanation for the unsolvability of the task: „It cannot be done because the 5-unit-bar can not be laid down in another way“.



We see that the pupils begin with the combination 5-unit-bar /4-unit-bar. This is a strong hint that they were guided by some logical considerations. After that, we see some actions with small pieces which may be seen as following the chosen logical path, or as gestalt reaction. From 0:30, the 5-unit-bar is lying on the puzzle all the time. We conclude that the pupils have realised, that this piece *must* be placed first. Since the pupils stop work until the interviewer starts talking to them (I2), we conclude that there answer - though very short - is a result of a clear insight *that* the task is unsolvable, and *why* it is unsolvable.

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"Why are we assessed in mathematics?": The views of students

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Students from elementary, intermediate and high schools were asked why they considered they were assessed in mathematics. The small sample (n=559) came from the US, the Netherlands, and New Zealand. Some responses were consistent across countries. Most students thought they were assessed in order that teachers might find out what they know/understand. Some cultural differences are suggested by the data. Illustrative comments are included.

The issue of assessment has long been important and on-going in mathematics education. Researchers have investigated the:

- historical trends in mathematics assessment;
- efficacy and efficiency of different forms of assessment;
- political issues surrounding the topic;
- assessment procedures that may be most fair to learners;
- contexts that might be used for assessment purposes;
- impact of various theories of learning on approaches to assessment;
- relationship between what is taught and what is assessed.

Examples of how assessment (and in particular certain forms of testing) may drive the curriculum are well documented (Leder and Forgasz, 1992). The often-quoted research of Schoenfeld (1988) is a powerful description and analysis of how students may have outstanding results as measured by examination papers, but have little conceptual understanding of the geometry (in this case) that was being learned. This study revealed a teacher, while on the one hand exhorting the students to 'think' during mathematics lessons, was on the other encouraging them to rote learn examples because these would appear in the examination paper. Ritchie and Carr (1992) comment on the same phenomenon in the context of 'mastery' testing in primary schools. In this case once students were grouped for instruction following a mathematics pre-test, teachers regularly consulted the approaching end-of-unit mastery test during the teaching of the unit.

The purposes of assessment , therefore, may turn out to be different from that which was originally intended. In this regard, we might say that assessment devices *per se* may be powerful influences on what mathematics is taught, and how the mathematical content is presented to the students.

> ...assessment should be recognised, not as a neutral element in the mathematics curriculum, but as a powerful mechanism for the social construction of mathematical competence...assessment influences student mathematical behaviour and determines mathematical performance, distorting the behaviours it was intended to monitor and to inform constructively (Clarke, 1992, p. 166)

The evidence suggests that the assessment regime that is in place in a school can influence how teachers view mathematics. Of course these views will vary from teacher to teacher. As well, there may be different teachers who hold different conceptions of the same assessment device (Clarke, 1992; Cohen and Manion, 1994). It seems likely, then, that the students themselves will hold varying conceptions of the assessment procedures that they encounter in the mathematics classroom. Clarke (1992) presents a small window on this when he notes how students are very accurate and consistent when ranking their peers in mathematics achievement. He also describes the effect that assessment can have on students' self-concepts. All of this suggests that students are actively reflecting on the assessment processes in mathematics.

The variety of assessment devices that students encounter in mathematics has probably become more extensive in the last decade. The strong recommendation in many curriculum and related documents (eg. Ministry of Education, 1992; National Council for Teachers of Mathematics, 1995; Australian Education Council, 1990) is for the use of a range of assessment procedures (whether or not these recommendations have been reflected in actual practice remains another question). Ellerton and Clements (1997) comment on the inadequacies of externally-set, pencil-and-paper mathematics tests (ESPPMTs), and note the dislocation between rhetoric and reality. They advocate more authentic methods for assessing mathematics, and present data that shows how even the most carefully constructed ESPPMT can yield "mismatches" of up to 28% - that is, up to this proportion of students may give correct answers while not understanding the concepts being assessed, or give wrong answers even though they possess a full or partial understanding.

Van den Heuvel-Panhuizen and Gravemeijer (1993), on the other hand, argue that students can be assessed fairly with pencil and paper tests if the contexts are realistic to the students and "offer the opportunity to test the children's abilities while avoiding obstructions caused by formal notation" (van den Heuvel-Panhuizen and Gravemeijer, 1993, p. 56). Test items need not remain restricted to flat one-step tasks that can be performed directly, without any preceding analysis. They can also be used to investigate more complex problem-solving activities. (van den Heuvel-Panhuizen and Gravemeijer, 1993, pp. 57)

It was with a background of these types of concerns that the writer approached the current study. In much of the above the central theme has been that of providing students with fair, authentic, realistic and varied forms of assessment in mathematics. As mathematics educators debated and researched assessment issues, students (in most cases) were foremost in their minds..

Turning to students, then, the general question arose of what they might think about the processes of assessment that they are subjected to in mathematics classrooms? To be more precise:

- what do they see as the purpose/s of assessment (in its various forms) in mathematics?
- in their views, why were they being assessed?

These were the questions that provided the focus for the present study.

The Present Study

The Sample

Students in this study came from three different countries. The data was collected in the period 1995 to 1997. The US sample (n=168) was obtained from three high schools in the state of Nevada during the autumn (northern hemisphere) of 1995. All three high schools were in urban areas. The Dutch samples (n=174) were from a high school which served a village and environs about 30 kilometres from the nearest city, and from an elementary school in the city of Utrecht. This data was gathered in the spring (northern hemisphere) of 1996. The New Zealand data was obtained during April 1997. These students (n=217) came from urban schools in the town of Hamilton. Four schools were sampled to get the NZ data - one high school, two intermediate and one elementary school. The schools from the three countries were quite different in character - no attempt was made to tap into schools (between countries) of similar characteristics, perhaps because this would, in any case, be an impossible task in a study of this type.

Students in the study followed the curriculum documents (mathematics) commonly used in that particular country. The US students were taught by three competent, highly qualified and motivated teachers who averaged eight years classroom experience. The Dutch high school sample was from a school where the 'realistic mathematics' approach was used (see de Lange 1992: van den Heuvel-Panhuizen, 1996) by a teacher who also was employed at the mathematics education research centre in the Netherlands. This experienced teacher was also an author of a Dutch textbook series in mathematics. The elementary school sample (Dutch) came from an inner city area which used a conventional mathematics programme. The New Zealand sample contained students from seven teachers - all experienced, averaging 24 years in the classroom.

The US sample (n=168) comprised 89 females, 79 males; the Dutch group (n=82) 43 females, 39 males; the NZ sample (n=217) 104 females, 113 males. All students in the study were from non-streamed classes.

Procedure

The study was approved by the School of Education's ethics committee, and the writer approached teachers in schools asking if they would be willing to have their students take part. The purpose of the study was explained to the students, and it was emphasised that participation was voluntary.

Each student was given a sheet of paper. On the top was written "As part of what you do in mathematics classes, sometimes you will be assessed. The assessment can take different forms. Why do you think that you are assessed in mathematics? In other words, in your view what is the point of this assessment?" This was read to the class. Students were asked if they had any questions, and once these were discussed they wrote their responses on the paper. The Dutch instructions were "Soms kryg je by wiskunde een toets. Waarom denk je dat je toetsen krygt by wiskunde? Schryf heronder op waarom". Dutch colleagues administered the task for the writer (and translated the subsequent responses).

The students were not given a time limit for the exercise. Most finished within 10 minutes. They were encouraged to work independently.

Results

As might be expected, the students constructed a wide range of responses to the open-ended question. Initially responses were difficult to categorise, but as more and more response sheets were considered, themes began to emerge. The writer consulted with two colleagues during this phase of the study.

The response categories in tables 1 to 6 are generic phrases that were generated to represent the views of the respondents without using the exact words of the student response.

Table 1:*Why are we Assessed? Elementary School Students NZ (n=51)*

		N	%
1.	To find out what we know/understand "I think we have maths tests because techers will have to know what you know and don't know" (female) "So that she can find out if we are learning" (female)	47	92
2.	So that we can be helped "...if I get the answer rong she will teach me more about	31	61

	the problem I got wrong" (female) "So we can learn to do our mastacks right" (male)		
3.	To find out my level/group "...so they can put you on the correct level of maths" (female) "I think we get tested because it helps both the teacher and us children understand what level we are at, so we children aren't struggling at a hard level or an easy level" (female) "They give us tests in case we need to go into a special maths group or to help us" (male)	17	33
4.	So we can get jobs later "When we're older we can get jobs easier and be able to add things up" (female)	4	8
5.	To make us try "Cause the teacher want to know if you are working hard.. " (male)	2	4

Table 2: ***Why are we assessed? Elementary School Students Netherlands (n=92)***

		N	%
1.	To find out what we know/understand "Because they need to know if you can do your maths" (female)	62	67
2.	For the report to parents	18	20
3.	In order to go up a grade (e.g. to high school)	16	17
4.	To see how clever we are	16	17
5.	Feedback to the teacher/student "To see what you can do the best - which one you do best - adding, take-away, multiplication, division" (male)	10	11

Table 3: ***Why are we Assessed? Intermediate School Students NZ (N=91)***

		N	%
1.	To find out what we know/understand "So she knows what I'm good at" (male)	59	65
2.	To find out my level/group "I think you are assessed so they can show what level or stage you are up to" (male) "So that you are not put into a group that is a lot more advanced than you" (male)	46	51

3.	So we can be helped “We are also tested in maths so that the teachers can pick up on our slack parts in maths” (female) “I think you are assessed to find out whether you know something or not and if you don't then your teacher can help you” (male)	37*	41
4.	To provide feedback to the teacher “Mainly so they know what to teach you and know that they're not teaching you something you already know” (female)	14	15
5.	To get a (good) job “If you get a good mark and it is an important test eg.School C it can effect your career as an adult” (male)	12	13
6.	To show how you have improved “So we can see how much we improve over so long” (male)	11	12

* $\chi^2 = 10.8$ $p < .01$ (females more often than males)

Table 4: *Why are we Assessed? High School Students NZ (n=75)*

		N	%
1.	To find out what we know/understand “The point of assessment is to see how well people learn mathematics according to age and sex” (male)	68	91
2.	So we can be helped “The point of assessment is to acknowledge what you can/can't do, and establish what you can therefore improve upon” (female) “It is so they know who needs help in what areas. Because there are so many people in the class it is impossible for the teacher to know we are coping with the work. Our teacher doesn't take our books in, and only glances at our work, so apart from the tests, and when people answer questions out loud, they have no idea who understands and who doesn't” (female)	48	64
3.	To find out my group/level “So that the teachers know how good you are and can put you with people with the same ability as you and so you don't get bummed out if everyone gets 10/10 and you get 2 out of 10” (female)	28	37
4.	to compare you “Also so you can be compared to others in the class” (female) “...to find out how we might compare with different people in our age group” (female)	13	17
5.	For jobs in the future “It also helps us with jobs in the future.	6	8

I sometimes don't like maths test but what can we do?
We have to go with the flow" (male)

Table 5: *Why are we Assessed? High School Students Netherlands (n=82)*

		N	%
1.	To find out if we know/understand	59	72
2.	For reporting to parents	44	54
3.	So we can be helped	25	30
4.	To see if we can apply our learning	11	13
5.	To identify talented and untalented students	5	6
6.	To make students learn	4	5
7.	"It's the law - it has to be"	4	5

Table 6: *Why are we Assessed? High School Students US (n=168)*

		N	%
1.	To find out how much we know/understand "It's kind of a way to say, O.K., I've taught this to you, now what do you know. Prove it to me that you know it" (male)	62	37
2.	For the purposes of grading "I think we are assessed for a few reasons. One is for a grade to get through school" (female) "Showing somebody a grade or what there grade is in math is showing them what they know or how much they know. It's a tool (talking about grading) that every teacher in America uses. A teacher will grade a student to show them what they know but in reality that grade doesn't tell how smart a person is, it just shows how much they try. Grades, your grades are based on how much work you do" (anon)	31	18
3.	To help us learn (motivation) "The grades are also motivation. Like if you have a certain grade to play (illegible) you will work harder, so that you can. That's why and how assesment is used in math" (male) "We take tests because the information sticks in your head better" (anon)	25	15
4.	So we can be helped "To find out what the students need help on the most" (male)	19	11
5.	To see if the teacher is teaching properly	9	5

6.	To reward us when we work hard	5	3
7.	It's a requirement (of the School Board)	3	2

Apart from in table 3 no significant differences existed (in terms of frequency) between the responses of males and females within each of the categories.

Discussion

The statements of the students show some consistency across the three countries. This is particularly evident when the most common response is considered. In this study students (at whatever stage of schooling) in each country gave the same most common reason for assessment in mathematics. The response - words to the effect that assessment was put in place so that teachers might find out what the students know and understand - is possibly reassuring for many teachers. This view, coupled with little mention of assessment being for the purposes of comparing and ranking students, perhaps indicates that the students in this study were able to discern an 'educational' reason for assessment in mathematics.

The second most common reason for assessment reinforces this view. In all samples 'So that we can be helped' suggests a 'healthy' orientation that many of the students hold towards assessment. As teachers of mathematics, we would all like our students to feel that we are there to help them when they need support.

An interesting comparison can be made with the responses of the US high school students' (Table 6) and those of other groups and the two other countries. The US predilection with grading is hinted at with this group. It would appear that many of these students are in classrooms where newer approaches to assessment may not be present (National Council of Teachers of Mathematics, 1995). As well, the relatively small number of students (19 out of 168) in this high school group who saw a purpose of assessment as getting help from the teacher is worth commenting upon. This may suggest that the more educational aspects of assessment, including assessment for better learning, are not present to any large extent in the three schools that these students attended.

Another interesting view is that expressed by the Dutch high schoolers. Over half the students thought one reason for assessment was to report to parents. No other group approached this proportion (apart from the younger Dutch children). Without the support of qualitative data it is difficult to interpret this result, but a cultural artefact is suggested

Many educators consider that assessment orientations should shift increasingly towards self-assessment. In constructing their responses the students in this study did not indicate that they were thinking about this view of assessment . The only exception to this was the elementary school sample in the Netherlands where ten out of the 92 students thought that there was an element of 'feedback' to themselves.

Self-assessment involves the learner in assessing the task and in assessing her or his actions in it. In other words learners are making a judgement about their own sense making. In order for it to operate successfully teachers must develop a climate of respect, understanding, honesty and tolerance in their classrooms. Anderson (1993), in giving examples of self-assessment in practice, describes how it can be used as a powerful tool that has the potential to empower learners in the assessment process. Given the dearth of responses to do with self-assessment it would appear that the teachers in the present study will face challenges in guiding their students along the path of self-assessment. The question can be asked if teachers of mathematics, generally, are incorporating self-assessment into their programmes.

Perhaps the main point of this study is that it provides us with further information on *students'* views of what we (as teachers) do. The results may confirm what we thought we knew. The results may provide us with some surprises. In either case they should help inform our practice, and remind us that learners have their own legitimate views of learning and assessment.

When asked why he was assessed in maths one ten-year-old wrote: "Because if you are big, you can go to work like a policeman and a teacher. Then you can have a family and have some money to buy anything for your children, and if you work very hard you can be the government." Assessment may also be confusing.

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Developing a framework for viewing affective and knowledge factors in teaching primary mathematics

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An extensive study of the literature on teachers' views of mathematics and mathematics teaching revealed inadequacies in the available frameworks for structuring and reporting on research. In this paper a new framework is developed for analysing and describing primary school teachers' knowledge and feelings about mathematics teaching and learning. The framework; 'Teacher Type Table', was developed after surveying 107 primary school teachers in suburban Melbourne. The teacher type table is presented here as a model for use in the professional development of teachers.

Introduction

The research which is reported in this paper arose in response to well publicised concerns about the knowledge and attitudes of primary school teachers with respect to teaching mathematics. This paper reports on some qualitative components of a study (Carroll, 1997) which was designed to investigate how primary school teachers view themselves as teachers of mathematics. The paper identifies issues from research on primary teachers and their mathematics teaching, analyses the results of a survey of primary school teachersí views and presents a new framework for analysing and describing teachersí knowledge and affective factors regarding their mathematics learning and teaching.

Issues in the Review of Research

An extensive review of the research literature on affective and knowledge factors in teaching primary school mathematics (Carroll, 1997), revealed a number of important issues which are discussed below.

Affective and Knowledge Factors of Primary Teachers

The research review supported the view that primary teachers *were* limited in their knowledge of mathematics (Cockroft, 1982, p.189; Fennema & Franke, 1992; Kanes & Nisbet, 1994). Beginning and preservice primary school teachersí knowledge was also found to be lacking in their understanding of methods for representing mathematical ideas in ways that are understandable for students (Ball, 1990, as cited in Fennema & Franke, 1992, p.154; Sullivan,1989, p.15).

Many studies reported that negative attitudes towards mathematics were held by primary school teachers and teacher trainees (Ball 1988, Bobis & Cusworth, 1994; Carroll, 1994; Cockroft, 1982, para.679; Hart, 1993; Relich Way & Martin, 1994, p.58; Sullivan, 1987). Researchers expressed their concerns about the high proportions of teacher trainees who are lacking in confidence in their ability to do mathematics and their knowledge of how to teach mathematics to children. The changes in attitudes and beliefs of trainee teachers during their training were well documented.

Lack of Participation by Classroom Teachers in Research Studies

Less well understood and studied, however, was the of evolution affective and knowledge factors when teacher training is completed and a teacher begins to teach in the classroom. A number of studies reported on knowledge and affective factors of primary school teachers who were undertaking further study or who were involved in professional development programmes focussed on their teaching of mathematics (Cobb, Yackel & Wood, 1992; Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996). Research which reported on the knowledge and affective factors related to teachers working in primary schools, who were not pursuing some form of further study was limited and signalled a need for research to be conducted that examines the views of primary teachers across the board, rather than just those training, beginning or returning to study.

Inadequacy of Models for Researching Teachersí Knowledge and Affective Factors

A further issue concerned the inadequacy of available frameworks for structuring and reporting research on the mathematics teaching and learning. A number of models related to teaching and learning were considered in the development of the present study and while useful in constructing a theoretical framework for reporting on the literature, designing the study and reporting results, these models were all found to be limited.

Koehler and Grouws (1992, p.117) presented a model for researching the mathematics teaching and learning process which involved a range factors including teachersí knowledge of content, pedagogy and student learning, and teachersí attitudes towards and beliefs about teaching and mathematics. Their model provided a way of structuring discussion and presenting research, however it failed to reflect the dynamic and interactive nature of the process of teaching and learning mathematics and also omitted an important component of the teaching process; the ëteacherís knowledge of mathematical representationsí (Ball, 1988; Shulman, 1987, p. 8; Bromme, 1994, p. 75).

McLeod (1992) in his comprehensive review of the research on affective factors, criticised many studies of mathematics teaching and learning which in the past had focussed on knowledge and tended to ignore affective factors. McLeod (1989b) had earlier attempted to redress this neglect when he proposed a model which divided the affective domain into beliefs, attitudes and emotions. His model showed relationships between beliefs, attitudes and emotions and cognitive involvement and also incorporated the notion that affective factors are changeable and linked to cognitive factors. The ideas presented in McLeodís model attempt to reflect the dynamic, interactive nature of cognitive and affective factors in mathematics education in ways which were lacking in the Koehler and Grouws (1992) model.

Examination of the limitations of some key theoretical models for research into the teaching of mathematics suggests the need for new frameworks to be developed in order to better understand the complexity of these factors and to design further study in the area.

Investigating Primary Teachersí Views of Mathematics Teaching

The study reported here was designed to address the question of how primary school teachers view their; knowledge of mathematics, knowledge of the approaches required for teaching mathematics, attitudes and beliefs about doing mathematics, and attitudes and beliefs about teaching mathematics

Method: The data was collected using a survey instrument; the ëMathematics Attitude and Knowledge Scaleí (MAK scale) which was designed to investigate how primary teachers felt about teaching mathematics, how they rated their knowledge of mathematics and the approaches required for teaching it and also how they felt about their own learning of mathematics and their understanding of it.

The scale was developed by identifying five constructs relevant to the literature on teachers' views of mathematics and mathematics teaching (Nisbet, 1991, p.37; Relich, 1995). The five constructs were:

1. knowledge, competence and understanding of mathematics;
2. knowledge, competence and understanding of mathematics teaching;
3. enjoyment and confidence in learning and doing mathematics;
4. enjoyment and confidence in teaching mathematics and
5. conceptions of mathematics and mathematics teaching.

Six items were used for each construct; three items were worded positively (for example, ìI have always done well in mathematics classesî) and three worded negatively (for example, ìIím not very good at mathematicsî). Some of the items were adapted from scales developed by Nisbet (1991) and others were adapted from a scale developed by Relich. (1995). The MAK scale consisted of 30 items requiring responses on a five point Likert-type scale ranging through strongly agree, agree, not sure, disagree to strongly disagree. Table 1 lists the thirty items and the five constructs are identified in column 3. The positive and negative signs in column 3 of the table identify whether the item was positively or negatively worded. The scale was also designed to collect biographical information.

Procedure: Twenty Department of Education primary schools were chosen at random for the study from the South Central Metropolitan Region of Melbourne. Permission to conduct the research was sought from school principals and received from 13 of the schools. Surveys were delivered to each classroom teacher in the schools and collected several weeks later.

Analysis of data

Coding of responses: The teachers responded to the MAK scale items using the five point scale; strongly agree, agree, not sure, disagree to strongly disagree. For positively worded items, responses were coded: strongly agree, 5; agree, 4; not sure, 3; disagree 2; and strongly disagree, 1. The negatively worded items were coded: strongly agree, 1; agree, 2; not sure, 3; disagree 4; and strongly disagree, 5. The teachersí coded responses were analysed using the SPSS statistical package.

Principal Components and Factor analysis: The MAK scale had been constructed with five scales which were based on the model proposed by Koehler and Grouws. Principal components and factor (Oblimin rotation) analyses were used to interrogate these scales; investigating and summarising patterns of correlations among the items and consequently testing the theoretical framework on which the original five scales were based.

Factor Scores: The factors identified by the principal components and factors analysis were used as a basis for the computation of factor scores for each teacher on each of the factors. The factor scores were computed by summing the teachersí coded scores for each item in a factor. A mean score was computed for each factor. An individual teacherís factor score was considered to indicate a negative view on a factor (negative factor tendency) if it was below the mean on that factor. A positive view on a factor (positive factor tendency) was indicated by a factor score above the mean.

Table 1. *Items Loading on 4 Factors*

Factor 1 (**F-** Feelings about teaching mathematics)		
No.	ITEM	Construct
24 *	I look forward to teaching mathematics	4 +
19 *	I do not enjoy teaching mathematics	4 -
4 *	I enjoy teaching mathematics to primary school children	4 +
14 *	I find teaching mathematics challenging and exciting	4 +
29 *	I do not feel confident about teaching mathematics	4 -
12 *	Teaching mathematics doesn't scare me at all	2+
26 *	Wherever possible I avoid doing mathematics	1 -
Factor 2 (**M-** Knowledge of Mathematics and feelings about doing it)		
21 *	At school my friends came to me for help in mathematics	1 +
1 *	I have always done well in mathematics classes	1 +
16 *	I'm not very good at mathematics	1 -
11 *	I have generally done better in mathematics courses than other subjects courses	1 +
3 *	I find many maths problems interesting and challenging	3 +
13 *	I enjoy doing mathematical problems	3 +
8 *	I feel frustrated doing mathematical problems	3 -
9 *	I get a sinking feeling when a child asks a hard question when I'm teaching mathematics	4 -
Factor 3 (**K-** Knowledge of methods and approaches for teaching mathematics))		
22	I am confident about my knowledge of current approaches to teaching	2 +
7 *	I feel that I need to know more about the methods of teaching mathematics to primary school children	2 -
27 *	I worry that I'm not using the right teaching methods in mathematics	2-
17*	I'm not sure about what to do when teaching mathematics to primary school children	2 -
Factor 4 (Conceptions of mathematics and mathematics teaching)		
20	Maths is mostly facts and procedures which need to be memorised	5 -
15	Each student should be encouraged to build on their own maths ideas even if there is much trial and error.	5 +
30	Discovery methods of teaching tend to frustrate many students who make too many errors before making any discoveries	5 -
25	There are many ways to solve a maths problem	5 +
10	Solving a maths problem usually involves finding a rule or formula that applies	5 -
The following questions did not have loadings of > .48 on any factor		
2 *	I feel secure about teaching mathematics	2 +
18 *	Mathematics makes me feel inadequate	3 -
23	Doing maths makes me feel clever	3 +
28 *	When I think of doing mathematics, I remember how bad I felt about doing maths tests.	3 -
5	Studying mathematics helps to develop the ability to think creatively	5 +
6 *	I have hesitated to take courses in mathematics	1 -

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Results

One hundred and seven primary school teachers completed and returned the Mathematic Attitude and Knowledge scale; 95 were female and 12 were male. The teachersí experience ranged from first year out to 32 years teaching, with the average being 14 years in the classroom.

Factor Analysis: The factor analysis using Oblimin rotation (Carroll, 1997, p.87) identified four factors which had items with loadings greater than 0.48. Table 1 lists the 30 items which were contained in the MAK scale and items which load significantly on each of the four factors.

The first factor contained items relating to the teachers' feelings about teaching mathematics. The second factor consisted of items related to the teachers' views of their knowledge of mathematics and their feelings about doing it. Items which addressed the teachers' knowledge of the methods and approaches required to teach mathematics to primary school children were clustered in the third factor while the fourth factor related to the conceptions of mathematics.

Principal components analysis: The principal components factor matrix showed that 22 items (asterisked in Table 1) loaded substantially (loadings greater than 0.5) on one factor. The 22 items all related to the teachersí knowledge and feelings about mathematics or mathematics teaching. This could be considered to represent the underlying construct for these items. The remaining items related mainly to the teachersí views of what mathematics is and how it should be taught.

The factor solution generated by factor analysis and the underlying construct identified in the principal components analysis were used to select the three factors which generated the Teacher Type Table. It can be seen in Table 1 that the items contained in factors one, two and three contributed substantially to the principal construct: the teachersí knowledge and feelings about mathematics or mathematics teaching. Items contained in factor 4 in table 1 showed insignificant loading on this construct. Consequently, for the purposes of analysing and discussing the teachersí views of their knowledge and feelings about mathematics or mathematics teaching, only the first three factors were considered in the teacher type table.

Development of a New Framework: the Teacher Type Table

A new framework for analysing and describing affective and knowledge factors in teaching primary mathematics was developed by considering the teachersí factor scores on the first three factors in Table 1. Teachersí scores which were below the mean on a factor were considered to indicate a negative factor tendency, while scores above the mean represent a positive factor tendency. The symbols; **F+, F-, M+, M-, K+, K-** summarise the factor tendencies. A profile of each teacher was developed by considering their tendencies on each of the factors. Table 2 shows the eight teacher types and percentage of teachers who were in each category.

Discussion

The Teacher Type Table is a descriptive tool which provides a new perspective on the teacher. It presents a framework for reporting and mapping changes in teachersí views of their knowledge of mathematics and feelings about learning and doing it, and their knowledge of the approaches for teaching mathematics and feelings about

Table 2. *Teacher Type Table*

F-M-K- *23%* Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating. Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children**	F+M-K- *3%* Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening. Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating. Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children**
F-M+K- *12%* Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children**	F+M+K- *8%* Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening. Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children**
F-M-K+ *11%* Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating. Knowledgeable about the methods and approaches for teaching mathematics to primary school children.	F+M-K+ *6%* Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening. Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating. Knowledgeable about the methods and approaches for teaching mathematics to primary school children.
F-M+K+ *5%* Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging Knowledgeable about the methods and approaches for teaching mathematics to primary school children.	F+M+K+ *31%* Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening. Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging Knowledgeable about the methods and approaches for teaching mathematics to primary school children.

teaching it. The Mathematics Attitude and Knowledge scale is readily administered and consequently, teacher type profiles can be quickly developed. The Teacher Type Table has potential in the area of professional development. It could be used prior to embarking on professional development in assessing teachersí needs and could also be of use in evaluating the effectiveness of professional development programs by charting changes in teacher type profiles.

The Teacher Type Table addresses the shortcomings of the Koehler and Grouws model discussed earlier in this paper. The Koehler and Grouws model was found

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lacking in its ability to represent the interactivity of cognitive and affective factors. It also neglected the teachersí knowledge of mathematical representations. The Teacher Type Table incorporates the notion that knowledge and affective factors are interactive and changing and factor three includes of teachersí knowledge of mathematical representations.

The Teacher Type Table requires further testing to fully investigate its potential. A limited number of mathematical life histories were analysed and tended to support the categorisations in the table, however further validation studies are required. It may be of use to have teachers self select their teacher types after completing the MAK scale to provide a comparison for the teacher types generated by their factor scores. The use of the mean as the cut off point between positive and negative factor tendencies was supported by the life history data, but further studies are needed to fully investigate the cut off points. As can be seen in Table 2, the extreme teacher types; F-M-K- and F+M+K+ were the largest groups, while some of the teacher types were quite small. Further investigation of the MAK scale and Teacher Type Table in conjunction with qualitative studies should provide increased understanding of the categories.

The Teacher Type Table provides a new way of viewing primary mathematics teachersí knowledge and feelings which is based upon their own reports. While further study will consolidate and perhaps provide modifications, The Table has considerable potential in identifying professional development needs.

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Teachers' understandings of the role of learning activities

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This study investigated teachers' understandings of the role of learning activities in secondary school mathematics. A qualitative study involving interviews and questionnaires was conducted in government and non-government schools. Most teachers in the study were supportive of the use of learning activities in the classroom for both teaching and learning purposes but were aware of constraints of time, adequate resources and curriculum requirements.

Selected Literature Review

In the changing face of school mathematics, the *National Statement* (AEC, 1991) and *The Cockcroft Report* (1982) envision that students develop within their capabilities, the mathematical skills and understanding required for adult life, employment and further study and training. Hence, mathematics can no longer be regarded as a 'chalk-and-talk' subject from the perspective of the teacher or as a 'pencil-and-paper' subject from the perspective of the student. The *National Statement* and The *Professional Standards for Teaching Mathematics* (NCTM, 1989) acknowledge the increasing diversity of clientele in schools, in terms of their abilities and personal experiences. These documents acknowledge that not every student will have the same interests and capabilities in mathematics, as some students will enjoy mathematics more than others, or a particular student may enjoy some aspects of mathematics but not others. To ensure that students leave school with a positive attitude towards mathematics, these policy and curriculum documents advocate that students must be exposed to a wide range of learning activities in mathematics.

There are several different ways in which the concept of 'activity' could be interpreted. 'Activity' can be understood as a concept within Activity Theory. In this particular theory of learning, mathematical understanding arises from the engagement in mathematising experiences with others and becoming a legitimate participant within a community where mathematical activity is accepted as part of competent membership (Lave and Wenger, 1991). In a mathematical learning activity in the classroom, learners become part of a community of practice interacting to help each other understand mathematical principles (Bishop, 1988). Activity could also be understood as a concrete experience used to illustrate a concept, or it could be a game which is associated with drill and practice. One of the purposes of this study, was to determine where teachers were located with respect to these different interpretations of activity.

There are a range of 'learning activities' that exist in mathematics education. These can range from independently completing a "fun" worksheet or playing commercially-prepared or teacher-made instructional games with peers in order to reinforce one's understanding of a concept, to getting physically involved in a whole class group activity to learn a new concept or skill. Bishop (1988) lists six activity-types commonly used for the teaching and learning of mathematical concepts. These are: Counting, Locating, Measuring, Designing, Playing and Explaining. Guided by policy and curriculum documents, Lovitt and Clarke (1988) have listed types of activities that may be worthwhile learning experiences. In the associated resource package are activities that students are able to become involved in: physical activities, creating a visual representation, participating in group investigations, problem-solving and making mathematical models. Sometimes, students also have the opportunity to write journal entries or stories to describe mathematical representations. Stephens, Lovitt, Clarke and Romberg (1989) contend that the specific classroom activities collected and documented by RIME (Reality In

Mathematics Education) and MCTP (Mathematics Curriculum and Teaching Program) represent "snapshot images of a quality learning environment" (Stephens, Lovitt, Clarke, and Romberg, 1989, p.229). The collection of activities in these activity packages forms a "gallery of the art of good mathematics teaching", and serves as a vehicle which "illustrate[s] new understandings and reshaped practice."

Mathematical learning is now viewed as a process that involves both active construction and enculturation. Cobb (1994) asserted that sociocultural and constructivist theories both highlight the crucial role that activity plays in mathematical learning and development. However, unlike the constructivists who give priority to students' sensory motor and conceptual activity, socio-cultural theorists typically link activity to participation in culturally organised practices; that is, activities being performed in groups. Ernest (1994) outlines some principles of social constructivism which provide a useful framework for the study to be reported in this paper. As students construct knowledge by learning together in a social mode, they are communicating their progressive understanding of mathematical topics by describing in their own language what the topics mean to them and how they apply their interests and personal experiences.

Teachers' beliefs (Cooney, 1985; Thompson, 1984) about mathematics and learning have a powerful influence on how they teach the subject. In turn, teachers' views of mathematics are shaped to a very large extent by their own mathematical experiences as students (Dorfler and McLone, 1986). These views may also be influenced by content policies, teachers' own content knowledge bases and their interest levels (Porter et al, 1988). Although the Queensland Years 1 to 10 Mathematics Syllabus (Department of Education, 1986) has advocates the use of activity-based lessons, many teachers do not feel secure about this arrangement, and feel that in such lessons, their authority is more apt to be challenged. Richards (1991) and Porter et al (1988) believe that this is because their years of teaching in a more traditional teacher-centred classroom provided them with a sense of security and comfort, in which they knew the questions that would be asked and the answers they would provide to these questions. In an activity-based lesson, teachers need to take on a different role. Rather than transmitting knowledge to students, teachers are required to facilitate students' investigation and construction of concepts. This is a more open environment, where the teacher may have less control over student behaviour and the kinds of things that students learn. Therefore, teachers may be reluctant to take on an activity-based approach to teaching.

Purpose of the Study

This study is an investigation of teachers' understandings of the role of learning activities in secondary school mathematics, and examines the three key questions listed below:

- What views do teachers have of activity-related teaching methods in mathematics ?
- What are teachers' views on the nature of student learning activities in mathematics ?
- What curricular issues do teachers associate with learning activities ?

It must be noted that the investigation did not endeavour to ask "What are learning activities ?", but instead, to find out about teachers' understandings of the role of learning activities in secondary school mathematics. In the context of the transformation of mathematics education from an expositional model to a more investigative one, teachers were asked to disclose their views of learning activities, what they thought these comprised, and what they believed should be taken into consideration when planning for a learning activity. Information was also sought on the reason for teachers' attitudes about the role of learning activities in the secondary mathematics classroom. The study endeavoured to seek an insight into individual teachers' views on various aspects of learning activities.

Methodology and Scope

The direction that this study took with regard to its questions and methodology was representative of interpretative social science and hence, the study used a combination of qualitative and quantitative research methods. Although qualitative methods were predominantly used, quantitative methods were also used to analyse questionnaire data. The centre of interest in this study was not collecting large volumes of data, but organising and making sense of the variety of thoughts and experiences, that were gathered.

The research study was carried out in two main stages of investigation: an initial study and a subsequent study (see Table 1). The findings that arose from the initial study were taken into consideration for the second phase of the investigation. While the aims of the initial study was to find out what secondary school mathematics teachers considered to be an 'activity' and whether these 'activities' were an effective learning and teaching tool, the subsequent study aimed to examine teachers' understandings of the role of learning activities in secondary school mathematics. In the initial study, the participants were mathematics teachers from selected non-government schools. However, in the subsequent study, the participants were teachers from both government and non-government schools. With regard to data collection methods, a questionnaire and a semi-structured interview were the main research instruments in both phases. In both phases, all of the teachers that agreed to participate were required to complete a questionnaire. For the interviews, however, a subset of the teachers were selected, based on their responses to the questionnaire. Interviews were audio-taped for the purpose of obtaining an accurate transcript. Qualitative and quantitative data analysis methods were used to present, describe, analyse and interpret the results from the questionnaire. For the interview, on the other hand, qualitative data analysis methods were used to present, analyse and interpret the data.

Table 1 Structure of the study

Stage of data collection	Aims of Research	Data collection methods	Data analysis methods
Initial Study	To find out: · What secondary school mathematics teachers considered to be an 'activity'; and · Whether these activities were an effective learning and teaching tool	· Initial Study questionnaire · Semi-structured interview (audio-taped and transcribed)	· Quantitative and Qualitiative · Qualitative
Subsequent Study	To examine teachers' understandings of the role of learning activities in secondary school mathematics	· Subsequent Study questionnaire · Semi-structured interview	· Quantitative and Qualitative · Qualitative

Participants

As the study was an examination of the central topic, teachers' understandings of the role of learning activities in secondary school mathematics, this paper focused on the results emerging from this phase in the investigation. For the subsequent study, 12

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government and 12 non-government schools were approached. It was anticipated that about 2 teachers from each school would participate. Of the schools selected, 7 government and 4 non-government schools agreed. From these 11 schools, 26 teachers (18 government and 8 non-government) agreed to be participants in the questionnaire. Of these 26 participants, 7 agreed to participate in the interview. Only 1 of these 7 teachers were from a non-government school. Teachers were coded using a special coding system. The first letter of the code was either G or N to indicate whether they were from a government or a non-government school. The second letter of the code indicated the order in which the schools agreed to participate. The number was based on the serial number of the questionnaire which they wrote their responses in. The questionnaire documents For example, Teacher GA2 was given her code because she was from a government school, her school was the first government school to agree to participate, and she was the second teacher who filled in the questionnaire. Through gathering the data on their demographic characteristics, it can be seen that the participating teachers varied greatly in the length of their teaching experiences. There were teachers who had less than 10 years of experience, teachers who had up to 25 years, and teachers who had over 25 years. However, it must be understood that the length of teaching experience does not always affect teachers' perceptions about learning activities, as these perceptions could have been reshaped by in-service and professional development programs.

Results

Both the questionnaire and interview data were analysed in terms of the three key topics for this study. Teachers' actual responses to the questionnaire and interviews were used throughout the analysis to illustrate or reinforce the findings.

Teachers' views of activity related teaching methods.

From the questionnaire data, it was found that a significant proportion of the teachers believed that activities which were cognitive-active in nature (i.e. such activities that required students to think and participate actively) were more effective as teaching strategies. Teachers' believed that their teaching was more effective when they got students to: engage in problems and activities; solve a problem independently; or discuss a concept and apply it to real life. Consequently, most teachers had a positive view of learning activities. Teachers, however, had different ideas about using learning activities. Teacher GC2 believed that teachers had different 'understandings' about activities, as "there were some teachers who will not go any further than using one of those worksheets with the codes at the bottom, and that's their learning activities". Her personal preference was to develop students' thinking skills with the use of student-constructed questions. She found this to be particularly useful for students, as it gave them the opportunity to work at different levels.

Teachers also found that when they used learning activities, they were able to recognise that effective learning was taking place. Teacher GA2, who encouraged students to write about their mathematics, explained how she recognised effective learning taking place in her classroom. She attributed the efforts of students to her approach to teaching.

> Because I tend to take....mathematical and scientific words, and get the kids to write it in their own words, like "Maths is the stuff that you find one object" or "My Math"...[W]hen the kids talk about them, because the year 8's tend to be "I", they're egocentric like...little kids... "My maths is how much there is inside my body, or the things that make up my body", and they might use some different other term in it in the exam, and that one word would throw the whole thing, and the kids would say, "Oh, is that what you wanted me

> to"...you know, [or] "I know how to do that. Why don't they say it that way ?".

What is particularly interesting about teachers GC2 and GA2 is their emphasis on student-centred activities. In the former case (GC2), the students are placed in the traditional teachers' role of devising questions and problems to solve; in the latter case (GA2), the teacher facilitates student engagement in writing about mathematics. These "activities" do not involve manipulation of concrete materials, but they do require deeper engagement by students in reflecting on how to think mathematically. In the case of GA2, she encouraged to talk and write about their mathematics, and when they did, she found that they were learning and constructing their own mathematical knowledge.

Teachers' views on Student Learning Activities.

Most teachers were supportive of activities which had an investigative nature. Such activities included physical involvement and group investigations and problem-solving. they also perceived 'hands-on' activities to be effective for students as these activities involved mathematical modelling or creating visual representations. Teachers believed that these were worthwhile learning experiences for students, as it exposed them to various learning processes. Thus, the teachers supported Guild's (1989) view that "[w]hen teachers plan for a variety of ways for students to learn, more students can use individual strengths to be successful" (p.17). Speaking of their classrooms, GA2 and GF1 provided evidence supporting Guild's view:

> So, you've got students who learn visually, they got to see it all. Students that learn by tactile, by moving stuff around. Students learn stuff by discussing it, because it was a class group discussion that also took place with the cards and dice and stuff. Um and then, we wrote it down. We took it straight from the concrete to the abstract and related it directly on the board, so nearly everyone....all the teaching styles were catered for. [GA2:32]

> Some students learn by actually writing, ... they can't take it in by listening. So, if you're just explaining something, they're not going to learn from that, because they can't conceptualise. So then, you know, with a lot of things I do, I use the whiteboard or the blackboard, I use the overhead transparency a lot...At times I prepare worksheets, so they're doing again, so I guess it's the saying "Monkey-see, Monkey-do, Monkey-don't-do"....there's the students that can just listen and take it in; and there's the students who must do it to learn....so I try to make sure that I'm doing all those things. You're talking, you're putting up things visually through overhead transparencies and worksheets...[GF1:22]

Teachers GA2 and GF1 differed in their perspective on catering for learning styles. Teacher GA2 believed that through the many learning processes that she provided, students were able to construct their own mathematical knowledge. Teacher GF1 on the other hand, had a more managerial perspective. As her priority was to keep the class under her control, she used activities, in which she did most of the talking and presenting. This difference could be attributed to the different values and beliefs that they held about teaching.

Curricula issues associated with learning activities.

Many curricula issues arose throughout the questionnaire and the interviews. The more pertinent issues, related to learning activities included preparation and planning, accessibility of resources, discipline and coverage of mathematical content materials. Many teachers considered that being prepared was an important characteristic for an activity. To them, being 'prepared' meant knowing what resources were available, ensuring that the environment was controlled, and ensuring that instructions for the activity were simple enough to be understood. In the questionnaire, teacher GE1 offered a valuable comment on the organisation and preparation for the activities:

> To get the ideas, then the resources, takes a little time, and is sometimes a little daunting, however, it is almost ALWAYS very much worth the initial extra time and effort.

Accessibility of resources was an important issue, because teachers' knowledge of these determined their choice of activities. In addition to the planning and preparation issue, one teacher raised an issue about coverage of content materials. The teacher claimed that with syllabus constraints there isn't always the time to conduct activities for every topic, especially in the senior school. Teacher NE1 reinforces this with the comment below:

> "...the only restriction in the senior school is purely time. We have certain restrictions by which we have to work, and therefore, we have to make sure that we cover what we have to cover, and there is time to do learning activities, but not all the time. We have to be careful that we're not harming the kids' chances of progressing into university at the expense of just basically learning [NE1:66]

The discipline of students was also another important issue. "Strong discipline" was a priority with some teachers. In the questionnaire, teacher GB1 made a comment that:

> Experience has taught me that strong discipline, good classroom management skills, setting parameters, expectations and rules etc, are necessary if learning activities other than the formal lesson are to be attempted.

This was supported by a comment made by GG2 in the interviews:

> I cite the case of a few years ago of a teacher. He was first year out. He had wonderful activities planned for the students, wonderful hands-on things for students to do outside the classroom....but unfortunately, in this particular instance, he was not able to manage the students, so they were an absolute disaster...he probably needed to start off in a more formal setting, a bit ..more towards the chalk-and-talk...more of a teacher-dominated style of lesson first of all, until he was actually able to manage the students. He let the students go, and it was really "hell for leather"... he was well-prepared, but he wasn't able to mange a group of students efficiently or effectively. [GG2:32-36]

Based on his knowledge of this case, Teacher GG2, who himself was a very experienced teacher, believed that if activities weren't controlled properly, they were ineffective. Consequently, he recommended that beginning teachers establish a way of managing student behaviour before integrating learning activities into their teaching repertoire.

The teachers who participated in the study were in favour of using activities for dual reasons. As a teaching strategy, activities enabled teachers to determine whether learning was taking place. As a learning experience, activities exposed students to various learning styles. Teachers, however, had different ideas about how learning activities to be used. Other pertinent issues that were raised were accessibility of resources and strong discipline, which implies that teachers need to be effective classroom managers. The findings also implied that teachers be thoroughly prepared before conducting activities. This means that they need to know what resources are available at the school; the student group they are preparing for; and the year level for which the activity is targetted. These implications, however, are based purely on the participating teachers' views of learning activities. Although separate views on learning activities and classroom management were supported by the literature, views on the role of classroom management in mathematical learning activities have not yet been supported nor contradicted in the literature.

Recommendations for Future Research

With an opportunity for future research, this area of interest (teachers' understandings of the role of learning activities) could be investigated in alternative ways. To achieve greater consistency within the results, a more extensive examination of the teachers in Queensland schools would be useful, if schools across the state (rural and urban schools) were examined as opposed to schools in the Brisbane metropolitan area (government and non-government schools). Alternatively, observations of teachers carrying out learning activities would have been useful, as they would have supported or contradicted the claims that teachers made in the questionnaire and interviews.

With the availability of a longer time period, this area of interest could be investigated through a case study where two 'case' teachers are surveyed, interviewed and observed. Another alternative method would be to conduct an ethnographic study on a few teachers from government and non-government schools on their views of the effectiveness of learning activities. This would involve observing and interviewing these teachers and comparing what they say to what they do. Both approaches are useful because they would indicate whether teachers' understandings' of learning activities have any relation to the way they conduct these learning activities.

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Showing and telling: Primary students' outcomes in data representation and interpretation

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Students, in triads in a near-classroom environment, were video-taped as they worked on interpreting and representing supplied data. Their responses were categorised using the SOLO taxonomy. Representation skills varied from copying out some of the data to relating two variables graphically; interpretation skills varied similarly. Moreover, there appeared to be connections between the two skills. The study also considered the nature and effectiveness of the collaboration which took place within groups.

The performance of students working in collaborative environments has received considerable attention in the past 20 or so years, and such studies have often involved mathematical tasks. In the past decade the study of chance and data has become an explicit part of the mathematics curriculum across all grades in most states of Australia. This study arose out of earlier research which was initially concerned with the abilities of students in finding and representing relationships in data and which later came to consider, in addition, the nature of the collaboration that takes place if they are working on such tasks in groups.

Background to the Current Study

The "data cards protocol," which is described briefly in the next section, was first used to investigate students' functioning in the chance and data area of the curriculum in the work of Watson, Collis, Callingham and Moritz (1995). The researchers in that report studied students working in small groups in an actual classroom, using a single roving video camera to record what happened, before analysing the graphs and tables produced by the students for evidence of different levels of understanding. They found that there were two three-step cycles of functioning: one where the student acknowledges associations in data sets with increasing sophistication, and the second involving the justification of those associations. Later Lidster, Chick and Watson (1997) used the data cards protocol with both individuals and groups of three students removed from the classroom, to consider the way that hypothesising and data representation together contribute to the ability to interpret the data and confirm hypotheses. Again a video camera recorded the actions of the students, to gain evidence of what they hypothesised and what they produced as output. In the process of considering the four groups and twelve individuals used in that study it became apparent that one of the groups was particularly productive: the group members' hypothesising and interpretation skills were sophisticated, they grappled creatively with the difficulties of representing the data to justify their hypotheses, and, most notably, their collaborative interactions were highly efficacious. This prompted a closer study of that group and the remaining three in order to examine aspects of collaboration and determine their effect on cognitive outcomes (see Chick & Watson, 1997, for a case study of the single group, and Watson & Chick, 1997, for consideration of all four groups plus some groups involved in a different protocol).

The 1997 studies suggested that working in groups had some advantages over working alone for the data cards protocol, particularly for the noteworthy group, but it was acknowledged that the environment in which the groups worked was "ideal" in a certain sense. Each group worked on the protocol away from the regular classroom, with the interviewer present providing occasional cues. This may well have influenced the levels of concentration and cooperation. Certainly, it is not obvious that such productivity and understanding would result from group work conducted in a standard classroom, notwithstanding some fairly positive results achieved in the 1995 study. This question provided some of the motivation for the present study. In the light of the growing emphasis on chance and data in the curriculum, a second question concerned the natural level of understanding that the students actually bring to the protocol without receiving specific instruction in data representation and interpretation. The final issue of interest, following from the first two studies, concerns the connection between different aspects of statistical cognition — in particular, whether or not there is a relationship between the ability to interpret and the ability to represent the data.

In general the verdict on the effect of collaborative settings on outcomes associated with mathematical problem-solving tasks is still open (e.g., Davidson, 1985; Good, Mulryan & McCaslin, 1992). On one hand, Lehrer and Romberg (1992) were impressed with the data modelling which occurred when Grade 5 children worked as a group of ten in one instance and as a pair in another. Stacey (1992), on the other hand, had some reservations about the outcomes when Grade 9 students worked in groups of two or three on algebraic problem solving. She felt that students often selected the easiest or simplest approach to a problem, with detrimental consequences when such choices were incorrect.

It should be highlighted that the data cards protocol used in the current study is an exceptionally open-ended task. Unlike most mathematical problem-solving activities, which typically have a discernible end-point so that students know when they have achieved "the answer," the data cards protocol allows students to explore a variety of questions using numerous possible approaches. From some students' perspectives this open-endedness may be somewhat disconcerting.

Method

The data cards protocol on which the students worked (described in more detail in Watson, *et al.*, 1995, or Watson & Callingham, 1997) involved the consideration of a set of 16 cards, each bearing name, age, weight, weekly fast food consumption, favourite activity and eye colour information for a fictitious but realistic juvenile. The students were asked to look for and attempt to show any interesting features of the data.

The 27 students who participated in this study were from a Grade 5/6 class at a suburban primary school. Their teacher informed the researchers that the average abilities of the students were generally lower than other classes in her experience. She was asked to divide the class into groups of three, and she claimed that the students had successfully undertaken collaborative activities in mathematics in similar groupings. They had not, however, participated in any data gathering, representation or interpretation activities with this teacher.

There were five sessions in all, conducted over three weeks. To facilitate video-taping of the individual groups, the first three sessions were run twice, back to back, firstly with five of the nine groups and then with the remaining four. These sessions were 45 minutes long and the researchers who conducted them (the first author and another researcher) reminded the students of the expectation to work in groups. The first session was introductory and allowed an exploratory examination of the data cards, with a view to identifying any interesting features. At the beginning of the second session students were shown a variety of different ways of representing data, such as bar graphs, pie charts, tables and scatter plots. The students were encouraged to prepare a poster to display their findings, a task which continued into the third session.

As one focus of the research was to determine the "natural" statistical abilities of a "standard" class prior to specific instruction, the researchers were deliberately vague in describing the task and their expectations to students. Students were merely asked to try to find interesting things, followed up by questions like "How might you show that?" with an overarching instruction that the group was expected to come up with a poster to show what had been found. Showing the students some examples of graphs at the beginning of the second session was intended to provide a clue and a focus for approaches that could be taken if desired. To reinforce the potential value of graphical representation the researchers supplied graph paper as well as blank paper.

After the completion of the posters, a fourth session was held with the whole class, where members of each group were given the opportunity to present and explain their poster to their peers. During the fifth and final session the second author — who had not been present for the earlier sessions — interviewed the students individually, asking them questions about their posters and their feelings about group work. One purpose was to determine the students' abilities to explain their approaches and understanding to someone who had not seen their initial working.

All sessions were video taped, and where the students were working in their groups there was a camera focussed on each group. The tapes were transcribed and both the transcriptions and the original video tapes, plus interim rough work produced by students and copies of the group posters, were used to analyse the students' collaborative behaviour and statistical outcomes. This categorisation will be described below. Where judgements have been made about the levels of statistical outcomes attained these were made by the authors independently, and when initial interpretations disagreed discussion took place until consensus was reached. A similar approach was taken for categorising the levels of collaboration exhibited by each group, but involved the first author and the transcriber. Most disagreements were a result of slightly different interpretations of definitions; excluding these, agreement was reached without discussion being necessary in over 80% of cases.

Analysis

There were two aspects to the students' statistical results: (a) Their ability to observe properties and interpret the data, which will be referred to as "interpretation," and (b) their skill in depicting the data through, for example, graphical or tabular means, which will be referred to as "representation." Both aspects were analysed using the Structure of Observed Learning Outcomes (SOLO) model (e.g., Biggs & Collis, 1991) which considers *modes* of functioning and classifies students' responses according to the level of complexity observed and whether or not relevant information is considered. In this case the students are generally functioning in the concrete symbolic mode of the SOLO taxonomy. Three main levels are distinguished within this mode: unistructural, multistructural, and relational. Each level represents an increasing order of complexity, as indicated in Table 1; moreover these levels are identifiable for both interpreting and representation. There is also a prestructural level preceding unistructural; it is likely that this indicates functioning more typical of the ikonic mode. Finally, there is a level following relational, namely extended abstract (EA). This is characterised by responses which are more general, not necessarily restricting themselves to the specific details of the data or problem which elicited them. It indicates that the response is moving into the next mode of functioning, namely the formal mode.

A wide variety of collaborative — and non-collaborative — behaviours were observed, to the extent that it may have been reasonable to define nine categories for collaboration, one for each group. Some groups had episodes in which two or all members collaborated effectively, and then their behaviour would degenerate for a time. Other groups worked reasonably harmoniously, but with no extensive development of relevant ideas. A member of one particular group was unwilling to engage in the task, which had a deleterious effect on that group, although he later became motivated about an aspect of the work on which he then worked alone. At times, particularly when the students were producing graphs for the posters, there was a tendency for a group to appear to be three individuals, rather than a collaborative group. Nevertheless, despite the range of collaborative behaviours it was possible to classify the groups into one of four categories.

C1 There was little or no collaboration between group members.
C2 There was substantial disruption and/or antagonism within the group; however, some non-trivial collaboration took place between pairs or even the whole group.
C3 There were no overt clashes between group members, notwithstanding the occasional negative undercurrent; some sharing of ideas occurred, but not in a very productive way.
C4 Considerable useful collaboration took place; furthermore, the social environment was generally harmonious.

Note that our interest is in collaboration which directly affects mathematical outcomes, so that a group which socialised amiably without discussing the task would not be regarded as having collaborated productively. Some reasons for the disparate behaviour of the groups are proposed in the discussion.

Table 1.
SOLO levels used to classify students' responses in interpretation and representation.

SOLO level	General Characteristics	Interpretation	Representation
Pre-structural/ Ikonic (P/IK)	Uses no relevant information; may refuse to engage in the task, or attempts to deal with it ikonically	Introduces patently irrelevant ideas to the situation	Does not consider the data at all; e.g., colours in a graph produced by someone else, or decorates the poster
Unistructural (U)	Uses only one relevant aspect of the information	Considers individual aspects of the data; e.g., suggests reasons for the results on a single card at a time	Depicts individual aspects of the data set or a subset; e.g., records some or all data values in a table with no attempt at aggregation
Multistructural (M)	Several aspects of relevant information are used, often in sequential fashion rather than with more complicated connections	Considers all the data, but considers only one variable; e.g., notes that more people like television than any other activity	Can represent a single variable aspect of all the data; e.g., by drawing a bar graph of eye colour
Relational (R)	Integrated understanding of the relationships among various aspects of the information	Proposes cause and effect relationships between two variables; e.g. between fast food consumption and weight	Can depict a two variable relationship; e.g., by drawing a scatter graph

Results

During the first session, when the students were exploring the data and recording some of the things that interested them, very few came up with the idea of using a graphical representation to depict the data. Some wrote out information from a subset of the cards, others recorded some summative information in tables, while others verbalised their observations. At the beginning of the second session, however, when one of the researchers showed various graphs and tables, nearly all students indicated that they had seen and produced a bar graph on some occasion, with many indicating familiarity with pie charts, line graphs, and 2 × 2 tables as well. The least familiar type of representation was a scattergram. The researcher's presentation of different kinds of data representation was enough to prompt most of the students to use graphical approaches to the data for their posters. It is not clear whether this shift in strategy was because the students realised the value of such representations for summarising data and illustrating their attributes or because they judged that since the researcher seemed to value them — as shown by her demonstration — then that was what they ought to do.

As indicated in the previous section, students were classified on three criteria: the SOLO level of their interpretation, the SOLO level of their data representation, and the level of collaboration exhibited by their group. The results for the 27 students are shown in Table 2. Entries in the table are individual students, identified by their group number; the four narrow shaded vertical columns (within the representation columns) represent the different categories of collaboration, ranging from C1 on the left to C4. Thus, for example, the last entry in the table is one of the three students from Group 6, a group whose collaborative behaviour was assessed as being in category C3, while the student's SOLO levels were extended abstract for interpretation and relational for representation.

Table 2.
Students' outcomes and levels of collaboration on the data cards protocol.

		REPRESENTATION																			
		Nil				Prestructural/ Ikonic (P/IK)				Unistructural (U)				Multistructural (M)				Relational (R)			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
INTERPRETING	U					G7															
	M						G3 G5								G3 G3 G5 G9	G2 G4	G8 G8				
	R			G1				G1				G1		G7 G7	G5 G9	G2			G9 G9	G2 G6 G6	G8
	EA																			G4 G4 G6	

Collaboration categories: ▨ C1 ░ C2 ☐ C3 ☐ C4

It must be pointed out that the *highest* level of functioning which was observed was the one which was recorded. Some of the students ventured an interpretation based on a few cards — a unistructural response — before responding multistructurally by putting forward more general analyses. A number of students made no statistical contribution to the final poster, instead doing headings or decorations. These students have been classified as having a prestructural SOLO level of representation, or nil in the case where nothing was done at all. The assessment of the students' representations was based on their work during the sessions and not just on what was on the final posters. One of the students in Group 6 had completed a scattergram (or equivalent) which was omitted from the final poster because it was similar to but judged as messier than one of the other student's efforts, while the third student in the group started a scattergram but was absent for the third session and so did not complete it.

As can be seen from Table 2, half of those who actually produced a representation of the data, did so at the same SOLO level as their interpretation, with either both outcomes being multistructural or both relational. The remaining thirteen students were able to interpret data at a higher SOLO level than they were able to represent it. Perhaps not surprisingly no students gave interpretations which had a SOLO level lower than that of their representations. It might be expected that valid and complex interpretations can be made even when there are shortcomings in the representation. On the other hand, if an appropriate representational form has been done well it cannot help making the interpretation easier. The power of suitable representations is such that they describe the data in an obvious way, and lend themselves to having interpretations made at a level commensurate with the depiction. During the researcher's illustration of a

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variety of graphical forms at the beginning of the second session, many students indicated an understanding of what the graphs and tables were telling them, yet very few chose to use the more complicated forms, such as scattergrams, to illustrate relationships such as that between weight and fast food consumption in their own analyses. It is thus proposed that there is a cognitive difference between being able to interpret data presented in a carefully selected representational form and being able to choose the appropriate form for oneself.

The data in Table 2 further support this hypothesis. Two of the students who produced multistructural representations but gave relational interpretations came from groups (Groups 2 and 9) in which another member had produced a relational representation. These students were able to interpret the graphs made by their colleagues, either by direct examination and interpretation of the graph, or as a consequence of discussions that took place among the students in the course of the collaborative group work. The remaining students who had relational interpretations and lower level representations, seemed to have based their interpretations on an extensive study of the cards, and thus proposed some relationships in the data despite the fact that no one in the group had depicted the data in a relational way.

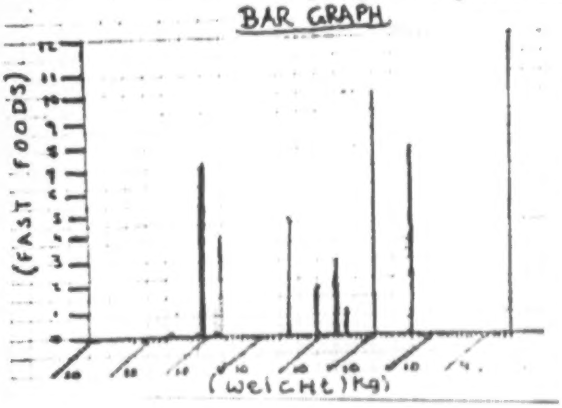

Figure 1. Two variable graph completed by a student in Group 6.

The remaining three students, whose interpretation level is indicated as extended abstract in Table 2, displayed a level of interpretative behaviour which went beyond the others. Two of them had completed a scattergram (or similar), while the third was a member of the same group as one of those who did so. All were able to discuss the fact that the data did not fit a theory of increased weight with increased fast food as well as they had supposed. Having had no explicit experience with variability in data sets one of them was prepared to reject his hypothesis on the basis of the lack of a more uniform linear trend in the data (see Figure 1). They were the only students of the 27 to indicate that the representation was allowing them to *prove* (or, in this case, cast doubt on) an hypothesis, rather than merely showing what had already been observed. One commented that "It doesn't show what I wanted it to show," which led to a discussion about other influential variables, such as age, and as a result he produced a weight versus age graph which was also included on the poster. For most other students the graphs seemed only to give a concrete representation of what they already knew. The SOLO level of these three interpretations is verging on extended abstract, because of the students' ability to interpret the chosen data beyond just making a summative observation.

Only one student interpreted the data at a unistructural level by recounting various facts from individual cards. She seemed unable to interpret the graphs produced by her colleagues in the final interview. Her own representation level was prestructural because she just wrote down data from a few of the cards that interested her.

Finally, it appears that the nature of the collaborative behaviours in the groups has little influence on the outcomes. Students with lower level outcomes came from groups whose remaining members did quite well, and this was true of all collaboration categories. Certainly it would be inappropriate to make any generalisations based on the data as analysed thus far.

Table 3 summarises the different kinds of representational form used by the students on their final posters. Some students produced two graphs for the final poster while others did not have any thereon for a variety of reasons. This explains why there may be more or fewer than three representational forms for a given group.

Table 3.
The different representational forms used by the students on their posters.

Representation type (approximately in order of increasing SOLO level, where P=Prestructural, U=Unistructural, M=Multistructural, R=Relational)	Students producing that graph representation (identified by group number)	Total
Table (partial information) (P)	G5, G7, G7*	3
Table (all aspects of all data) (U)	G1	1
Bar graph (partial information) (M)	G9	1
Bar graph (frequency) (M)	G2, G2, G3, G3, G4, G4, G7, G8, G9, G9, G9	11
Pie chart (M)	G8	1
Line graph (but equivalent to a bar graph) (M)	G5	1
Bar graph with grouped data (weights) (M)	G9	1
Bar graph with an extra variable (girls v boys) (R)	G5, G9	2
Pie chart with information about an extra variable (R)	G8	1
Scattergram, but with lines rather than dots (such as shown in Figure 1) (R)	G6, G6	2
Scattergram (R)	G4	1
Two comparative scattergrams (girls v boys) (R)	G2, G4	2

* This table consisted of a very specific subset of the data — those who watch television — and was adjudged to be a multistructural representation.

Discussion

It should be pointed out that there was a range of performance levels even within the various SOLO categories. Some students, for example, made relational connections but were not convincing in their discussion of them, while others considered more general issues such as whether the results of the data analysis apply to other people. The ability to employ SOLO classifications for both the interpreting activity which took place and the representations (graphical or tabular) produced is very useful in this setting. The fact that half of the students performed at the same level on both aspects of the task indicates they can combine understandings in two related domains consistently. The remaining students were able to interpret the data at a higher level than their representation skills, suggesting further research is needed to determine reasons for this.

As mentioned earlier, collaborative setting seems to have no discernible effect. Discrepancies such as the fact that individuals in some groups do not do as well as others in the same group point to the need for a closer consideration of the other factors involved in collaboration in the classroom setting. It is certainly true that these groups did not work together as well as the three boys who formed the focus of the study of Chick and Watson (1997). Indeed, the researchers had a definite feeling that the groups often behaved more as three individuals than a genuinely collaborative team.

It should be reiterated that the groups were formed by the teacher, and that the students had worked collaboratively on mathematical tasks before this study was conducted. Nevertheless, the groups formed for the data cards protocol did not appear to be the same as those used on a problem-solving task observed by the researchers prior to the data cards sessions. This was backed up by a comment made by one of the students during the course of the activity. All the groups were mixed, by gender in all cases and by school grade in most. The authors feel that this probably had a strong influence on the outcomes of the study, and that it would have been possible to form "better" groups. It must be emphasised, however, that one of the purposes of this study was to consider the realities of the classroom, and that teachers may not always be able to choose the best groups in the first instance or, indeed, in any instance if there are constraints to be taken into account. In this case, the researchers had imposed the requirement that the groups have three members, as this duplicated the group size used in the earlier studies and it also made viable the video taping of individual groups. Furthermore, the teacher apparently consciously decided that she wanted the students to work in mixed-gender groups, as this was observed during other visits to the classroom.

In this study the students did not readily bring their previous knowledge and experience to bear on this problem. As mentioned in the results section, despite having encountered and produced various representational forms prior to the sessions involving the data cards protocol, virtually all students were unable to transfer that experience to this situation without being cued by the researchers. This points to the need for teachers to make these connections explicit in the upper primary years.

There is scope for further research on the data obtained from this class of students. As mentioned earlier a wide variety of collaborative behaviours was observed, not all of them constructive. It would be interesting to determine whether or not useful ideas were lost because of detrimental behaviour, and to examine the students' perceptions of the collaboration which took place in their groups.

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Representing the connectedness of mathematical knowledge

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An important area of mathematics teachers' expertise is their content knowledge. One way to characterise the quality if this knowledge is to examine the organisation of the various components that form the content knowledge base. In this paper we describe the procedures we have used to generate estimates of the state of connectedness of mathematical knowledge of teachers. We illustrate the procedures by focusing on a single teacher's knowledge of the concept of square.

Introduction

During the past ten years the role of mathematics teachers has changed considerably. The current thrust in teaching strategies places emphasis on understanding mathematics by helping students make sense of what is taught in the class. These changes are articulated in the recommendations such as that there is a need to 'shift from dispensing information to facilitating learning' (National Council of Teachers of Mathematics, 1989, p. 128). This change in emphasis for teachers' actions recognises the crucial role that teaching plays in influencing what knowledge is constructed by students. Even though the ultimate control of the knowledge construction process rests with the student, it is clear that the actions of the teacher in a lesson do have substantial impact on the outcome of that process.

In the wider project in which this study is embedded we are interested in drawing links between the knowledge and actions of mathematics teachers and the mathematical knowledge and actions of their students. In pursuing this objective we have examined the knowledge states of teachers and students, observed the planning and delivery of lessons and then investigated the knowledge states and problem solving performance of students. We are interested to see how the actions of the teacher might facilitate the establishment of a well-connected structure of knowledge in students. We have assumed that the generation of these connecting, or linking, actions by the teacher is affected to a substantial extent by the state of connectedness of the teacher's own knowledge. In this paper we describe the procedures we have used to generate estimates of the state of connectedness of mathematical knowledge of teachers. We will illustrate the procedures by focussing on a single teacher's knowledge of the concept of square.

Background to the study

In recent years a number of studies have begun to address the above question by focussing their efforts on providing a rich description of knowledge and skills that teachers bring to the classroom. A review of the discussions of the quality of teaching (Merseth, 1993) and of the research literature on expert and novice teachers of mathematics (Leinhardt, 1989) clearly indicates that teachers need to be able to access and exploit three types of knowledge both before and during teaching.

The first type of knowledge is knowledge about learning, about learners and about the context of learning. This includes information that the teachers hold about the processes of learning, knowledge of learners' repertoires of conceptual and procedural knowledge about the topic (Leinhardt et al., 1991; Shulman & Sykes, 1986; Tamir, 1988); knowledge of the learners developmental states and conceptions of the learners' motivational patterns (Leinhardt et al., 1991); and their conceptions about learning of the subject matter (Pramling, 1992).

The second type of knowledge required by the teacher is knowledge about how to teach the mathematical content to the learners. Pedagogical content knowledge (Shulman & Sykes, 1986) includes components such as : understanding of the central topics in each area of subject-matter as it is generally taught to children of a particular grade level; knowing the core concepts, processes and skills that a topic can convey to the students; knowing what aspects of a topic are most difficult for the students to learn; knowing what representations are most effective in communicating the appropriate understandings or attitudes of a topic to students of particular backgrounds; and knowing what student misconceptions are likely to get in the way of learning (Tamir, 1988); and the teacher's perceptions and beliefs about the role of a teacher and the teacher's perceptions about students. Finally the teacher must have a store of appropriate mathematical content knowledge. It is this mathematical content knowledge that is the focus in this paper.

Mathematical content knowledge

Included in the teachers' repertoires of subject-matter knowledge are: (1) substantive mathematical knowledge, such as facts, ideas, theorems, mathematical explanations, concepts, processes (and connections between these elements) (2) understanding of knowledge about the nature and discourse of mathematics, (3) knowledge about mathematics in culture and society, and (4) dispositions towards the subject (Ball and McDiarmid, 1990).

Both the quantitative and qualitative characteristics of this knowledge are of interest. In a crude sense, the breadth of mathematical content knowledge has relevance for any assessment, since the mathematical actions of the teachers will be influenced by what knowledge they have available. So quantity of knowledge is important.

However, sheer quantity of knowledge is not all that is of relevance in influencing teaching action. The state, or quality, of organisation of that knowledge also makes an impact on the accessing and use of knowledge. Poorly organised knowledge may well not be accessed at the appropriate time during teaching or problem solving (Prawat, 1989; Schoenfeld, 1985). Lawson and Chinnappan (1994) showed that a substantial part of problem-relevant knowledge could well remain inert during problem solving. In that research the problem solvers for whom access was difficult also showed evidence suggesting that the extent of connectedness of knowledge was limited compared to that shown by successful problem solvers.

Knowledge connectedness

The quality of connections among knowledge components is assumed to influence the ease with which the presence of one element aids in the retrieval and use of another in a problem environment. Schoenfeld (1985) observed that development of mathematical thinking requires not only mastering various facts and procedures, but also understanding connections among them. He suggested that there is value in providing detailed descriptions of these structures that support such thinking. As yet we have very few detailed discussions of the ways in which the quality of connectedness of elements of knowledge can be investigated and represented.

Mayer (1975) represented the accumulation of new information in Long Term Memory as adding new 'nodes' to memory and connecting the new nodes with components of the existing network. He utilised this nodes-network framework to examine learning outcomes along three dimensions. In two of these dimensions, the notion of connectedness was employed to evaluate prerequisite knowledge and the activation of assimilative knowledge structures. 'Internal connectedness' was referred to the degree to which new nodes of information were connected with one another to form a single well-defined structure. The degree to which new nodes of information were connected with information already existing in the learner's cognitive structure, was called 'external connectedness.'

In a further analysis of mathematical knowledge structure, Greeno (1978) noted that within the schema the learner could also be expected to establish connections of how the concept relates to specific problems and situations. This notion of connectedness can be seen as an attempt to establish a link between knowledge of a concept and knowledge of how to use that concept, thus being concerned with the proceduralization of declarative knowledge.

We have used the notions of internal and external connectedness as a conceptual framework for representing the qualitative characteristics of the teachers' knowledge. We see this as useful for two main reasons. First, it seems conceptually sound in that it allows us to represent the complexity of a knowledge base in a way that focuses on the state of organisation of that knowledge. Our second reason for use of this connectedness framework is that it provides a basis for establishing relationships between knowledge states and teaching actions. In relation to this second purpose we suggest that one of the key motivations of the teachers, and one of the most important sets of their actions, can be seen as attempting to facilitate the establishment of both internal and external connections in the knowledge bases of the students. In this way we argue that it makes sense to seek out relationships between the teachers' knowledge, those of their actions designed to improve students' knowledge connectedness, and the resultant state of connectedness of the students' knowledge base. Thus we will focus here on trying to represent the connectedness of a teacher's knowledge, using the square as a focus concept.

In representing this connectedness we will lean heavily on use of the concept map. Since their popularisation by Novak (eg. Novak & Gowin, 1984) concept maps have been seen as useful ways to represent the spatial structure of knowledge relationships (Lawson, 1994). Other related mapping systems have been developed for essentially this purpose, such as Chi, Hutchinson and Robin's (1989) network model of knowledge coherence and McKeown and Beck's (1990) semantic net representation of students historical knowledge. In this paper we will describe how we have used the concept map and its characteristics to generate an assessment of the organisational state of a teachers' knowledge of the concept of a square.

Method

Participants

In order to get a pool of participants for this project we approached a number of organisations within the states of Queensland and Western Australia including the Education Departments, the Mathematical Associations. A male high school teacher who has twenty years experience in mathematics was among the group of volunteers involved in the project. The teacher was the head of the mathematics department of his school.

Tasks and procedure

Three tasks were developed for the purposes of assessing teacher's knowledge about geometry. The first task, Free Recall (FRT), required the teacher to talk aloud about any idea that he could associate with the topic of plane geometry such as shapes and their properties. The second task, the Problem Solving task (PST), consisted of four problems. All the four problems were related either to plane and/or coordinate geometry. The first problem involved the use of Pythagoras's theorem in a rectangular coordinate system. Solution of the second and third problems required an understanding of the relationship between properties of squares and rectangles, segmentation of their areas and gradient of straight lines in a rectangular coordinate system. The fourth problem, again necessitated an integration of knowledge of properties of square and right-angled triangle. The final task, Geometry Probing (GPT), was designed to obtain further information about teachers talked about during the Free Recall task. In the GPT the teacher was required to respond to a number of questions each of which aimed at probing what was said during the first session. The probes were designed to provide the teacher with opportunities to display his available knowledge of features of the target concept and his knowledge of relations between this target concept and a list of other concepts relevant to this area of mathematics. The checklists of features and related concepts that had been established in pilot work with expert mathematicians and are listed in Tables 1 and 2. For example, if the teacher mentioned symmetry of squares in the FRT but did not elaborate on this idea he was asked to explain this idea in the probe session. This strategy was expected to provide more data about what the teacher knew about the squares. The use of the above techniques have been argued to provide rich data about interconnections that exists between knowledge units, and their structure (Royer, Cisero & Carlo, 1993).

The teacher met with us on three occasions and undertook one of the tasks in each session, in the order indicated above. In the first session he was invited show his knowledge of geometry by talking, drawing and writing in response to the FRT. During the second session he was asked to complete the PST which involved generating solutions to four problems. He was encouraged to talk aloud during the solution attempts. In the third session we sought the teachers' responses to the probe questions. All three sessions were video-taped and transcribed for subsequent analysis.

The data analysis focussed on accumulating as much data as possible about the teacher's knowledge of squares. The use of three different contexts in which the teacher was required to search for knowledge helped maximise data generation. Two experienced teachers of mathematics analysed the transcripts and video-recordings. Any differences in their interpretation of the data was resolved in consultation with a third person who was also an experienced mathematics teacher and teacher educator. In order to generate a more complete picture about the extent of the teacher's knowledge we developed a concept map and devised a system for scoring of the concept map.

Structure of the concept map

The concept map was designed to allow identification of four types of information in the teacher's performance:

1. The features of the target concept, both the essential, or defining and other related features;
2. Relationships that were established within the features of a concept;
3. Relationships established between the target concept and related concepts.
4. Other representations of the concept, such as in analogies, illustrations or real life examples.

The concept map shown in Figure 1 was structured to record these types of information. Information related to the features of the concept was recorded in the bottom left corner and elaborations on these features were recorded in the lower right section of the map. Links to related concepts were recorded on the upper right part of the map. Type 4 information was included in the top left part of the map. Where relationships were expressed they were labelled on the map.

Scoring of levels of connectedness

Table 1 shows the types of judgements made by the teacher that were internal to the schema for square. These internal connections made about of the square were separated into two categories, *essential* and *related Features*. Essential features refer to properties that were defining properties of the target concept. Related features refer to properties that were derivatives of squares, such as area and symmetry of square.

The teacher's verbal and physical actions in the three tasks were examined for identification of the features and any elaborations, or expansions, on any of the identified features. In Table 1, the teacher mentioned that squares had line symmetry, which was a counted once under the column, Initial Link. The teacher proceeded to expand on this point about line symmetry by giving us four distinct elaborations about symmetry. This is noted in the Elaboration column.

In addition to counting the nodes, we were interested in making judgements about the quality of these connections. This was achieved by scoring the teacher's responses in relation to a scale of connectedness that had three levels. Level 1 connectedness recognised that features had been noted. A level 2 rating was given if a feature was both noted and elaborated upon. If the noted and elaborated features were further discussed by establishing properties of an elaboration a level 3 rating was recorded. The levels of connectedness are intended to represent the richness of connections associated with the target concept.

Any connections made between a square and other geometric figures were the subject of a separate external connection analysis. A significant aspect of a teacher's knowledge base could be expected to be concerned with relations between squares and other geometric figures such as triangles and parallelograms. The node frequency and levels of connectedness of these external connections were scored in the same way as internal connections.

Results and Discussion

Concept map

Figure 1 shows the concept map that was constructed for the focus concept of square for our expert teacher, Gary. The most visible aspect of this map is that Gary has built up a reasonably large number of connections involving squares as shown by the number of arrows emanating to and from the focus concept. The bottom left hand corner of Figure 1 shows that his knowledge about the properties of squares is extensive. The top-half of the map indicates that Gary has been able to link squares with other polygons such as rhombus and isosceles triangles. Gary's understanding of defining properties of a square was extended to more advanced features such as symmetry and area. This component of his knowledge is depicted by the right-half of Figure 1. The upper left section of the concept map provides information relevant to the contexts in which the above knowledge components were activated and the ways that these would be expressed by this teacher.

The concept map of the expert teacher revealed that he has a rich store of knowledge as manifested by the multiple links he had constructed with many facets of a square. These links suggests that the teacher not only has a well-developed understanding about the properties of square but more importantly, his has an extensive network of connections between squares and other geometrical figures. For example, he was able to represent a square as a polygon and as a quadrilateral. A square was seen as a subset of rhombus, though the relationship was not seen as symmetrical. His ability to recognise a right-angled triangle as forming part of the square also indicated that the teacher was able to use basic properties such as right-angles to create other figures. Gary's knowledge base indicates that he was able to highlight differences and similarities that exist between square and related figures. That is, he has an advanced relational understanding of geometric figures.

Gary commented that a square has symmetrical properties and that this property can be created by more than one way. This understanding of symmetry is extended to his observations that squares tessellate. Here one can detect not only an understanding of the geometric properties but also practical applications of squares.

Analysis of connectedness

Results of analysis of the focus concept in terms of number of links and quality of the links are provided in Table 1 and 2. As explained earlier, Table 1 shows internal connections whereas Table 2 shows external connections made by the teacher. Gary made a total of eight links with information that was directly associated with square and five links with information which can be seen as extensions of his knowledge about square. Gary was able to access both knowledge related to each of the essential features of the square and knowledge of each of the related features.

The 'Level of Connectedness' column in both tables provides a measure of the richness of connections associated with this schema knowledge. This analysis suggests that Gary's knowledge of square is quite well elaborated. Five of the seven judgements about level of connectedness were at level 2, with one each being at levels 1 and 3.

The analysis represented in Table 2 suggests that Gary has established a wide range of links from square to related concepts. The only omission from his discussion was the description of square as a plane figure. The level of complexity of these external connections from square is generally lower than those observed for the internal organisation of knowledge about a square, with only two of the six judgements receiving level 2 or 3 ratings.

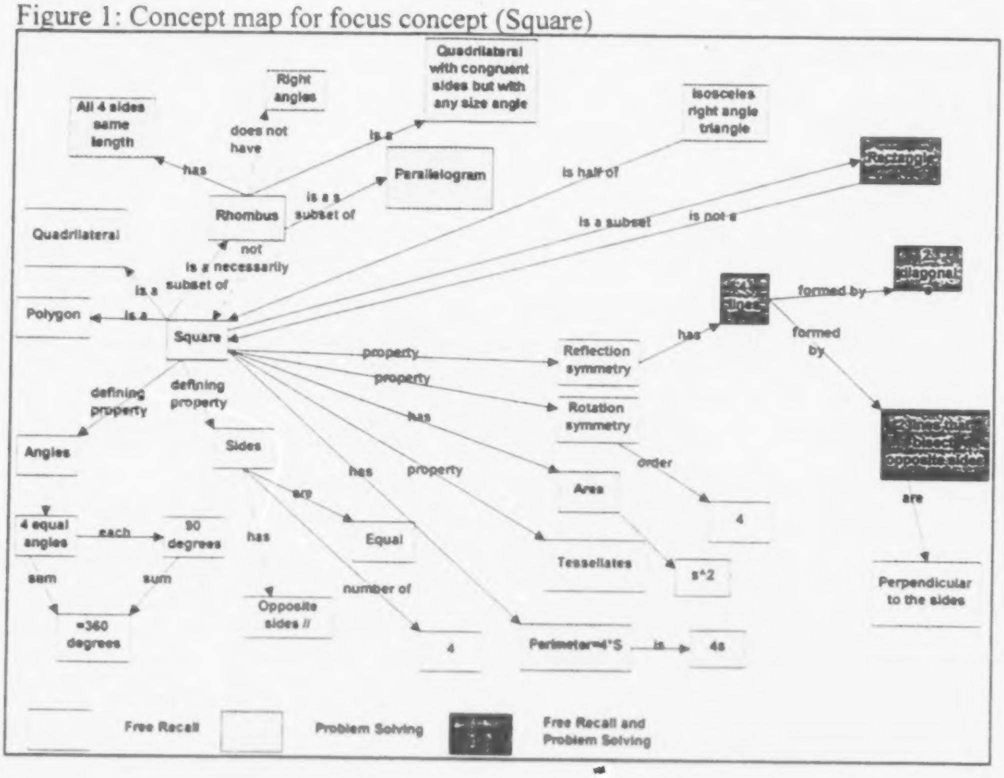
Table 1- Focus Concept: Square (Internal Connection)

Features	Node Frequency		Level of Connectedness
	Initial link	Elaboration	
Essential Features			
Angles			2
Four	1		
All equal	1		
All 90 degrees	1		
Sides	1		2
Four	1		
Equal length	1		
Opp. parallel	1		
Sub total (essential)	8		
Related features			
Area	1	1	2
Perimeter	1	1	2
Line symmetry	1	4	3
Rotational symmetry	1	1	2
Tesselates	1		1
Subtotal (related)	5		
TOTAL	13	6	

Table 2 - Focus Concept: Square (External Connection)

	Node frequency		Level of connectedness
	Initial link	Elaboration	
Plane figure			
Rhombus	1	4	3
Right-angle triangle	1		1
Rectangle	1		2
Polygon	1		1
Quadrilateral	1		1
Parrallelogram	1		1
TOTAL	6	4	

Figure 1: Concept map for focus concept (Square)


Quadrilateral with congruent sides but with any size angle
Right angles
All 4 sides same length
does not have
is a
has
Parallelogram
Isosceles right angle triangle
is half of
Rectangle
Rhombus
is a s subset of
is a subset
is not a
Quadrilateral
not
is a necessarily subset of
is a
Polygon
is a
Square
diagonals
lines
formed by
has
formed by
property
Reflection symmetry
property
Rotation symmetry
has
defining property
defining property
Angles
Sides
has
property
order
Area
lines that bisect opposite sides
are
4 equal angles
each
90 degrees
has
are
Equal
Tessellates
4
s^2
Perpendicular to the sides
sum
sum
=360 degrees
Opposite sides //
number of
4
Perimeter=4*S
is
4s
Free Recall
Problem Solving
Free Recall and Problem Solving

Conclusion

The purpose of the study reported here was to provide data about an expert teacher's knowledge about concepts in plane geometry. In the present study we focussed on knowledge the teacher has constructed about squares by providing three different contexts in which the teacher could have displayed his understanding of square and its applications. The focus of our representation of this knowledge was on recognition of both the presence/absence of details of the knowledge and judgment of the richness of connectedness of this knowledge , both internally and externally.

Experience with this procedure suggests that there is value in use of the idea of connectedness to make judgements about the qualitative features of a knowledge network. The procedures we have developed here extend the utility of Mayer's (1975) distinction between internal and external connectedness. The levels of connectedness framework allowed us to make a distinction between the richness of the internal and external

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connections associated with square that were made by this teacher. In a practical sense this analysis seems to have provided a way for use to establish a qualitative estimate relevant to the teacher's knowledge base that can be used in other parts of the project where relationships between mathematical content knowledge, pedagogical knowledge and teaching actions are of concern.

Our expert teacher has a wide body of knowledge related to squares and other geometrical figures in the area of the plane geometry. Beyond this quantitative judgement our analysis provides a basis for making statements about the quality of organisation of this knowledge. In this regard we suggest that this teacher's knowledge is richly organised, having a high degree of internal connectedness and a moderate degree of external connectedness. Analysis of the teacher's output in other parts of the interview, where other concepts were the target concepts, will allow us to gain more information that might be used to validate our judgement about the richness of the external connections. Experts organise their domain knowledge in meaningful clusters such as schemas (Marshall, 1995). Because experts construct multiple connections with core concepts, they can be expected to access and use that knowledge in different contexts. Thus, organisation improves assimilation of new information and accessing of prior knowledge as evidenced by our expert teacher.

The variety of contexts in which the expert teacher could display a particular concept can also explain the richness of explanations such teachers are able to generate during the course of a lesson. Recent work in the area of arithmetic (Leinhardt, 1988) has shown that expert teachers draw on a well-developed content knowledge base in order to make inferences and generate explanations during the course of their instruction. The results of this study is consistent with the these studies.

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Beginning teachers problems with fundamental mathematics

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A Ministerial Report has questioned again whether beginning teachers are adequately prepared at universities. The Report proposes measures to ensure this does happen such as a testing regime in mathematics and literacy at the completion of courses. This paper provides data that indicates some beginning teachers do not show competence in the mathematics they will soon be teaching. Hence there is a basis for concerns raised in the Ministerial Report. However it is argued that the solutions in the Report are somewhat naive and an inadequate response to this problem.

'Tough new test plan for teachers' (Jones, 1998, p.1) was the front heading on page one of The Age, Melbourne's broadsheet. The article was commenting on a Victoria government Ministerial Report which examined teacher training. Two of the recommendations of this report are, to raise the quality of students entering teacher preservice courses, and the need for a demonstration of minimum literacy and numeracy skills for graduating preservice teachers. Clearly there is ongoing concern in the community as to whether teachers can demonstrate a comprehensive command of the material that they are expected to teach.

This concern of entry and exit standards for preservice teacher courses is not new. It has been expressed by the community before, and indeed has been an ongoing concern within the profession. In the late 1980s the mathematics and science components of preservice teacher courses were examined and resulted in the Speedy Report (1989). This Report recommended that entry students should have completed years 11/12 mathematics (their interpretation was years 11 and 12, not as a number of institutions interpreted the recommendation as year 11 or 12), and that all primary preservice students should complete one unit of tertiary mathematics to ensure a good grounding in the subject.

An examination of past Mathematics Education Lecturers Association conference books show that there has been a concern within our profession, not only with our students' affective responses to mathematics, but with their mathematical achievement. So Southwell and Khamis (1987) noted that;

> It is reasonable to assume that teachers in Years K-6 have a sound grasp of the concepts and skills related to elementary mathematics in order to provide their students with the kinds of experiences in mathematics which will enable them in turn to develop strong mathematical skills. Consequently, some measure of the performance of pre-service teachers is desirable. (p.120)

In a number of other papers and reports lecturers report on various approaches to deal with students who were at risk because of their lack of skill and understanding (e.g. Bell, 1989; Perry, 1993; Redden & Clark, 1991).

The starting point of this paper was my ongoing concern with my students in the preservice course I help teach who seem to have difficulty with fundamental ideas of mathematics. These students complete a compulsory semester unit in mathematics, which I believe is innovative and excellent (for a description of the unit see Clarke & Clarke, 1996). Then follows two semester units in mathematics education. The teaching of spatial, measurement, and number curriculum ideas, among other topics, are dealt with as part of the first of the two mathematics education units by lecture, tutorial and workshop activities. No claim is made that these units are exemplary. They are, I understand, rather typical of comparable units in other university courses.

Part of the assessment for this unit is an examination. A number of items on the examination deal with the mathematical concepts covered in the unit. Clearly this group of items do not of themselves constitute a reliable 'test', in the normally accepted sense. This paper reports on how some students performed on these items. It is rather an audit of the pattern of results that occurred, but this I think is of interest for what it suggests.

The report is broken into two sections. The first deals with number and some measurement items, and the second with spatial items.

Number and Measurement Items

The results from one cohort of 63 students taught in 1997 were available. The vast majority of these students were either in their second year of a combined BA/BT program. Some six students were in their first year of a BT degree, open only to students who had previous tertiary qualifications. As is typical of this group of students, 90% were female. Many were from non English speaking backgrounds. This group of students did not appear to the author to be any different to groups who had been taught over the years.

The students' responses are shown in Table 1 for the number items, and Table 2 shows their responses to the measurement items. The students were not permitted to use calculators for these items on the basis that as teachers, although in their teaching we will want them to encourage the appropriate use of calculators, they should still be able to process such skill items without a machine.

Table 1
Number Items and Facility for 63 Students

Items	Facility
368 + 229 + 43.45	92
5000 - 225.8	76
435 x 59	74
680.3 ÷ 7 (correct to one dec. place)	31
258 + 56 x 3	53

Table 2
Measurement Items and Facility for 63 Students.

Items	Facility
What metric units would you use to measure:	
a large area of land (answer: hectare or Km^2)	36
a very small capacity of water (answer: *ml*, cm^3)	87
Using metric units estimate the:	
height of the door (answer: 1.76 - 2.25 *m*)	56
temperature of air in room (answer: 16° - 20°)	48
253 *gm* = … *kg*	59
2 *l* =…*ml*	75
31 *mm* =…*m*	34
round off to nearest centimeter: 3.568127 *m*	33

Clearly these results are not the source of great joy to me as the lecturer concerned. It can be reasonably claimed that none of these items appear to be very hard and all could be found in the primary curriculum. Indeed all items should be completed correctly by many grade 6 pupils. These results are little different to scores on a similar set of items reported in the 1980s from students completing the equivalent unit in that pre service education course (Clarkson, 1987/88). Anecdotal data from some colleagues suggests that such results would not be atypical for their students.

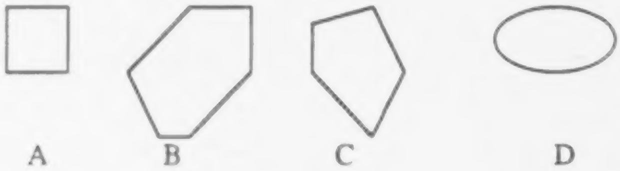
Spatial Items

The other group of items dealt with spatial relations. It was interesting to note when scanning through past papers that very few comments or test items sampled this vital area of the curriculum at all. Hence in reporting the students' responses below, a number of incorrect answers are also listed, and some more extended discussion is included.

Results were available from two cohorts of students. The first cohort were a group of 67 students in their second or third year of a combined BA/BT four year course taught five years ago. The responses for these students are shown below each of the following items in the first column. The second group of students were the 63 taught in 1997 and their responses are shown in the second column below each item. There was little difference in the gender or ethnicity make up of these two cohorts.

The spatial items are listed below with the percentage or students who chose the shown alternative. Normally only the correct response is listed. When more than one response is presented, the starred one is correct.

Item 1: Give the following shapes their correct mathematical name.



A.	Square	97	87
	Other students suggested a cube.		
B.	Hexagon	46	51
	Other answers given were rhombus, pentagon, polygon, trapezium, sextagon, rectangle, heptagon, oblong, octagon and irregular.		
C.	Pentagon	40	33
	Students also chose hexagon, rhombus, diamond, trapezium, polygon, quintexagon and irregular		
D.	Ellipse	16	5
	Oval	73	84
	Other names given were sphere, polygon, spirolateral and oblong.		

Comment: It would have been hoped that tertiary students would be able to name common mathematical shapes. Leaving aside whether 'polygon' and 'oval' are acceptable, the responses are worrying.

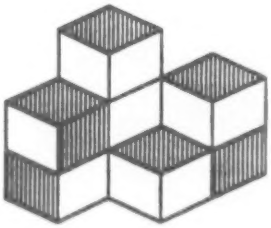
Item 2.

Imagine you were helping a child build a pile of blocks using this diagram.

(i) The parts of the blocks which are shaded are all .... of the blocks.

(ii) Indicate on the diagram an edge of a block.

(iii) If there are no blocks hidden behind this pile, how many blocks will you need?



(i) Sides/faces/surfaces/ends 64 83

Other answers given were edge, tops, square, widths and dimensions.

(ii) Edge 78 80

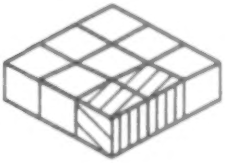
Other answers given were a corner indicated, a side indicated and some gave no answer.

(iii) 8 67 77

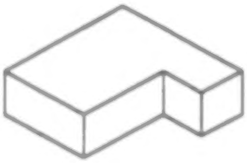
Most other students gave 7 as the answer, the number of blocks in view.

Comment: The first two parts of this item again involve naming, but in a different way to item 1.

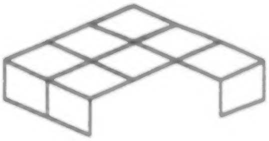
Item 3: This building has been made from cubes. The two shaded cubes are taken away. Draw what remains. (The drawing had an isometric dotted background. The position for completing the drawing was on isometric dotted background.)



Original drawing



Answer - Outline correct



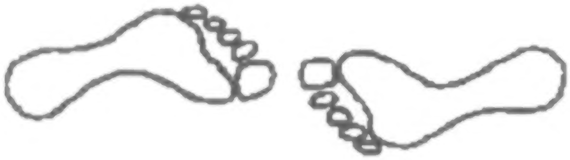
Answer - Missing edges

Correct	88	84
Outline correct	0	0
Missing edges	10	10

Comment: This item required more than naming or counting. Again leaving aside whether drawing the outline is sufficient, students who left out edges in their response appear not to have grasped the notion of a diagram.

Item 4: These two foot prints were seen on the sand. Circle the correct answer. They were made by:

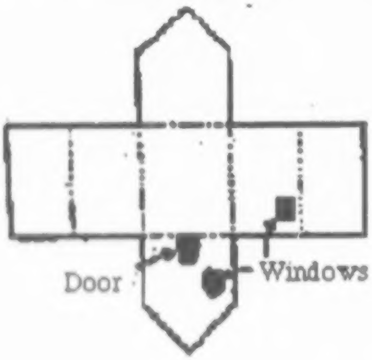
A. a left foot and a right foot, or B. two left feet, or C. two right feet.

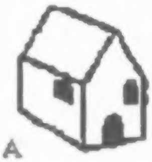


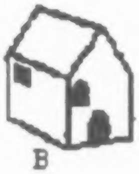
B 93 97

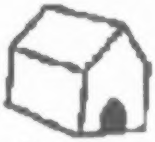
Comment: The wrong answers were spread evenly between the other alternatives.

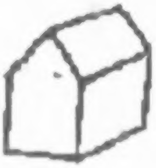
Item 5: The plan of a house which has only one door and two windows is shown below. Which of A, B, C, D, E could be the house?

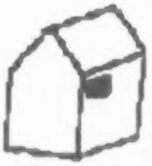



A


B


C

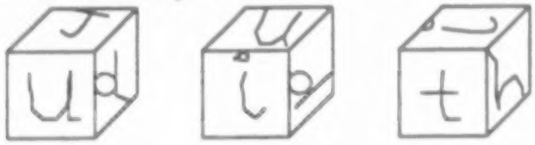

D


E

D	67	67

Comment: This item proved to be hard.

Item 6: Judith wrote the letters of here name on the six faces of a cube. Three different views of the cube are given.



Which letter is on the face opposite the u?

A J　　C t

B d　　D h

t	40	38
* h	54	53

Comment: This was perhaps the most difficult of all items. Apart from 'u', all other incorrect responses had their supporters with 't' being by far the most popular.

Item 7: Draw the possible two dimensional shapes which are constructed using four squares where each square shares at least one side with another square.

* 5 Correct	25	36
4 Correct	13	13
3 Correct	4	8
2 Correct	15	18
1 Correct	19	8
Totals- One or more correct	*76*	*83*

Comment: This item clearly relies on respondents being aware that a number of correct shapes are possible to fulfill the criteria. 25% in the first cohort and 20% in the second drew 3D diagrams. This implies that a number were not familiar with the term 'two dimensional'.

Item 8: There is a pile of 27 cubes stuck together in such a way to make one large cube which contains no holes or empty spaces. The surface of this large cube is painted. How many of the small cubes have;

only 1 side painted? .., 2 sides painted .., 3 sides painted .., no sides painted ..?

Totally Correct	37	34
1 side painter (6)	52	51
2 sides painted (12)	43	43
3 sides painted (8)	58	64
no sides painted (1)	43	53
Total no. of blocks (27)	51	48

Comment: This item proved to be difficult for all respondents. There were comparatively a large percentages who did not check to see whether they only had 27 small cubes in total, most of these having more than 27.

The first three items were based around diagrams, but to answer the question correctly the diagram could remain static. The first item and parts 1 and 2 of the second simply required naming of shapes and parts of diagrams. The knowledge of such terms is fundamental if teachers are going to adequately communicate with pupils. Later in item 7, respondents needed to understand the technical term 'two dimensional'. The responses to these items taken together give some signs of cause for concern. The last part of item 2 required that a block not shown be counted. Item 3 required that blocks be removed and the remainder drawn. There is cause for concern here given the percentages of errors made by the students.

Items 4, 5 and 6 were also based around diagrams, but for a correct answer to be obtained this had to be manipulated in some way. Item 4 required the rotation of part of the diagram. Responses of the students hint at concern. However the next two items which are clearly more complex in the manipulations needed, led to even higher error rates.

The last two items proved to be challenging. The multiple nature of the responses clearly contributed to this, but did not explain all errors. Images needed to be formed and dealt with in different ways for correct solutions to be obtained. It is outside the scope of this short paper to speculate for example on why in item 8 the edge blocks with the two sides painted proved to be the most difficult.

This brief analysis indicates that the beginning teachers were not handling the items at all well. It should be remembered that these items need skills which many would assume all entry tertiary students would process. Indeed many would assume such skills would be available to many exit primary school pupils. It may be that the spatial errors that are observed with school students (e.g. Clarkson, 1994) are in part related to teacher abilities in this area.

The other obvious point to note is that there was little difference in the results for these two groups of students. Hence changes to entry conditions to this course may have had little effect, a point taken up below.

Discussion

One could criticize these set of results on the basis that they only come from one university, and for half of the results presented, from only one group of students. However it seems to me that they are worth reporting because of what they suggest. They suggest that these students have not shown they are wholly in charge of what would appear to be reasonably straight forward mathematics that they certainly will be expected to teach within two years time.

It will be noted that these students have already completed an innovative semester unit of mathematics at university. To the author these students do not seem to be as worried by the subject matter as students who presented in the mid 1980s. This suggests that the preliminary unit is certainly helping overcome some of their fears for mathematics. But the teaching of that unit does not seem to have allowed students to come to grips with fairly fundamental mathematical skill notions. Hence there is some reason to think that the reasoning behind one recommendation of the Speedy report has not come to full fruition. In that Report it was implied that an extra unit of mathematics would bring all students up to an acceptable achievement level. These results suggest that this is not so. It is probably the case that a number of students who have not studied this mathematics for some years are helped to remember and refine their ideas while completing the first mathematics unit. However it also appears there are a number of students who have such deep misunderstandings that the completion of one such unit, although in some ways very helpful, is simply not sufficient to help them come to grips sufficiently with their long term problems.

The other recommendation from Speedy noted earlier in this paper, that is that entry students should have completed year 11 and 12 mathematics, was never fully implemented for my university. The 1993 students whose results on spatial items were shown above would have been able to claim a bonus percentage for their university entry score if they had have completed some mathematics in year 12. However, by the time the 1997 students applied for entry to this university that bonus had been discontinued because of administrative ease, and because we were battling for students. But, as noted above, there seems to be little difference between these students, at least on the scores on the spatial items. Here then is some indication that manipulation of entry criteria to the course in this way has little direct benefit as had been hoped. This suggestion is in line with an earlier study which in part looked at students' secondary school studies in mathematics and their performance at tertiary level. There were a number of students in that study who were not performing well at tertiary level, even when they came with year 12 mathematics (Clarkson, 1987/88).

What then are we to do? What obligations do universities have in this area, given that many of my students will exit their course and make fine primary school teachers? It should be noted that virtually all of them will gain employment within six months of graduation. Principals of the schools that they go to will also report for most of them that they 'are an asset to my school'. We know this for a fact from survey work completed by the Faculty.

Should we as a profession continue to be concerned about these issues of lacking in particular areas, in this instance mathematics although the same argument could be run for art, or science, or physical education, since students do graduate and get jobs? Are we caught up too much with just one small facet of their preparation, given the extensive matrix of parameters in which primary teachers, on the whole, should have some expertise?

Clearly the Ministerial Report is based on information that to some degree matches the data reported here. Are the concerns expressed in that Report 'for real', or is it just political expediency and another opportunity to kick the profession? Will the macro lines of action that the Report suggests be more beneficial than what Speedy put forward?

It seems to me that students who have long term problems with mathematics still make fine teachers, but they can cause their pupils harm if their mathematical insights are not grown. But in these days of downsizing in universities, and my own in particular, we are no longer in a position to help with long term solutions for these mathematically at risk beginning teachers. Hence the problem must be addressed at the entry point of the course. However the use of macro tools working with data readily available, such as the level of mathematics studied at secondary school, is not sufficient. Other assessment strategies need to be developed and implemented. If governments are really serious about this matter, then one could expect to see tenders which seek expressions of interest to address these issues being advertised. I for one am not holding my breath.

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Investigating students' understanding of the relationships among quadrilaterals

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This study is the second in a series designed to explore how class inclusion concepts evolve. Class inclusion concepts in this context is the ability to have an overview of possible relationships that exist among figures, and is an important characteristic of Level 3 thinking in the van Hiele Theory. Indepth interactive interviews were undertaken with 24 capable secondary students concerning how they view linkages among six different quadrilaterals. A developmental pattern is identified.

This paper presents the results of the second phase of a larger study involving a focused analysis of the understanding and development of class inclusion notions, in a geometrical sense. This is carried out within the framework offered by the van Hiele Theory (van Hiele, 1986). This theory hypothesises a five-level framework to describe students' thinking in Geometry, with class inclusion considered an important component of Level 3 thinking. This is a level where students are able to achieve an overview of the properties of figures, and relate (or order) the properties so that particular properties flow from, or are a consequence of, an individual property or group of properties.

While it is recognised that the development of class inclusion concepts is a prerequisite, which strengthens and deepens students' understanding for deductive reasoning, studies have highlighted the difficulty of the task faced by students in achieving and appreciating sub-class relationships (Burger & Shaughnessy, 1986; Fuys et al, 1985; Mayberry, 1981; Pegg & Davey, 1991). Some studies, which have measured levels of thinking via a written test, assign Level 3 thinking to a response if the student accepts a single statement involving class inclusion with no means for justification (e.g., Usiskin, 1982). Other studies (Burger & Shaughnessy, 1986; Pegg & Davey, 1991), involving short answer questions and interview tasks, have enabled a more detailed identification of the components of Level 3, highlighting that importance should not be placed on "the acceptance of class inclusion but the willingness, ability and the perceived need to discuss the issue" (Pegg, 1992, p.24).

To date, some limited work has been undertaken in the general area of class inclusion associated with quadrilaterals. However, it has basically revolved around answers to such questions as those which require the recognition of squares as rectangles (Usiskin, 1982; Mayberry, 1983). In contrast, an earlier stage of this current research, as presented at MERGA 20 (Currie & Pegg, 1997) addressed an investigation of students' understandings of class inclusion, as related to different triangle types. The results of this study revealed that class inclusion appeared to be a gradually acquired skill which follows a developmental path. The hierarchical framework began with links based on the spontaneous identification of a single similar feature or property, without the formation of triangle classes. Following this, three classes of triangles emerged and became a workable identity. Further growth was indicated through a stronger focus from each class as separate identities, to the relationships that exist between classes. The culmination being the ability to incorporate subsets.

These results act as a useful preliminary baseline for the current study due to the less complex structure of relationships between triangles as compared to quadrilaterals. Also, the design features used in the previous study which included indepth task orientated interviews were seen as very appropriate for clarifying students' attempts at trying to understand the relationships between six different quadrilaterals.

Three research questions guided the study. They are:

1. What was the nature of the links students formed when grouping different quadrilaterals?
2. Was there evidence of some developmental pattern in the structure of the different responses?
3. What similarities exist between the nature of the links students form among triangles and those formed among quadrilaterals?

Design

Twenty-four students, six from each of Years 8 to 11, were selected from two secondary schools in Armidale. The students were of above average ability, and there were equal numbers of males and females.

The purpose of this study was to have the students identify and justify relationships among six quadrilaterals, namely, square, rectangle, rhombus, parallelogram, kite, and trapezium. The interview format, as contained in Table 1, outlines the student tasks and questioning focus which enabled the students to work with familiar recalled information, supplemented information, individual tree designs, and discussion involving prompts and probes from the interviewer. The continual revisiting of the same issues, in slightly different forms, as drawn on different maps by the students, provided a vehicle for extracting further information, as the maps were used as a catalyst for discussion concerning the reasons for the existence of links (or relationships). The investigation required the analysis of responses to questions (ii), (iv), and (v) in Table 1 which deal with the relationships between the six quadrilaterals.

Quadrilateral Relationships	
(i) Int:	Write a list of all the quadrilateral names you can think of. Draw each quadrilateral.
(ii) Int:	Design a tree diagram which links the different quadrilaterals. Draw a sketch to link each type. (Discussion to follow concerning the reasons for links or lack of links. If required the following three points are addressed.)
(iii) Int:	There are some quadrilaterals that we can add to this list (names of quadrilaterals not recalled are provided). Draw a sketch of each new quadrilateral.
(iv) Int:	Design a second tree diagram incorporating all quadrilaterals on the list. (Discussion follows concerning the reasons for links or lack of links.)
(v) Int:	Taking your first tree add the new quadrilaterals to your original tree, again include relevant links. (Discussion follows concerning the reasons for links and lack of links.)

Table 1. Summary Interview Structure

Results

Overall, the students found the ideas familiar, but the questions were seen as non-routine. Analysis of the twenty-four interviews, which considered the links described, and the justification of these links, made evident the significance of a developmental path and allowed a comparison between the growth identified previously with triangle relationships and that identified here with quadrilaterals. Below is a detailed analysis of the main categories of responses, including a description of one typical student response for each type, and relevant samples taken from the interview transcript.

Type A

These responses include links based on the identification of a single similar feature or property, such as 'at least one set of parallel lines', 'equal sides', 'right angles', 'two sets of equal length sides', and links based on some aspect of a given shapes appearance. While the groupings of quadrilaterals formed in this way are based on a unifying characteristic, the most telling feature is that a particular quadrilateral may occur in several different groups. There were three responses coded as Type A. The differences evident in the students' responses were attributable to a gradual decrease in links based on overall appearance and an increase in links associated to different properties.

One of the better Type A responses was provided by Tracy. She was able to form groups on the spontaneous identification of 'parallel sides', 'right angles', 'diagonal parallel lines', and 'two sets of equal lines'. Tracy's discussion below illustrates her concentration on a single particular feature or property to link the figures. When prompted to discuss other links, Tracy accepts that they may connect for more than one reason, but forms the new group in isolation to links identified previously.

Int: How does your kite link to the square?
Tracy: Because they both have two sets of equal lines which means that those two are the same. This is going to get really messy. I have got all those ones that have at least one set of parallel lines.
Int: Can you tell me about these other ones?
Tracy: Well this is just connecting the ones that have two sets of equal length sides, that is, already linked.
Int Which one do you mean?
Tracy: Well that is linked because it has parallel diagonal lines.

Overall, a Type A response is characterised by the spontaneous formation of groups based on the identification of a single unifying feature or property. The properties of the quadrilaterals are treated in isolation, and links based on other properties are not considered within that group of quadrilaterals. The groupings change as frequently as the identifying feature changes.

Type B

These responses appear similar to Type A in that links are based on similar properties. However, the lack, or existence, of a perceived significant property precludes a given link. For example, whereas in Type 1, squares and parallelograms would be linked because of parallel sides, in this grouping they would not be linked because of the absence of a right angle in the student's image of a parallelogram. This comes about because in this grouping, the particular named figure, such as a square, stands for certain known properties. For example, links, such as square to rectangle, square to rhombus, rectangle to parallelogram, parallelogram to rhombus, parallelogram to trapezium, will be made on the basis of similar properties, such as right angles, parallel sides, equal sides, and opposite angles equal. When describing the links identified, the student also articulates the differences between the shapes which preclude certain links. An important feature of the eight responses coded as Type B, is that a link is not made between the parallelogram and the square, or the rhombus and the rectangle, unless students are prompted to do so.

Andrew's response illustrates the links typical of a Type B response. The names of known shapes is represented by properties. The differences between the shapes are expressed as well as the similarities, and negative instances preclude the formation of links.

Andrew describes the properties in restricted terms, namely: 'the sides are longer'; 'length and angles are the same'; and, 'almost the same length and parallel'. The properties of the parallelogram and rectangle are described using language which restricts the inclusion of links based on similar properties. For example, concentration on right angles, and the 'same length' is utilised, and does not include opposite angles equal and opposite sides equal.

Int: Do you see the square connecting to the rhombus at all?
Andrew: Yes because they are all the same. See they are the same length and they are parallel and they have the same angles. Except that one is 90° and that is about it ...
Int: Tell me about your rhombus to your parallelogram.

Andrew: Well they (rhombus and parallelogram) are almost exactly the same except one (parallelogram) has only one side as longer, and that (rectangle) can go to that (square) because that is square but one side is longer so that is just like that.

In summary, Type B responses link quadrilaterals on the basis of similar properties while acknowledging a number of the properties represented by that shape. Although links may exist due to one similar property, other properties of the quadrilateral are also noted, and either reinforce the connections, play no role in the connection, or are dominated by negative instances over other similar properties. For example, the rectangle would not be able to link to the rhombus, as the rectangle is described as having two sets of equal sides, where the pairs are of different lengths, and, the rhombus has all sides equal. Another dominating difference, which precludes this link, concerns the rectangle being described as having four right angles, while the rhombus is described as having two obtuse and two acute angles.

The responses in this category are also characterised by restrictions placed on property descriptions. For example, an inability to describe the square as having opposite angles equal, or opposite sides equal precludes its link to the parallelogram. The majority of responses in this category also include responses based on overall appearance which accentuate the differences between shapes.

Type C

Type C responses are very similar to Type B descriptions with the addition of tentative links suggested between the square and the parallelogram. Similar properties are noted and described in a more inclusive manner. However, the differences expressed result in an indecisiveness concerning this relationship. There was only one student (Alice) coded as Type C. This response illustrates the conflict evident between inclusive descriptions of figures, and the differences identified between them.

The significant difference between Alice's response and the Type B responses lies in Alice's ability to discuss the possibility that the parallelogram can have four right angles, as the sides will still be parallel. Alice continues by stating that if the sides were the same, the parallelogram would be a square. When prompted to discuss the possibility that the square is a parallelogram, Alice is restricted by other properties of the square and describes parallelism as an unimportant characteristic of the square.

Int: So do you think that they can go together? (parallelogram and square)
Alice: Um, no it is not important enough. That one (rectangle) there has got four right angles so it can go to the square. That one (parallelogram) can have four right angles as well.
Int: The parallelogram?
Alice: Yes because all of these sides are parallel and these sides are parallel and if they are all the same then it is a square.
Int: So does that mean that the square is a parallelogram?
Alice: No because parallel isn't important to a square.

The indecisive nature of a link between the square and the parallelogram coincides with a willingness to describe the parallelogram as encompassing right angles and four equal sides. The square, however, is restricted in nature and is dominated by the right angle property, hence, the link remains only a possibility.

In summary, the Type C response includes links based on similar properties while taking into consideration known properties of each figure. The differences between the figures are not as dominant in nature compared to the Type B responses, allowing a tentative link to be made between figures, such as the parallelogram and the square, which in Type B would have been precluded. The parallelogram is described in less restricted terms, thus, acknowledging the possibility of including right angles and equal sides.

Type D

This group of responses is characterised by the addition of links which were not included in the Type B responses, and which were tentatively linked in the Type C response. Differences observed, such as right angles and equal sides, do not hinder the links that exist between the parallelogram and square, and/or, the rhombus and rectangle. These links are described in terms of inclusive language. For example, students use, 'opposite sides parallel', 'opposite angles equal', and 'two sets of equal sides'. Links exist between all parallelograms in the majority of responses based on similar properties, but the classes of parallelograms, rectangles, and rhombuses, are not inclusive of other classes of figures. For example, the rhombus, rectangle and square are linked to the parallelogram but none are seen as a subset of the class of parallelograms. There are four responses coded as Type D. A typical Type D response is provided by Jenny's answers.

Jenny's response illustrates the use of language which allows the formation of more links when compared to the Type B and C responses. Jenny's response is not focused upon differences between the quadrilaterals, thus, similarities described in terms of properties are not dominated by the properties with strong visual cues. Jenny acknowledges that the square has opposite sides equal and parallel, but does not describe the link in terms of opposite angles equal.

Int: Can you tell me why you have the square, the rhombus, the rectangle and the parallelogram all linking up together?

Jenny: Well it is more of a progression I suppose as the square has all right angles and so does the rectangle that is why they are linked. And also they have opposite sides equal and parallel lines, the opposite sides are parallel. With the rhombus all the sides are equal and the same with the square and the parallel lines also um these are obviously not right angled as this doesn't have right angles either but the opposite sides are parallel and the opposite sides are equal. These two are linked because everything is the same except this one doesn't have right angles.

When prompted to include the square, rectangle, and rhombus within the parallelogram class of quadrilaterals, Jenny accepts that they are a group but is unable to place these shapes within the generic category carried by the word parallelogram.

Int: Do you see these as fitting into a group together?
Jenny: Yes.
Int: What would you call it?
Jenny: I don't know.

In summary, Type D responses are characterised by the addition of links between the parallelogram and square, and/or the rhombus and rectangle. The links based on similar properties acknowledge all the known properties of the generic category represented by a quadrilateral name. The links are not hindered by the properties dominated by appearance, and the descriptions encompass a range of property subsets as opposed to the exclusion evident in the Type B responses. Each of the responses in this category give none or few justifications based on overall appearance. This emphasises the reliance on the known properties of the different quadrilateral types, and the reconciliation between similarities and differences made possible via the inclusive nature of the language used.

Type E

This group of responses includes the class of parallelograms as an important feature of the quadrilateral relationships. The class of parallelograms is described succinctly, and is based on similar properties. The parallelogram encompasses other classes of figures such as square, rectangle, and rhombus, with justifications based on more than one similar property. There are links made to the trapezium and kite which are based on

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similar properties. There are three responses coded as Type E. The links and justifications described by David are typical of a Type E response.

David's initial comment concerning the design of the tree diagram indicates that the class of parallelograms is comprised of subsets. David is not content with his diagram, as it suggested that the square and rectangle are not part of the parallelogram class. He then alters his diagram to rectify this problem.

> David: I was going to start with has right angles and doesn't have right angles but a parallelogram would end up over there, but a rectangle and a square are parallelograms, but I don't think I can differentiate between the two because they are parallelograms because they do have parallel sides. This is a tricky one ...

David's discussion demonstrates an ability to distinguish between the different quadrilateral types within the class of parallelograms. He describes the properties characteristic of the class of parallelograms using inclusive language, such as, 'opposite sides parallel and equal', and 'opposite angles equal'. David is able to maintain a distinction between figures represented by different names through the use of language such as 'all sides equal'. When prompted to address the class of parallelograms, David describes in detail the justification for this, and states that the square is a parallelogram but the converse of this is not true.

> Int: Can you tell me why this parallelogram has all these lines coming off everywhere?
>
> David: Because it has almost all the characteristics of the other shapes like it has the parallel sides, the set of equal opposite sides, it can have angles that equal 90°, but it can't be um, you have to remember that a square is a unique parallelogram, as in a parallelogram can't have sides of equal length. As in a parallelogram can be a rectangle but it can't be a square.
>
> Int: Can a square be a parallelogram?
>
> David: Yes it is a parallelogram.

David's response accentuates the links between the rectangle and square, and rhombus and square, by describing the additional similar properties, however, the notions of the rhombus class inclusive of the square, and the rectangle class inclusive of the square is not evident.

In summary, Type E responses are characterised by the formation of a parallelogram class inclusive of other quadrilateral types categorised by a different name. The language used to describe the relationships is inclusive, while different figures within the class of parallelograms are distinguished by their additional properties.

Type F

This type of response includes the class of parallelograms characteristic of a Type E response, with the addition of the rectangle class, inclusive of the square, and/or the rhombus class, inclusive of the square. The justifications for these are based on similar properties with links also made to the trapezium and kite. There is only one response coded as Type F. Brendan's response includes: the class of parallelograms which encompasses the square, rectangle, and parallelogram; the class of rectangles which encompasses the square, and, links based on similar properties.

Unlike David, Brendan begins his discussion of the tree diagram by describing in detail the individual links between the different quadrilaterals using similar language to the Type E responses. When Brendan is prompted to discuss the parallelogram class, as suggested in the design, Brendan begins with a description of the rectangle class as incorporating the square. The rhombus is described incorrectly as a special square which suggests that the rhombus subset is evolving. The parallelogram class is described as the basis from which the square, rectangle, and rhombus come from.

Int: Is there a reason why these three shapes all go to the parallelogram or why these three all go to the rectangle?
Brendan: It is do with the parallel sides and a square is a rectangle but it is special and the same as a rhombus is a square but it is a special one.
Int: What about your parallelogram?
Brendan: A parallelogram, um, a rhombus certainly comes from a parallelogram and as I say a square is also a rhombus.
Int: Is a square a parallelogram?
Brendan: Yes.
Int: What about a rectangle?
Brendan: Yes it is but it isn't flat ...

Although Brendan's description of the classes with subsets are not described with the same detail as David, the nature of the links formed, when compared to a Type E response, are more complex. The class of parallelograms includes other figures while also mentioning subsets of the rectangle class, however, Brendan describes incorrectly the rhombus as a subset of the square class.

In summary, the Type F response is categorised by the class of parallelograms including the subsets of square, rectangle, and rhombus. Within the class of parallelograms, two classes exist which incorporates subsets, namely, the rhombus with the square subset and/or the rectangle with the square subset. Each of these classes can be justified on the basis of similar properties.

Summary

The students' responses were coded into six types. These can be summarised as;

Type A: One single similar property identified to link the quadrilaterals. The groupings depend upon the unifying feature chosen and this property remains the single focus of the link.

Type B: The formation of quadrilateral classes known by name and categorised by a group of properties. The classes have no subsets. Relationships are described between classes when a perceived significant property is evident which often has visual cues. Links are described using language which often precludes links. For example, four equal sides does not encompass opposite sides equal. Links do not exist between the square and parallelogram, and rhombus and rectangle.

Type C: Characteristic of a Type B response with a tentative link made between the square and parallelogram. The similarities are noted between the shapes but the differences observed hinder the formation of a link.

Type D: The relationships are formed across the quadrilateral classes and are based on similar properties using inclusive language. The link is made between the square and parallelogram, and, the rhombus and rectangle, as the properties have developed an encompassing quality. Relationships are not dependant upon visual cues.

Type E: The class of parallelograms includes the figures known as square, rectangle, and rhombus. These subsets are justified on the basis of similar properties.

Type F: The class of parallelograms described in Type F acquires further development. The subsets also contain figures known by a different name. Within the parallelogram class the rectangle is inclusive of the square, and the rhombus is inclusive of the square.

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When this overview of the results of the different types of responses are compared with the results of the previous study, involving relationships among different triangle classes, three issues stand out. First, there was a marked similarity between the findings of the two studies. In both cases, clear categories were identified as were transitional responses where students appeared to be struggling for a more sophisticated answer. Associated with this, is that the similarity occurred even though there was a subtle difference in focus. In the case of the study involving triangles, a number of the triangles were variations of a specific class, e.g., there were three types of scalene triangles, namely, acute-angled, obtuse-angled and right-angled triangles, whereas in this current study the focus was on general representatives of a class. Second, early responses in both studies saw the transition from a shape having certain characteristics/properties to the properties characterising the shape. Third, certain descriptions based on known properties precluded the formation of links in the case of different triangle classes (e.g., three sides equal did not prompt two sides being equal), this was also evident in attempts to link quadrilaterals (e.g., four sides equal did not prompt opposite sides equal).

Conclusion

Overall, the results of this study provide substantial support for the tentative findings of a previous study of a developmental pattern in the acquisition of class inclusion concepts. Also highlighted are the difficulties that students must face and the cognitive hurdles that must be overcome to reach such an understanding. The good news is that with this information, teachers may have, for the first time, a structured framework in which to plan ways forward to help students develop such skills.

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Rethinking what it means to understand: the case of combinatorial problem solving

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This paper argues for the need to address children's structural understanding in dealing with mathematical problems. In support of this argument, a study that investigated children's structural understanding of 2-D and 3-D combinatorial problems, via a range of thought-revealing tasks, is reported. The results raise a number of issues for further attention, including the discrepancy between children's accuracy and their structural understanding, and the lack of significant correlation between children's graphic and symbolic representations on most of the problems. Individual profiles of response highlight the importance of rethinking our interpretations of understanding.

With the increasingly widespread use of national and international comparative studies of school mathematics (e.g., Literacy and Numeracy Benchmarks; Third International Mathematics and Science Study [Stigler & Hiebert, 1997]), it is imperative that we do not fall into the trap of equating accuracy with understanding. The current testing frenzy is all the more reason to focus our attention on what it means to understand mathematical ideas, how we might best foster this understanding, and how we might assess for its presence. In this paper, I argue for the need to address children's *structural understanding* in dealing with mathematical problems. To illustrate my argument, I report on aspects of a study that implemented a range of tasks designed to assess this understanding in fifth-grade children's combinatorial problem solving (English, 1998a).

Defining Structural Understanding

This section addresses the nature of structural understanding, as used in the present study. First, it is necessary to define problem structure. The structure of a problem or problematic situation refers to the ways in which its mathematical ideas relate to each other, irrespective of the context in which the ideas are set (English, in press a; Novick, 1992). Structural understanding extends beyond a recognition of problem structure. For children to have developed a structural understanding of a given problem type (e.g., Cartesian products), they need to be able to:

1. explain the meaning of a problem;
2. represent the problem in different modes, including concrete, graphic and symbolic forms;
3. apply the processes of analogical reasoning to:
    (a) identify the structural elements of the problem;
    (b) detect the structural similarities and differences within, and between, related problems;
    (c) solve more complex cases of the given problem; and
    (d) pose new problems from the given problem. (English, 1998a)

Some might argue that aspects of (3), above, are an application of problem solving, rather than a fundamental component. On the contrary, a failure to apply the processes of analogical reasoning is one of the major causes of students' difficulties in dealing with mathematical problems (English & Halford, 1995; Holyoak & Thagard, 1995; Novick, 1995). Indeed, this was pointed out by Polya (1954) several decades ago.

Simply put, analogical reasoning entails understanding something new by analogy with something that is known, and is a fundamental process in children's mathematical learning and overall development (English, in press b). When applied to problem solving, reasoning by analogy first requires the solver to recognize and understand the structure of a given problem (known as the "source" or "base"), whether the problem be recalled from memory or supplied by an outside agent. Knowing the source structure can be of assistance in solving a new, related (target) problem, because the source structure can be mapped onto the structure of the target problem (Gentner & Gentner, 1983; Gentner, 1989; Holyoak & Thagard, 1995).

Despite the fact that reasoning by analogy contributes significantly to children's conceptual development during problem solving, it has received little attention from the mathematics education community (English, in press b, Holyoak & Koh, 1987; Novick, 1988, 1995; Silver, 1990). This is surprising, given that one of our major goals of mathematics education is for children to see the connections and relationships between

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mathematical ideas and to apply this understanding to the construction of new ideas and to the solution of new problems (English & Halford, 1995; Fuson, 1992; Hiebert, 1992; National Council of Teachers of Mathematics, 1989, 1991). It is thus imperative that children's problem experiences include a focus on the processes of analogical reasoning.

Combinatorial Problem Solving

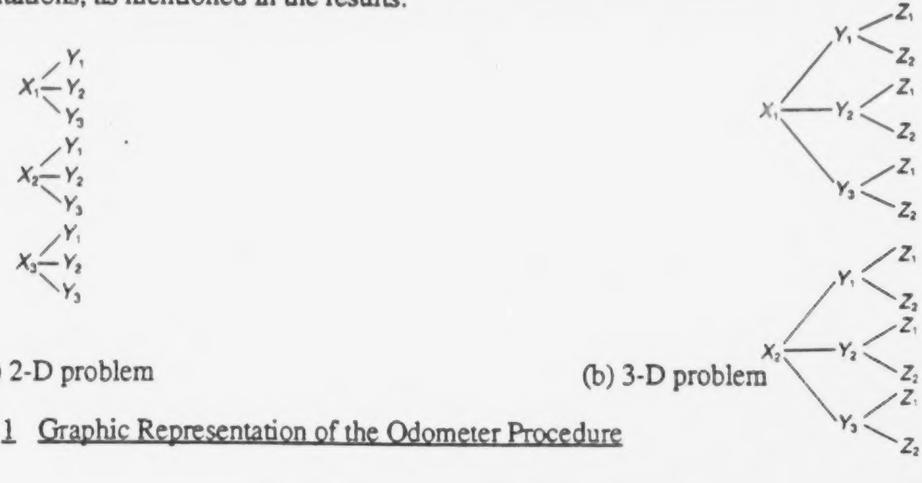
The present study investigated children's structural understanding of 2-D and 3-D combinatorial problems (i.e., X x Y [Cartesian products], and X x Y x Z). An analysis of the mathematical structure of these problems is presented in English, 1998a. In contrast to the focus placed on the other elementary problem types (e.g., Clark & Kamii, 1996; Kouba, 1989), combinatorial problems have received little research attention. This is a serious omission in the mathematics education literature, especially since they are one of the most difficult of the multiplication types for elementary school children (English, 1997; Harel & Confrey, 1994; Mulligan & Mitchelmore, 1997; Nesher, 1988; Outhred, 1996). Furthermore, combinatorics is a significant component of the curriculum, comprising a rich structure of significant mathematical principles that underlie several other areas such as counting, computation, and probability (English, 1993).

The existing literature has indicated that children employ a range of strategies in solving combinatorial problems (English, 1993, 1996a; Mulligan & Mitchelmore, 1997), that they frequently view the problems in terms of additive, combine problems (Nesher, 1992), and that, in general, they do not like dealing with problems of this nature, despite their colourful contexts (English, 1996b, 1998a). What seems to be lacking are studies that have undertaken a comprehensive analysis of children's structural understanding of these problems. To redress this situation, the present study employed a diverse range of "thought-revealing tasks" (as described in the methodology) that involved children in describing, explaining, representing, constructing, and justifying their ideas. (Lesh, Hoover, Hole, Kelly, & Post, in press; Lesh & Clarke, in press; Lester & Kroll, 1996).

To provide some background for examining the children's responses, the next section reviews briefly some approaches to representing and solving combinatorial problems.

Approaches to Representing and Solving Combinatorial Problems

Research on children's strategies in solving combinatorial problems has not been prolific. Previous studies (e.g., English, 1993; English, 1996a) identified 10 different strategies children use in solving 2- and 3-D cases when presented in both concrete and word problem form. These strategies range from trial-and-error approaches to the efficient odometer procedure (cf. odometer of a vehicle). As indicated graphically in Figure 1 (a), this latter strategy involves holding constant an item from one set (set X) and systematically combining it with each of the items in the other set (set Y). When this procedure is applied to 3-D examples, two items are held constant simultaneously (from sets X and Y), while the third item (from set Z) is systematically varied (see Figure 1 [b]). It is beyond the scope of this paper to address all of the different solution strategies children have been observed to use. Of relevance here is the efficient odometer strategy, which was reflected in the children's representations, as mentioned in the results.



(a) 2-D problem

(b) 3-D problem

Figure 1 Graphic Representation of the Odometer Procedure

There is also a variety of graphic representations, including diagrams, sketches, and tables (Cunningham & Hubbold, 1992) that may be used both in representing and in solving combinatorial problems. As discussed later, these include systematic listing/drawings of combinations, hierarchies or branching structures ("tree diagrams;" Novick, 1996; see Figure 1), and matrices or rectangular arrays (Behr, Harel, Post, & Lesh, 1994; DeGuire, 1991; Diezmann, 1998; English & Halford, 1995; Novick, 1990). It has been argued that the ways in which children represent problem situations, whether these be in concrete, graphic, or symbolic form, are indicative of the ways in which they construct mathematical relationships (Davis, 1992; Diezmann, 1998; Hiebert & Wearne, 1992; Mulligan & Mitchelmore, 1996; Outhred, 1996). On the other hand, as Mulligan and Mitchelmore's (1996) study suggested, children may choose to structure their representations in a way that minimises cognitive load. Their study found that, although children's initial pictorial recordings reflected the semantic structure of a problem situation, some children restructured their representations to simplify calculation.

The important point here is that we cannot assume children understand combinatorial structures just because their problem representations might suggest this. Not until we observe children's responses across a range of problem situations, such as those described next, can we draw any conclusions.

METHOD

Participants

Thirty-two fifth-grade children from two schools participated in the study (mean age of 10.2 years). The schools were located in contrasting neighbourhoods of Brisbane. The children had been exposed to combinatorial problems in their mathematics curriculum.

Tasks and Procedures

The children were interviewed individually in two sessions, each of about 20 minutes duration. Whenever possible, the two sessions were conducted on consecutive days. Each child's responses were videotaped and transcribed for subsequent analysis. The problems presented in the two sessions appear in the appendix: Problems 1 - 4 are 2-D cases, while Problems 5 and 6 are 3-D examples. The following procedures were used.

Session 1

Each child was presented with Problems 1 and 2, which were placed on separate cards and read aloud for the child. When asked if they had experienced problems like these in class, all but four of the children said they had done so. The children were then directed to: (i) explain, in their own words, what each problem means, (ii) indicate how the two problems are similar to each other, and (iii) indicate how the two problems are different from each other. With respect to the latter, if the child simply referred to surface or contextual features, the question was asked, "Can you see any other way in which the problems are different from each other?"[1]

The children were then directed to solve each problem, using the paper and colored pencils provided. The children were asked to show all of their working, and then to write a symbolic statement (number sentence) that represented their solution. The children were also asked whether they considered their problem to have been solved, and why. Finally, the children were invited to pose their own combinatorial problem, using one of the given examples as a base.

Session 2

The second session commenced with Problems 3 and 4 being presented along with two other problems that had different structures from Problems 3 and 4, but the same context. These problems were as follows: (a) In an activity class, Jenny and her friends made 6 blue skirts, 4 red skirts, and 6 white skirts to sell at the school fair. How many skirts did they make altogether? and (b) Tom has a big, hungry family. His family started the week with 5 loaves of white bread, 6 loaves of brown bread, 3 chickens, and 15 slices of corned beef. Towards the end of the week, the family had 1 loaf of white bread, 1 chicken, and 5 slices of corned beef left. How much of each item had his family eaten? The four problems were displayed randomly and the children were asked to select those problems that had similar structures (i.e., would be solved in a similar way) and to justify their selection.

Next, the children were directed to solve the two Cartesian product problems (Problems 3 and 4), using the paper and colored pencils provided. The children were asked to show all of their working, and then to write a symbolic statement (number sentence) that represented their

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solution. The children's solving of Problems 3 and 4 served as the source for the remaining activity, in which the 3-D target problems (Problems 5 and 6) were introduced. For these two target problems, the children were asked: (i) how the target problems are similar to, and different from, the two source problems (3 and 4), and (ii) how the source problems might help them solve the target problems. As before, the children were asked to solve each of the two target problems, to show all of their working, and to write a symbolic statement that represented the solution.

RESULTS

Children's responses to the present tasks have been examined in terms of the key components of structural understanding presented earlier, in addition to their ability to solve the problems. It is not possible to include all of the results within the present page limit; these will be presented at the conference.

Children's Accuracy

The children had few difficulties in solving the six problems. The percentages of children who solved each problem are as follows: 94% for each of Problems 1 and 2, 100% for Problem 3, 90% for Problem 4, 87% for Problem 5, and 77% for Problem 6. It is interesting that the children performed better on Problem 5 than 6, even though the latter is less complex computationally (2 x 2 x 2, compared with 3 x 2 x 2 for Problem 5).

Applying the Processes of Analogical Reasoning

Substantial evidence of the children's structural understanding of 2-D and 3-D combinatorial problems was gained from their ability to apply the processes of analogical reasoning. Included here are the children's: (i) identification of the similarities and differences between Problems 1 and 2, and likewise, for the remaining problems (as presented in Session 2); (ii) recognition of how source Problems 3 and 4 could assist in the solution of target Problems 5 and 6, and (iii) posing of new problems using Problems 1 and 2 as the base. These results are touched upon in the discussion.

Children's Graphic and Symbolic Representations

The nature of the children's graphic and symbolic representations provided further insights into the extent of their structural understanding.

Graphic Representations

There were five types of graphic representations identified in the children's responses (examples of these will be given in the conference presentation). In increasing order of abstractness, these representations are:

(i) Random and/or partial listing of items, including the use of drawings or words. These responses involved a random and/or partial listing of individual items (or combinations). Sometimes, such a response involved only a few single items being recorded randomly on the child's page.

(ii) Systematic listings or drawings. Responses in this category involved lists or drawings of combinations that were generated in a systematic manner. In the majority of cases, this listing reflected the use of an odometer procedure (as described previously), but occasionally a cyclic approach was evident (English, 1993). That is, in contrast to holding an item constant, the latter approach involves "cycling" through items in each set (e.g., *white bread/ham, brown bread /chicken, multigrain bread/corned beef, white bread/chicken, brown bread/corned beef* ....).

(iii) System of Paths. As described by Novick (1996), a system of paths does not have a formal structure, and does not necessarily have a unique starting or ending node, as in a branching structure or hierarchy (described next). The items in a system of paths have identical status, and the links between them can be associative, unidirectional, or bidirectional. There are no constraints on the linking of items (i.e., any item may be linked to any other item). When a child uses a system of paths in working the present problems, it is usually difficult to discern how the links between items have been made and also, whether a random or systematic procedure has been used in forming the links.

(iv) A Branching or Hierarchical Structure. This form of representation is commonly referred to as a "tree diagram," an example of which is given in Figure 1. Among the key features of this representation is the unidirectional nature of the links between items, that is, the path flows from a beginning node to an endpoint. Items at the same level cannot be linked, and neither can items in non-adjacent levels (Novick, 1996). Only a single path can enter each node but multiple paths can leave the node. Furthermore, as Novick pointed out, each node includes implicit information about how to reach it. For the present problems, the odometer procedure is clearly reflected in the design of the branching structure.

(v) Quasi-matrix. As the name implies, this form of representation displays the basic features of a standard matrix, in that it has a rows-and-column format representing distinct sets of items. However, a quasi-matrix, as applied to this study, lacks some of the standard notations of a formal matrix. This may include an omission of the main grid lines or a failure to fully label them, and a failure to complete all cells. Nevertheless, these quasi-matrices display a factorial combination (Novick, 1996), where all the values of one variable (e.g., one type of bread) have the values of another in common (e.g., combined with a one type of filling to represent one type of sandwich). There is no linking of items within the one row or column (e.g., one type of bread cannot be linked with another type of bread).

No formal matrices were identified in the children's responses. This is not surprising, as children usually require specific instruction in using a standard matrix (Diezmann, 1998), and it is unlikely that such instruction would have been included in the children's fifth-grade curriculum. Table 1 displays the percentages of children who used each of the above representations in working each of the problems.

Table 1
Percentage of Children Using Each Form of Graphic Representation

	Form of Representation				
Problem	Random or partial listing of items	Systematic listing/drawing	System of paths	Branching structure	Quasi-matrix
1 (3 x 2)	37	19	9	13	13
2 (3 x 3)	13	22	6	22	22
3 (3 x 2)	13	25	9	16	22
4 (3 x 3)	16	22	13	13	22
5 (3 x 2 x 2)	9	28	19	19	16
6 (2 x 2 x 2)	16	31	16	19	9

Note. N = 32 The remaining children did not respond or gave a non-descript response.

Although it is difficult to discern clear trends in the children's use of graphic representation, there are some points worth noting. First, there appears some improvement in the sophistication of the children's representations, particularly between Problems 1 and 2. Second, very few children used a system of paths when dealing with the 2-D problems (although there was a small increase on the 3-D problems). Third, the use of systematic listing/drawing was favored overall, and, in particular, in dealing with the 3-D problems. Children painstakingly listed each of the combinations for Problems 5 and 6, instead of making use of a more efficient, albeit abstract, representation.

Also worth noting is the lack of consistency in the children's use of each type of graphic representation. Only nine children consistently used the one type of representation across all problems. Five of these children used systematic listing, three used a branching structure, and one randomly listed items. Of course, this lack of consistency may also be indicative of the children changing their graphic representation to suit the problem structure (e.g., adopting a different form of representation for the 3-D problems.). Ten children changed their form of representation when they moved to the 3-D problems, with this change not always being the most appropriate. Of these 10 children, three used two different representations in solving the 3-D problems.

Symbolic Representations

Children's symbolic representations of the problems included one-step and multi-step addition statements, one-step and multi-step multiplication, and the recording of an answer only. The proportions of children displaying each type of statement on each problem appear in Table 2.

Table 2
Percentage of Children who Recorded Each Type of Symbolic Statement

	Type of Symbolic Statement				
Problem	1-step addition	Multi-step addition	1-step multiplication	Multi-step multiplication	Answer only
1 (3 x 2)	22	16	37	3	6
2 (3 x 3)	3	12	59		9
3 (3 x 2)	10	3	50		9
4 (3 x 3)	3	25	59		6
5 (3 x 2 x 2)	6	22	53	6	9
6 (2 x 2 x 2)	25	12	37	9	6

Note. N=32 The remaining children did not respond.

Although multiplication statements were favored on each problem, there was, nevertheless, a considerable proportion of children who recorded addition statements, including multi-step addition. On Problems 1 and 6, in particular, the proportion of children who recorded addition statements was comparable to the proportion who gave multiplication statements. It is interesting that there was a noticeable decline in the use of multiplication between Problems 5 and 6. This reflects the previously cited decline in the children's accuracy between these problems.

Sixty-nine percent of the children were consistent in their symbolic representations across the problems. That is, if the children recorded an addition statement for Problem 1, they did so for all of the remaining problems; they did likewise if they commenced with a multiplication statement.

It is clear that the "primitive" repeated addition model of multiplication (Fischbein et al., 1985) still dominated the thinking of many of the children. Of particular concern, however, is the lack of multi-step multiplication statements for the 3-D problems (e.g., 2 x 2 x 2 = 8). The majority of children who wrote a multiplication statement for Problems 5 and 6 used only one step, such as, 4 x 3 = 12, and 4 x 2 = 8. This is an interesting finding and warrants further research (the work of Behr et al., 1994 would be valuable here).

One last finding of interest in this section is that there were only two significant correlations between children's graphic and symbolic representations, these occurring on Problems 3 and 5 ($r_s = .559$, $p < .01$ for Problem 3, and $r_s = .403$, $p < .05$ for Problem 5). For these problems, as children adopted the more abstract representations their symbolic statements changed from additive to multiplicative.

DISCUSSION

Children's responses to the problem tasks raise a number of issues. One of these pertains to the discrepancy between children's accuracy and the extent of their structural understanding. The majority of children could solve the problems, although a decline in accuracy occurred on Problem 6. Further research is needed to determine the reasons for this decline, whether it be the problem context, or the equivalent sets of items (2 x 2 x 2), or some other factors. In contrast to their accuracy across the problems, the children's responses suggest weaknesses in their structural understanding. For example, only a small proportion of the children could provide a comprehensive explanation of the structure of Problems 1 and 2. Although some children might have had difficulties in expressing their understanding in words, this did not seem to hinder them in justifying their solutions to these first two problems. Here, they showed a greater awareness of the 2-D combinatorial structure.

Nevertheless, this awareness did not extend to detailing the structural similarities and differences within, and between, the 2-D and 3-D problems. The cross-multiplication feature of the 2-D problems was rarely highlighted, and likewise, the subtle structural differences between the 2-D and 3-D problems remained undetected by most of the children. This could partly explain why the children rarely wrote a multistep multiplication statement for the 3-D problems. At least the children were generally able to recognize how their solving of the 2-D problems could assist them with the 3-D cases.

Another issue relates to the finding that the children's graphic and symbolic representations rarely correlated. This raises the question of the extent to which children's graphic and symbolic representations reflect their structural understanding of a given problem type (cf. Hiebert & Wearne's, 1992, argument). As Mulligan and Mitchelmore (1996) have noted, the multiplication notation children use in representing their solutions does not necessarily mean that they conceive of multiplication as a binary operation; they may simply be using it as a summary of their repeated addition process.

An analysis of individual cases highlighted the above issues. For example, Sally displayed a knowledge of the odometer procedure yet recorded addition statements only. Sarah, on the other hand, did not demonstrate this knowledge, yet recorded a multiplication statement for each problem. Kate and Siobhan used sophisticated graphic representations and also recorded multiplication statements. However, neither girl could explain the meaning of Problems 1 and 2, and neither could give a detailed account of their structural similarities. On the other hand, they were more adept at detecting the structural differences among the different problems. Kate was able to pose a solvable problem, while Siobhan could not.

This study has demonstrated the need to implement a broader range of thought-revealing tasks both in promoting and assessing children's development of structural understanding. We cannot assume that children fully understand problems of a given type just because they can solve them or can represent them in some acceptable format. This paper has attempted to illustrate the importance of looking beyond these skills to address the components of structural understanding that have been advanced. We should then have greater confidence in our children's attainments on comparative tests of school mathematics.

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Appendix

Problems Presented in the Two Sessions

Session 1

Problem 1

Jenny has a blue skirt, a red skirt, and a yellow skirt. With these, she can wear a white T-shirt or a green T-shirt. How many different outfits can she make?

Problem 2

Tom is making sandwiches for a picnic. He has white bread, brown bread, and multigrain bread. He can fill these with ham, chicken, or corned beef. How many different sandwiches can he make if each sandwich has one type of bread and one type of filling?

Session 2

Problem 3 (source problem)

Mrs. Jones is trying to decide on a flower arrangement. She can choose from roses, carnations, and daffodils. She can use a tall vase or a short vase. How many different arrangements could she make, if she were to use only one type of flower and one type of vase?

Problem 4 (source problem)

Mark is making crazy animals. For the animals' heads, he can choose from a rooster, a monkey, and a duck. For their bodies, he can choose from a donkey, a rabbit, and an elephant. How many different crazy animals can he make?

Problem 5 (target problem)

Marina is making boxes of greeting cards to sell at the fair. She has blue paper, red paper, and green paper. She also has striped ribbon, and spotted ribbon. For the lettering, she can use gold ink or silver ink. Each box will have different greeting cards. How many different greeting cards will she put in each box, if each card has one colored paper, one ribbon, and lettering of one color?

Target Problem 6 (target problem)

Mrs. Jones needs a new car but cannot decide what to buy. Here are her choices. She can choose from a 2-door or a 4-door car. It can have luxury seat covers or standard seat covers. It can also have metallic paint or regular paint. How many different choices has she?

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New times for mathematics in vocational education and training

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This paper will explore some of the effects on mathematics education in the VET sector where economic rationalist ideologies prevail, and hegemonic industrial values subsume educational values. In a sector which carries considerable investment of public and private funding; it may be expected that mathematics, which has a high public profile generally and with sections of business and industry in particular, should be an important focus. Somewhat paradoxically, there is an apparent lack of status.

Introduction

In terms of its relationship to vocational education and training (VET) the term *mathematics* is used in the broadest sense, encompassing all curricula which include mathematics, numerical and spatial calculations, as well as statistical components, whether officially designated by recognisable titles or incorporated into other vocational modules. There is considerable debate in some countries over the use of the term *numeracy*, but this will not be pursued here: the term will be subsumed under the broad concept of mathematics. For further discussion of the terminology of mathematics and numeracy see FitzSimons, Jungwirth, Maaß, and Schloeglmann (1996).

Kogan and Tuijnman (1995) identified several levels of educational research arising from the focus on the socialisation process, including those of "the individual learner, institutions and organisations, and the wider sociocultural, economic and political environments of societies" (p. 32). In the past I have addressed mathematics in the VET sector from the perspective of the micro- or operational level (FitzSimons, 1994a; 1994b) focusing on the activities, interactions and behaviours of the individuals themselves; and from the meso- or structural level (FitzSimons, 1995; 1996a; 1996b; 1997a; 1997b) focusing on the production and selection of knowledge as well as structure of learning relations. In this paper I will explore from the macro level the conflation of social, economic and educational values which is tending to define how, and what, mathematical competences are offered to VET students to meet perceived requirements of Australian society.

New Educational Times

Policy formation under economic rationalism. In Australia policy making for VET, typically of many English-speaking countries, has been characterised by political expediency and a consequent short-term outlook (Wiltshire, 1993). The management of education has shifted from experienced professionals to ministerial control (Bourke, 1994), informed by advice from career public servants. Economic rationalist ideologies seem to have provided the solution to the disjuncture between economic goals and the situations in which nations such as Australia have found themselves (Dwyer, 1995). There have been social shifts towards individual responsibility and competition, as the distinction between public and private good has been blurred (Kell, 1993); a range of social and economic agendas have been transferred onto education (Apple, 1993). The VET sector has been increasingly deregulated in the devolution of responsibility and accountability to the local level, at the same strictly controlled through legislative processes of federal and state governments who have imposed severe financial constraints.

Since the mid-1980s there has been a policy agenda to reform industry and training in the interests of improving national productivity. The policy discourse has privileged the voices of business and industry (Kell, 1992; Sachs, 1991) and marginalised practitioners and students (Anderson, 1997a; Fooks, 1996; Stevenson, 1993); policy advice on the provision of VET is sought exclusively from the privileged (e.g., ANTA, 1997). This means that the VET sector of education in Australia is informed by individuals or groups representing the interests of capital, apparently consonant with national economic (and presumably social) interests. The human capital paradigm, which has replaced that of social and personal development over the last two decades, has clearly shifted the focus of VET away from the students to industry, although such shifts appear to be cyclic (Stevenson, 1995; White, 1995).

Teachers' work in an open training market. Economic rationalist policy decisions have created an open training market for vocational education and training, with financial encouragement for private providers to compete with established institutions, albeit without sufficient thought for the ultimate structural and strategic consequences (Billett, Cooper, Hayes, & Parker, 1997; Fisher, 1993; NREC, 1997). There has been a consequent deregulation of teaching within the sector with the removal of any necessity for holding teaching qualifications, and the reduction of initiation of new teachers to an optional basis with as little as a three-day "train-the-trainer" course. Exacerbating the existing problem of a highly casualised workforce is the trend to outsourcing, resulting in the subcontracting of "outworkers" (Kell, 1997). Professional development has been devolved as a responsibility of individual providers, with deleterious effects (see, e.g., FitzSimons 1995; Kell, Balatti, Hill, & Muspratt, 1997). Under this scenario anyone with mathematical knowledge at the minimum level of the course requirements and no teacher training can be employed as a teacher or curriculum developer; the likelihood of appropriate professional development being offered, let alone taken up, remains slim.

Curricular issues. Until 1997 all VET curricula were the responsibility of national Industry Training Advisory Bodies (ITABs) and this has had serious consequences for mathematics education since there was neither a mathematics nor an education ITAB. Decisions about mathematics curricula have been distributed around a plethora of ITABs whose major focus is their particular industry grouping, hence it is unlikely that those making curriculum decisions have been informed by findings of recent research into mathematics education. The recent introduction of the National Training Framework (NTF) means that the focus is now on the training of students for particular industries, or more narrowly, particular enterprises (Anderson, 1997b), so that their mathematics education is unlikely to be considered a priority. This is of course contrary to the goals for school students (Niss, 1996) and runs counter to the NBEET (1995) report recommendation for higher education students to be supported in their studies by high quality service mathematics or statistics subjects — on an age-basis, to say nothing of developmental basis, there is an overlap of many VET students with their counterparts in other sectors and questions of equity may arise. Even on behalf of the school sector, mathematics educators (researchers and teachers) have found it difficult to be meaningfully involved in policy formation, and have resisted the subterfuge that they be used as "consultants" to give their imprimatur to decisions already taken (e.g., Ellerton & Clements, 1994). If one accepts the thesis that mathematics education can make a contribution to the well-being of a nation, important research questions are *what* mathematics is selected and *how* it may be taught. These questions, which have not been given due attention in the VET sector, must seriously address the needs and interests of both students and industry.

Competency-based education and training (CBT) was mandated as a requirement for all nationally accredited curricula, originally linked with the structural reform of industry. It was popular with administrators as a means of control over teachers and of providing accountability to the government and other stakeholders (Jackson, 1993). It was also popular with some sections of industry because of its apparent simplicity, although there has not been a large uptake of this system in Australia or the United Kingdom (Raggatt, 1997) — possibly due to the minimalist nature of the competency format, and its insistence on following national standards which are not necessarily relevant at the enterprise level. It has had the tendency to stifle or restrict creative and innovative teaching and to deprofessionalise teachers, although some teachers have managed to operate in the interstices (see, e.g., Clemans, 1997; Mulcahy, 1996). It would be reasonable to assume that the large majority of mathematics teaching practice in

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the VET sector follows a transmission paradigm based on the atomised series of competencies, in face-to-face direct instruction or flexible (i.e., self-paced) delivery which is frequently delivered off-campus or in composite classes on-campus.

Teaching and learning in VET. Flexible delivery is promoted in policy documents as offering improved access to education. It can be advantageous to students with good organisational and language skills, but is problematic for many other VET students (Misko, 1994). It is considered by many teachers to be merely a cost-saving device (Kell et al., 1997). Taken together with the policy of Recognition of Prior Learning (RPL) and the new NTF, the trend towards focus on assessment of outcomes rather than the processes of learning runs the risk of overlooking, even undermining, the rhetoric of lifelong learning. In order for new learning and its transfer to take place students must have a strategic framework or metatheories of learning (Billett, 1996) — strategies unlikely to develop spontaneously in a majority of VET students.

The importance of discipline knowledge has been subsumed under more visible work-related skills, although many writers (e.g., Kell, 1997) suggest that this strategy will not be appropriate for future employment scenarios where a combination of 'hard' (technical, practical) and 'soft' (strategic, communication, etc.) skills will be needed. Unlike the school sector where the focus has been on key learning areas, national curriculum and national testing in mathematics, the situation of mathematics per se in the VET sector has been virtually ignored by government policy makers, and not given any great priority by ITABs or providers such as TAFE institutes. In Victoria there are few if any remaining designated mathematics departments in TAFE; most mathematics teachers are scattered across vocational departments, and non-TAFE teachers are even more isolated. Across Australia generally there are no longer central support units for teaching and curriculum. The teaching and learning of mathematics appears unproblematic to policy makers who have engendered this situation as well as to managers at the provider level. There is no definitive career structure for a mathematics teacher who wishes to remain discipline-based; many former mathematics teachers have chosen promotion to managerial positions or left public education altogether to work in industry where they are more valued. One approach to this question of the inferior status assigned to mathematics in the VET sector may be from the perspective of mathematics as a socio-economic good.

Mathematics as a socio-economic good. Addressing the question of high status knowledge, Apple (1992, p. 420) noted that "business and industry, as well as government, place a high value on knowledge that is convertible ultimately into profits and control." He continued that mathematics was valued "not because of its beauty, internal characteristics, or status as a conservative form of human knowing, but because of its socioeconomic utility for those who already possess economic capital" (p. 423). This sentiment is reflected in a report to the Australian government which highlighted the utilitarian values of mathematical sciences, as well as a degree of self-satisfaction on the part of its authors: "Modern mathematical science is a supreme creation of the human intellect; it is also critical for economic competitiveness, and a basis for investigations in many fields" (NBEET, 1995, p. ix).

In fact, although the report's main focus was on the university sector (with one sub-section addressing the teaching of mathematics in the school sector), it totally ignored the VET sector. However, the inclusion of the VET sector would have been pertinent for the following reasons: (a) VET mathematics curricula span school and some undergraduate offerings; (b) the strongly grounded assumption of the importance of the links with industry is no less important in the VET sector, particularly for mathematics; (c) the report's emphasis on quality teaching in mathematics is equally if not more imperative to meet the particular needs of VET students; and (d) articulation between VET and university sectors is not uncommon and even encouraged by policy documents which advocate a "seamless web" of educational provision. The reasons for the exclusion of the VET sector, not an unusual occurrence in Australian mathematics education research, can only be surmised. Following Bourdieu's notion of cultural capital which sees education as a commodity, Apple (1992) suggested that mathematical and scientific knowledge function to support the unequal pattern and distribution of social and economic benefits. He argued that it may not be considered by the interests of capital to be a worthwhile investment to distribute such knowledge too widely at the expense of high status institutions such as universities which produce the technically oriented knowledge that can be utilised and controlled by them. The result, however, is that the provision of VET mathematics is apparently devalued and at the same time excluded from valuable opportunities through reports (e.g., NBEET, 1995) for public scrutiny and consideration of recommendations for improvement. I propose that this continued marginalisation of VET mathematics in government-sponsored reports and symposia, together with the conspicuous dearth of university-based research, contribute to the difficulties currently facing mathematics education in the VET sector.

Mathematical perspectives of stakeholders. In order to gain some purchase on the frame of reference for stakeholders (providers and users) who make decisions which influence the quantity and quality of mathematics education in the vocational education and training sector I investigate possible sources of ambiguity through a consideration of values, following Bishop (1988). Mathematics clearly has a high status in our society (Apple, 1992; Martin 1988/1997; Popkewitz, 1988) and yet the values which establish its power and invincibility serve to alienate or intimidate many people. Its ideological values of rationalism and objectism, separating objects from ideas and allowing abstractions to be treated as objects, have vested mathematics with power and authority but are based on a particular kind of theorising associated with modernist projects emanating from the Enlightenment. Such projects have been problematised by Walkerdine (1994) as normative, based on a limited selection of attributes reflecting the dominance of white European middle class males, marginalising all other groups. Martin's discussion of professional mathematicians supports this view of mathematics as a bureacratised, masculine domain.

The seemingly paradoxical sociological values of openness and mystery in mathematics have, in first case, been linked, albeit tenuously, to democracy and in the second to widespread feelings of ignorance and confusion. The latter could be attributed to the use of abstraction with consequent loss of contextualisation and distancing of meaning derived from perception and intuition (Bishop, 1988). Although mathematics can be used to enlighten, transcending immediate experience, it can also be used to obscure and mystify through its use of specialised knowledge and language (Martin, 1988/1997; Popkewitz, 1988). Linking mystery with control, mathematics can be used to support vested interests by providing seemingly objective and rational models, yet possibly with inbuilt biases, for predetermined outcomes (Martin, 1988/1997). Fischer (1993) observed that although mathematical models were not intended as facsimiles of reality, they can in fact be used to construct new social realities such as in economic and social planning.

Although the values of control and progress underpinned by technology have resulted in improvements in many aspects of society this has not been unproblematic. Mathematics has provided a tool for gaining control over physical and social environments, but humans are now entrapped in a technological society. Control is exerted dialectically with social perceptions of mathematics researchers and privileged interests of sponsors operating in the direction and utilisation of results of mathematical research (Martin, 1988/1997). However, most people are unaware of the subjective, systemic side of mathematics inherent in humans, thereby allowing the domination of the objective, controlling and explanatory side (Fischer, 1993).

Each of these six values, valorised by mathematicians and their supporters, can engender negative responses among other members of the community (who possibly hold simultaneously conflicting views when assuming different social roles). Whether held consciously or unconsciously, these values must impact on the decisions made about mathematics in the VET sector.

Conclusion

Since the mid-1980s Australian governments have embarked on policies informed by economic rationalism to reinvent the VET sector as a corporatist entity — devolving responsibility for management to the local level, and for matters relating to curriculum, assessment and credentialling to representatives from business and industry. In the formation of an open training market, as well as in the imposition of the competency agenda, governments have conflated social and economic purposes with those of education (Apple, 1993; Sedunary, 1993).

With an overall enrolment of 1.7m students the sector carries considerable investment of public and private funding, with only a modest amount of corporate support (Anderson, 1997a); it may be expected that mathematics, which has a high public profile in society generally, and with sections of business and industry in particular, should be an important focus. Its apparent, but somewhat paradoxical, lack of status can be explained by a range of factors, stemming from political economic rationalist ideologies, as well as ambiguous values ascribe to mathematics by decision makers. The sector's emphasis on industrial values have assigned a pre-emptive good to education and training (Stevenson, 1995), privileging the perceived interests of business and industry while effectively silencing the voices of teachers. The residual status assigned to VET mathematics by government, industry, and university research arms has exacerbated the situation. These are, indeed, new times!

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Exploring avenues of reflective practice with pre-service Mathematics teachers

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Pre-service teachers participated in peer and self assessment activities during a seminar series focused on current issues in mathematics education. Their feedback comments to each other and the various assessment forms they used were analysed to determine what changes took place in what they perceived to be important issues, how the quality of their comments changed over time, what they felt they learned from peer and self assessment, and how they developed in their capacity to reflect critically.

Introduction

Notions related to teachers as 'reflective practitioners' are not new in education, and their importance in the personal and professional development of teachers has received considerable attention by researchers and teacher educators (for example, see LaBoskey, 1994; Posner, 1993). Further, one of the five areas of competence explicitly emphasised in the recently released Australian *National Competency Framework for Beginning Teachers* (Australian Teaching Council, 1996) is that of "Reflecting, Evaluating and Planning for Continuous Improvement". The purpose of this study was to investigate how these 'reflective' processes can be developed with pre-service teachers through involvement in peer and self assessment activities. This 'assessment' avenue for exploration of reflective practice was chosen for two main reasons: (i) it provides opportunity for individuals to experience first-hand some of the 'alternative' assessment practices that recent trends and research in mathematics education support, and (ii) it is different and less time intensive than the common practice of using journals for reflection, thereby providing an avenue for reflection which it was believed people would be more committed to and would find more accessible. In addition, since the activities to be assessed focussed on issues of current concern in the mathematics education community (for example, multicultural classrooms), the assessment and related research program allowed examination of development of pre-service teachers' awareness of mathematics education issues. Specifically, the study focused on the following three research questions:

1. What types of feedback and evaluative comments do pre-service teachers provide when given opportunities to assess their own and their peers' teaching?
2. How do pre-service teachers' capacities to give feedback and constructive criticism develop through involvement in a peer and self assessment program that itself is continuously evaluated and revised?
3. What are pre-service teachers' reactions to involvement in a peer and self assessment program?

Considering the challenges teachers will encounter in their profession, including dealing with students from diverse cultural, language, geographic and socio-economic backgrounds, for ongoing improvement of teaching it is essential for teachers to have capacities to reflect, on and communicate about, their practice. Effective mechanisms by which these capacities can be developed in pre-service teachers at more than a superficial level, and in the context of authentic teaching tasks, need to be examined and understood. This study took steps towards addressing these challenges in ways that would be perceived by the pre-service teachers themselves as valuable and realistic, and in ways that address issues surrounding mathematics teaching and learning in today's society.

Theoretical Framework

A major impetus for this study was the research and development project, the National Project on the Quality of Teaching and Learning (NPQTL). The research-based and teacher-centred development of the resultant *National Competency Framework for*

Beginning Teachers (Australian Teaching Council, 1996) thereby provides validity for an emphasis on reflective practice in teacher education programs. While the *Framework* provides an expression of the need to develop reflective competency and some indicators to aid recognition of this competency, it does not articulate a mechanism to ensure its development. It was with this issue in mind that this study was developed.

A second area of research that informed the focus and development of this study was that of the 'reflective practitioner' literature. This field encompasses teacher education as well as professional development in a wide range of endeavours (for example, see Dewey, 1933; Schön, 1983; Brookfield, 1995). Reflective practice can be conceived of as a collection of strategies that are used as a mechanism to induce teacher growth (Frid, Reading & Redden, 1998). They range from deliberation about the practical aspects of teachers' work to critical appraisal of the meaning, in a moral or ethical context, of the teaching activities. The process of reflection requires explicit and deliberate action as distinct from impulsive action or action required by bureaucratic decree. Also contributing to the framing of this study was LaBoskey's (1994) conceptual framework for reflective teacher education in which the act of reflection is viewed in terms of content, processes, attitudes and teaching conditions. The first three of these categories are reflected in the foci of the research questions, while the conditions are determined by the context of this particular research setting (topical issues in mathematics education).

Finally, the research literature further supported the notion of development from practical to more global concerns in the work of Furlong & Maynard (1995). After working with pre-service teachers, Furlong & Maynard postulated the existence of a developmental path. In this path each of the four stages of development is characterised by a shift in the focus of teacher concern. They assert that initially teachers model themselves on others without considering the implications of their own practice, while in the second stage they focus on themselves as teachers with a concern about their skills and how these skills appear to others. Movement to the third stage then requires a shift from focusing on oneself as teacher to a focus on students as learners. The final stage of development is characterised as autonomy in which the focus shifts to exploring the theoretical underpinnings of one's actions as an educator. This four stage model served in the present study as a guide or framework for exploring shifts in LaBoskey's (1994) notions of content, processes and attitudes that occurred for pre-service teachers involved in reflecting on the mathematics related teaching activities of themselves and others.

Method

A cohort of 120 third (and final) year teacher trainees in a primary school Bachelor of Teaching program were the focus of the study. These students represented regions of the country in which there is often a strong presence of people of aboriginal descent as well as an increasing presence of persons of non-English speaking backgrounds. They were undertaking a compulsory course in mathematics education which has the aim of developing a set of theoretical understandings that could be used to support teaching activities in mathematics classrooms. For example, ideas related to various learning theories in mathematics, and various teaching and assessment practices. As part of the course, the students were required, via a seminar series, to explore ideas related to a number of current issues in education. The current issues included multiculturalism in the mathematics classroom, aboriginality, gender, attitudes towards mathematics, the role of language in mathematics learning, and consideration of the needs of gifted and talented students. The seminars were presented to tutorial groups of approximately 30 students by sub-groups of 3-4 members. It was the evaluations of the seminar presentations that provided the data source for this study.

To provide feedback and an evaluation for each seminar presentation an iterative process was developed. Each seminar group and supervising lecturer provided comments and a mark out of ten on a weekly feedback form and the presenting group of students used these evaluative comments to prepare a presenting group seminar report.

The presenting group also had to design a new feedback form for use in the following week by the next group of presenters. This process was repeated through eight iterations. No attempt was made to provide communication and sharing between the four tutorial groups, nor was there any formative evaluation shared between the lecturers involved in supervising the groups. Hence the data were seen as independent across groups. This provided a restricted form of triangulation to the study. The claim of triangulation is however limited since no guarantees can be made that the students did not share experiences outside of the classroom with students from other tutorial groups.

As a consequence of the above design the data for this study consisted of eight by 120 individual weekly feedback forms, eight by four group seminar reports and eight by four designs of weekly feedback forms. In addition, all students in the study were surveyed at the end of the semester to ascertain their views of the process. This survey data also provided triangulation of previous findings.

The analysis of the predominantly qualitative data involved three phases which could be described as exploration, integration and confirmation. In the exploratory phase the data from three of the tutorial groups were analysed by the relevant lecturers independently of the other lecturer. Specifically, the lecturers aimed to identify categories for students' evaluative comments, typical examples of reflective comments, what appeared to induce or inhibit reflection, and evidence of any longitudinal development in the identified categories. These independent analyses were synthesised in phase two of the data analysis. This integration process required vigorous discussion and clarification of category definition until a consensus between the lecturers was reached and the concepts were well understood by the other lecturer. Thus, the process provided to the analysis a degree of inter-coder reliability. In the final phase the emerging conceptual framework was tested with the data from the fourth tutorial group to provide confirmatory support for the two earlier data analysis phases.

Results

Data analysis provided the following information concerning the three research questions.

Types of Feedback and Evaluative Comments

Much of the feedback provided was superficial in quality in that it focused on whether or not particular aspects of the seminar were included, rather than commenting on the nature or impact of these aspects. For example, when asked to comment on 'What worked well' students often wrote such things as "the brainstorming activity", or "good pace", or "well organised". Similarly, when asked to comment on 'appropriateness' they said such things as "everything was appropriate", "very appropriate for the topic", "yep" and "good".

Early in the semester, comments were generally lacking in substance, consisting of single words or short phrases related to what students thought was 'good' or 'bad'. An extreme example in this regard is the student whose entire written responses to eight prompt items on a feedback sheet appeared as follows: (prompt items that appeared on the feedback form are indicated in italics)

Appropriate: very beneficial
Group encouraged: at times
Originality: yes
Preparation: well prepared
Class participation/class management: everyone involved
Useful strategies provided for future use: yes
Were presenters enthusiastic?: yes
Appropriateness of materials: good

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Although these comments are brief, they give some superficial feedback on the seminar when read in conjunction with the item prompts. In comparison, more elaborate comments, although relatively few in number, included such things as "made us think about differing strategies" and "very effective in demonstrating the different attitudes [towards mathematics] that teachers/students can develop". An example (from Week 2) of more extensive feedback than was generally given by students is:

> ***What worked well?***
> Introduction - great, got us interested & illustrated the point well.
> Eg task - Teddy bears. Purposeful, prompted discussion and cooperation & we had ownership of learning.
> - attention from presenters good - they went around the room and talked to us about what we're doing & why.
> - regularly changing presenters - everyone contributed & seemed to know what they were on about.
> - all points & info were related back to the topic [attitudes towards mathematics] - didn't get off track.

In general, the comments made by students showed a greater tendency to praise than criticise, and when criticism was given it was not always constructive. However, constructive criticism, although not as extensive in nature, was sometimes evident. An example (from Week 1) of a student's feedback that included constructive praise and criticism is:

> Good structure, I liked the way you had the lesson plan on the board, so we knew which direction we were heading in.
> A bit more time perhaps [could be] spent on giving instruction to the group, especially the writing task.
> Use of games was great, and very appropriate. Perhaps analyse games into appropriate year levels.

Overall, the comments tended to fall into three main categories: presentation skills, teaching methods, and content. These categories are consistent with LaBoskey's (1994) conceptual framework for reflective practice.

Comments on presentation skills, of which there were many, included such aspects as level of voice, quality of overheads, and organisation and involvement of group members in the presentation. The comments on teaching methods, more likely to be given when the methods were 'different' or 'original', tended to mention various approaches used such as brainstorming, case studies, games, work stations and role play. Most comments on content were concerned with very simple summaries of what was presented in the seminar or identification of particular content areas of interest. Whenever the quality of the content was mentioned it was generally only to the extent of whether is was "good" or "interesting". Typical comments included:

> We went through different maths curriculum from the ages. (Week 1)
> Had a brief talk on language & maths. Then looked at Reading & Writing & Talking & Listening and did some activities and came up with some Teaching Strategies. (Week 2)
> Gender issues is a somewhat controversial/interesting topic. (Week 4)
> Good list of things to do with ESL students' difficulties. (Week 4)
> Web sites excellent idea. (Week 6)

On occasion students presented one or more of these components in conjunction with supportive reasons. Comments involving any sort of synthesis of components of the seminar were sparse and mostly focused on the usefulness of the seminar presentation. Occasionally there was some identification of personal benefit to be gained. Examples are:

> Good to talk about ESL etc so that everyone knows a little bit of background. However, maybe it would have been better if it was related more to maths. (Week 4)
> The resources that [Rachel] showed us and the ideas that we came up with were really good. These were useable in a classroom situation. I would use some of the resources in my classroom for the benefit of all the class not just ESL or Aboriginal students. (Week 4)

Few of the students appeared to have made the transition from a focus on the practicalities of teaching to a focus on student learning as described by Furlong & Maynard (1995). That is, although there was some shift towards a focus on the impact of the seminar on their learning about mathematics teaching and learning issues, it did not become a major focus.

Another aspect of the students' feedback was the quantitative evaluation of each seminar by a mark out of ten. These assigned marks generated some speculation, discussion and ill feeling. Students repeatedly expressed in informal discussion concerns about the ability of peers to be objective and discriminating. Others felt that the actual mark should be assigned by the lecturer, not the students. Thus, in spite of workshop, lecture and seminar experiences aimed at broadening students' concepts of mathematics assessment tasks and the role of the teacher in a 'constructivist' oriented classroom (ie. the teacher as *not* being the sole authority), students did not easily change from their previous views.

Development of Ability to Provide Feedback and Constructive Criticism

Evidence of students' development in these areas was examined in relation to two components of the seminar process: the changes to the weekly feedback forms and the reports prepared by the presenting groups.

Changes made to weekly feedback forms: The changes made by students to the format of the weekly feedback forms highlighted a number of aspects. The structure of the items included changed as the weeks progressed. Initially students used one word or a short phrase for a prompt item (ie. a category or question to which the evaluators/students were to respond). For example, one Week 2 form had six items for student response, five of which were one word such as "Research" and the other was the short phrase "Use of Resources/Relevance". As the weeks progressed more items were included, and they became longer in structure and more of them were worded as questions. By Week 6 seminar groups were producing forms that were almost entirely comprised of questions as prompts. However, an overall examination of the sets of feedback sheets indicated that by the end of the seminar series a balance had been struck between the shorter and longer prompt items and between statements and questions.

The content of the items included also changed as the weeks progressed. Most items included on the earlier forms pertained to presentation skills or teaching methods. For example, items such as "organisation", "preparation" and "Was the seminar entertaining?" allude to presentation skills while "discussion", "activities" and "original" invite comments on teaching methods. There was a later shift to inclusion of items related to learning, such as "Did the seminar have a practical use for the classroom?" (Week 7). However, even the final weekly feedback forms for each group still had a high proportion of the items related to presentation skills and teaching methods and very few items suggestive of a concern with participants' learning about mathematics or issues related to mathematics teaching and learning.

Overall, more items on the form or the length of the items did not necessarily lead to more in depth or comprehensive feedback by the students. However, aspects of changes in the forms did indicate some development of the ability to provide feedback and constructive criticism. The changing structure, balancing questions and statements, of the items suggests that students were attempting to develop tools that they felt were most suitable for providing useful feedback. Of note is that the content of the items changed from predominantly presentation skills and teaching methods to inclusion of some references to actual learning. The overall impression that the forms' development gives is that students' focus of attention changed from the specific mechanical aspects of the seminar to the impact and usefulness of the seminar.

Report prepared by presenting group: The presenting group seminar report produced each week as a result of reading the feedback from fellow students varied greatly in quality. The simplest was a summary of good and bad points from the seminar as identified by peers' comments. Many reports merely listed peers' comments, arranged under the categories provided by the prompts on the feedback forms. Students often listed their peers' feedback comments without any attempts to summarise, categorise or integrate them. Further, there was no attempt to reflect upon the appropriateness of the feedback in light of the thoughts and reflections of the participants themselves. That is, although students were instructed to "summarise" their peers' feedback and to "reflect" upon the seminar by considering their own perceptions and reactions to the presentation, this seldom occurred.

The few reports of a higher quality showed some evidence of self-evaluation, evaluation of the seminar presentation experience, or reflective analysis of peer feedback. For example, part of one group's report was the following:

> We feel that by having four work stations with three different activities at each station we provided a wide variety of activities that were well received. Due to time restrictions we realised that students would only be able to visit three stations. Structuring the seminar so that no one missed out on a station would have been ideal but was not possible in the time allowed. This was compensated for by the handout giving a brief description of all the activities. Our planning of the number of activities was ambitious but by sticking strictly to our timetable we finished within two minutes of the scheduled time.

It must be noted that this sort of more detailed, comprehensive reporting was rare, and when it did occur it was consistent with the lecturers' perceptions of the students. Thus, it must be concluded that the value to students' reflective capacities of preparation of the presenting group seminar report must be questioned. The reports did not appear to provide a mechanism by which the students were enabled to develop reflective analysis skills. Hence, a weak feature of the entire seminar process was that only the four presenters benefited from the feedback each week. A better process would involve all students having access to the group seminar reports each week. In this way more material would be available to encourage and support development of reflective actions.

Reactions to Involvement in the Peer and Self Assessment Program

Comments made by students on the end of semester survey showed that they identified both strengths and weaknesses of the seminar evaluation process. These were mostly global comments such as: "It was helpful to hear what your peers thought". In a few cases comments on aspects of both teaching and learning were reported. An occasional comment focused on the learning aspect of the process: "Great idea. You look at the seminar in a different way, more as a teacher. What works and what doesn't, and at the same time you are learning." On the more specific question relating to the issue of receiving feedback from peers, most students felt it was a useful process despite drawbacks such as lack of anonymity of comments and lack of effort by some students.

Students were divided on the benefits of producing a group report on the evaluations produced by their peers. Some saw the process as worthwhile, as it helped them to reflect on possible improvements for their teaching. However, there was a general lack of support for the idea of a follow-up group report, and this appeared to result from the degree of effort or commitment to the process rather than from a problem with the process itself. That is, students indicated they had not entered whole heartedly into the process, making comments such as: "Was difficult as a group, we left it up to one person. Workload has caused it not to be done."

This apparent lack of commitment to the task was supported by students' responses to the end of semester survey item for which they were asked to indicate on a 5-point Likert scale the degree of effort they put into "recording your thoughts/comments about each seminar". For each of the four tutorial groups the mean score fell between "little" and "some". In comparison, when asked to report similarly (ie. on a Likert scale) their "effort" put into "thinking about what is happening in each seminar" the mean score for each of the groups fell between "some" and "much". This indicates that some students were making a superficial effort at reflection, while overall students had not accepted either the concept of professional growth, or the role of reflection in promoting professional growth. The issue seems to be associated with the observation that it is not enough to provide an opportunity for developing the skills associated with reflection because the value of reflection needs to be explicitly developed. In making these rather 'negative' observations it needs to be said that some students had indeed recognised the value of the process of reflection, making comments such as: "It would defeat the purpose of peer evaluation if we did not consider the information given us." However, overall there was little evidence of a commitment to the process of reflection except at a relatively trivial level. Certainly there was no evidence of what has been described in the literature as 'critical' reflection.

Many students thought the process of changing the evaluation form was worthwhile, but they were unsure of the quality of the forms produced. This might have been due to the difficulties students experienced in revising the form. For example, they expressed much indecision in determining criteria on which to base changes. Additionally, there appeared to be an equity issue in which students found it difficult to understand how consistency in criteria for awarding grades could be achieved with a constantly changing evaluation form. An example of a comment in this regard is: "Changing criteria could have led to marking problems or inconsistencies." These sorts of comments indicate that students were more concerned with a 'fair', norm referenced assessment system.

Further support for this view was provided by the emphasis students placed on the system of awarding marks. While appreciating the opportunity to hear the views of their peers, many thought that the lecturer should provide the final grade. It was as if the lecturer was a superior who was the sole source of 'valid' grades. These beliefs are indicative of Furlong & Maynard's (1985) lower stages of teacher development and do not reflect the 'autonomous teacher" stage in which lecturer and student collaborate on an equal basis. Nearly all students were 'troubled' by the suggestion that they should calculate their mark by finding the mean of the marks awarded by their peers.

Conclusions

Students' seminar presentations displayed capacities to use a range of teaching methods appropriate for mathematics learning (eg. brainstorming, hands-on activities, group work, discussions, writing tasks). There was however little evidence that students had 'integrated' the foci of the study, the use of peer and self assessment and the use of reflection as a learning tool, into their personal repertoire of teaching and assessment strategies. Considering the research findings in the context of the conceptual frameworks outlined previously, some additional tentative conclusions can be drawn. First, with regard to the National Competency Framework, there was little indication that these beginning teachers would meet the criteria specified for "Reflecting, Evaluating and Planning for Continuous Improvement". Second, it would appear that there is a need to address explicitly the value, content and processes of reflective practice if students are to

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be supported in their growth to shift from a focus on mere teaching skills to a focus on student learning and the overall contexts of mathematics education.

A number of issues for further consideration arise from the findings of this study, including:

1. The degree to which 'time' as well as the nature of experiences is a factor in the development of reflective capacities needs further examination. It was not expected that the time frame of this study would be sufficient for substantial progress to be made by students, but it was also not expected that progress would be so minimal in either the quality of the seminars presented or the nature and extent of reflective skills developed. Although students had a range of experiences both at university and in the classroom upon which to examine topical issues in mathematics education, they did not appear to have developed much capacity to do so. Hence, the degree to which the total time frame of a student's pre-service education impacts upon the extent of progress also needs further examination.
2. Implicit in the design of this study was an assumption that ideas and skills in one area of education (eg. alternative assessment ideas in mathematics education) might 'transfer'. Also, there was an assumption that skills involved in analysis of others' teaching transfer to self-analysis capabilities. The nature of such transfer, and in fact the validity of such assumptions, need further consideration because they did not appear to happen to any degree in this study.
3. Students' attempts at reflection were more superficial than those that had been expected. Their lack of reflective capacities, in spite of previous experiences, is indicative of the non-triviality the reflective process and the non-triviality of the mathematics education issues selected. In addition to being provided with activities focused on developing specific reflective skills, students need to develop an awareness of the value of reflective practice, be it related to mathematics education issues, or any other area of education. Development of reflective capacities is 'hard work'.
4. Related to issue (3) above, yet worthy of independent mention is the notion of the 'valuing' of reflective practice. That is, this study highlights that, to effectively develop students' reflective capacities, an awareness of the value of reflective practice needs to be developed. This notion could be linked to, or seen as a subset of, LaBoskey's (1994) attitudinal dimension.

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Identifying the Dilemmas in Early Mathematics Teaching

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This study gathered teachers' descriptions of dilemmas they faced when teaching young children mathematics. It was considered that asking teachers to describe teaching dilemmas would enable them to identify a problematic aspect of their teaching practice, and lead to reflection, evaluation and refinement of their practice so that their students' mathematical learning would be more powerful. In analysing the dilemmas described by teachers, key elements were identified. These elements have important implications for providing effective professional development.

Introduction

The introduction of the National Numeracy Benchmarks seeks to encourage schools to identify whether each Year 3 and Year 5 student has reached a critical level of numeracy, without which the student may have difficulty making sufficient progress at school. The benchmarking process is part of a strategy to achieve the National Goal, *that every child leaving primary school will be numerate, and be able to read, write and spell at an appropriate level.*

Achieving this goal focuses the attention of school communities on one of their important challenges: effectively supporting the numeracy development of all students. Further, the process of identifying those students who are not making sufficient progress will impel teachers to examine the effectiveness of opportunities they are providing for numeracy learning in the early years of schooling. This may lead to the need for teachers to explore more deeply how young children learn mathematics and to develop new and more effective teaching practices.

In many cases, facilitating teacher change has been based on the assumption that changes in practices follow changes in teachers' beliefs. Professional development programs, therefore, had firstly to change teacher beliefs before any associated change in practice could occur.

Guskey (1985) argues the opposite position. He maintains that changes in beliefs depend on successful changes in teaching practice, and further contends that attempts to foster teacher commitment to new practices prior to their implementation are generally unsuccessful. He believes that teachers become committed to new practices only after they have been using them in their classrooms and observed positive changes in student learning outcomes that resulted from specific changes they had made to their teaching practice. These changes in student learning outcomes include: involvement in lessons, motivation for learning, attitudes towards school, the class and themselves, as well as cognitive and achievements outcomes. Guskey therefore advocates that we need to find better and more efficient methods of providing teachers with feedback on the learning outcomes of their students resulting from the changes they are implementing.

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An opposing view is presented by Cobb, Wood and Yackel (1990) who conclude from their research that it is the problems and surprises which teachers encounter in their own practice that give rise to opportunities to re-organise beliefs. Therefore, to enable change, teachers must first perceive their current practice as problematic, and then search for new beliefs and practices to overcome the problems.

Based on this finding, the challenge remains for those providing professional development for mathematics teachers to assist teachers reflect on their practice to identify the problems they have experienced. One way of achieving this is to ask teachers to write about a dilemma they have faced when teaching. Such case writing, as proposed by Judith Shulman,

> *enables a teacher to describe a critical incident or dilemma encountered in personal practice. Collaborative reflection on the writing enables a teacher, and colleagues who read the case, to understand and interpret the described practice and thus to propose action for change or improvement* (Western Melbourne Roundtable Innovative Links Project, 1997, p 29).

It may be argued that it is reflection on the dilemmas that arise in the course of teaching that lead teachers to conclude that their practice may be problematic and in need of change. This often leads to the examination and refinement of beliefs.

Apart from anticipating that the process of writing about a dilemma would act as a prompt for reflection and discussion, and would be an important way of assisting teachers develop their understanding of children's mathematical learning, it was expected that any common characteristics identified through an analysis of the dilemmas would have important implications for the content of a professional development program, *Supporting Children's Early Numeracy Learning*. Furthermore, it would be helpful for those developing and implementing professional development programs to be aware of the dilemmas and difficulties teachers are facing when supporting early mathematics learning.

Gathering Teaching Dilemmas

The teaching dilemmas used in this project were gathered over a nine month period from 76 Years K-2 primary teachers employed in Melbourne schools. These teachers all participated in a professional development course developed by the writer, *Supporting Children's Early Numeracy Learning*. The participants ranged in experience from those in their initial year of teaching, to those with more than 20 years teaching experience. The participants were each asked to respond to the following task:

> *Describe a dilemma you have faced when teaching young children mathematics. Include what you remember of the context and any other relevant details.*

The participants each had one week to reflect on this task and return the response to the writer. Participants were informed that the dilemmas would be used to inform the content of the professional development program, and that they would have the opportunity to discuss their dilemma with other participants during the program.

Analysing and Interpreting the Dilemmas

The dilemmas described by participants were examined in order to draw from the descriptions the key elements of each dilemma. Each description was annotated with the key features identified. The key elements were next analysed in order to determine whether there were any similarities. In some cases, the key elements were re-named to be more inclusive of a group of similar elements. The descriptions were then grouped according to the key elements and a tally made to calculate the number of times each of the key elements was identified in the dilemmas. The key elements were finally analysed and grouped to determine whether any common themes existed.

Teaching Dilemmas

In reading the dilemmas gathered during this project, it became clear that in most cases teachers were not actually describing perplexing situations in which a choice had to be made between alternative courses of action, but were in fact describing problematic situations where a solution was not immediately obvious. The following teaching dilemmas demonstrate the range of dilemmas gathered during the study. The key elements identified for each dilemma are also described.

Dilemma 1: The language demands of mathematics

> *For the past two years I have taught grade one students in a school with an extremely high percentage of children from an ESL background (95 %). This has immediately brought about many problems associated with the language concepts involved in mathematics. I have found that most of the children have difficulties understanding "more than" and "less than as well as "before" and "after." When I have taught this area of mathematics, I have begun with lots of oral and hands on experiences before moving on to written work. This I find benefits many children, however there are still quite a few who require one to one assistance with both oral and written work and they still struggle to understand these concepts.*

The key elements of this dilemma are teaching students who have difficulty coping with the English language demands of learning mathematics concepts, and understanding how children learn mathematical concepts.

Dilemma 2: Children often come to school counting

> *When you begin exploring 2 digit numbers , some children will reverse the number e.g. 41 for 14. Even though they know the number consists of 1 ten and 4 ones, as place value is taught in conjunction, because of the way they say the number, e.g. Four/teen, they write the 4 first and then the 1 (ten) because teen is like ten. Often they will count...7, 8, 9, 10, 11, 12, 30, 40,........90, 20. What can we do?*

The key elements of this dilemma are teaching students who have difficulty counting and teaching children who have difficulty with numeral recognition and writing numerals.

Dilemma 3: Children who struggle with grouping

> *At least two to three children in my year one class experience great difficulty in grasping the concept of grouping. This topic grouping usually runs for three weeks, which allows for one solid week of manipulative discovery with groups as the focus. This generally means the children are using concrete materials to make groups irrespective of the number in the group. All I would like at this stage is that they become comfortable and aware of what a group looks like.*
> *In the second week I would hope that the children feel confident enough to make different groups eg. 2 groups of 4 (still using concrete materials). It is only in the final week and again only if the child is ready that we move towards written statements. The multiplication sign (x) is not introduced in year 1.*
> *The mathematical dilemma is those children who struggle with recognising grouping in the initial stages and only become more confused in future years when they are asked to tackle times tables and equations but still do not fully understand the fundamentals. How do I remedy this situation?*

The key elements of this dilemma are teaching the multiplication process, understanding how children learn processes such as multiplication, and taking time to develop children's mathematical understanding.

Dilemma 4: Fear of Failure.

> *This term we have been focusing on the* problem solving process. *My P/1 children have participated in many input sessions where I have provided them with many strategies to solve problems e.g. draw pictures, write a number sentence, make a model, use a calculator, look for a pattern, work it out mentally. We have also worked groups and individually to solve problems using all these strategies. My dilemma is that I have a student who has a fear of failure and starts to cry as soon as the words "problem solving" are mentioned. She is a perfectionist and enjoys the comfort of structure otherwise she feels threatened and withdraws from the grade. She becomes so overwhelmed and distressed that not even my assistance is any reassurance for her. She is a fairly bright student who is usually independent and able to cope with most of the classroom activities.*

The key element of this dilemmas is teaching students who lack confidence in learning mathematics, particularly in the area of problem solving.

Dilemma 5: Teaching to all need and levels

> *In a grade 2 class of 32 children, I am finding that my main teaching dilemma for mathematics, lies in the problem of teaching to all needs and levels. I have children at the 'basics' level and those who need to be extended. As well as the mixed maths levels, there is a huge language need for 2/3 of the class. Main language spoken is Vietnamese; of the 32 children, only 3 speak English at home. I would like some strategies on how to extend my more advanced students - so that time can be given to others - and how to set activities which the others can work on independently while I work with the advanced students.*

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The key elements of this dilemma are catering for the learning needs of all students within a mixed ability setting and teaching students who have difficulty with the English language demands of learning mathematics.

Dilemma 6: Organising group work

> *A mathematical dilemma which I find challenging nearly every year in the Prep classroom, is trying to organise and manage co-operative, rotational group work. I run an effective program in terms 3-4 as I have a lot of parent support, however, when trying to manage this on my own it always seems to create some difficulty. Listed below are a few concerns, "dilemmas" I seem to face:*
>
> - *Keeping independent groups on task for the session.*
> - *What grouping is better (multi ability or ability groups?*
> - *The ability to work with a clinical group without disruption.*

The key elements of this dilemma are related to classroom organisation, particularly grouping students effectively for learning and managing groups.

Dilemma 7: Assessing each child

> *My dilemma is trying to assess each child's ability as assessment needs one to one interaction. With thirty children and numerous tasks to be assessed it becomes almost impossible.*

The key element of this dilemmas is effectively assessing students' mathematical learning and managing this assessment process.

Dilemma 8: I'm not confident teaching maths!

> *My dilemma is that I can teach language with ease. I can quickly think of a task analysis for e.g. teaching a genre, and can execute clear, sequential sessions. I can easily slip in and out of 'English." BUT with Maths I am not nearly as confident with the content/big picture/global aims/parts of the whole. When I ask my peers some say, I just use last year's program or I just work through this text book. I don't want to be this kind of teacher. I don't even think I know how to do this successfully so where do I go from here. Is there a great Maths scheme which could guide me. I find using the CSF a little daunting; or am I doing it all without realising it. I think I need help with task analysis for Maths.*

The key elements of this dilemma are understanding children's mathematical learning and coping with the curriculum demands of mathematics as described in documents such as the Victorian Curriculum and Standards Framework (CSF).

Dilemma 9: Parents don't see the importance of games

> *When apparent asks, how can they help their child in maths? my suggestion is always – through board games, card games, dealing and counting money or playing games with them. The parents do not see the importance of these activities. They want worksheets.*

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The key element of this dilemma is working effectively with parents so that they can support their children's mathematical learning.

Key Elements of the Teaching Dilemmas

Having read through the dilemmas experienced when teaching young children mathematics and drawn from the descriptions the key elements of each dilemma, twelve key elements emerged (see Table 1). These mathematical key elements related to three aspects of mathematics teaching: understanding learning; organising and planning for teaching; and working with parents.

The key elements of the teaching dilemmas identified most often were: teaching children who have difficulty with counting; the need for a deeper understanding of how young children learn mathematics; teaching students who have difficulty with numeral recognition; catering for the mathematical learning needs of all students within mixed ability settings; difficulties with teaching arithmetic process and algorithms; and teaching students experiencing difficulty in coping with the English language demands of mathematics (see Table 1).

Discussion

Analysis of the teaching dilemmas indicated that there were several common characteristics of the dilemmas faced when teaching young children mathematics. It was interesting to note that during discussions about the dilemmas that took place during the professional development program, many teachers prefaced their responses with the comment that they had also experienced this dilemma. It was therefore clear that although teachers described only one dilemma they had faced when teaching mathematics, they had also experienced many of the dilemmas described by other teachers.

Two thirds of the key elements of the dilemmas related to the need for teachers to develop a deeper understanding of how young children learn mathematics. Indeed, the teaching dilemmas often pointed to teachers' lack of understanding of how young children learn mathematics and therefore a difficulty in knowing which teaching strategies most effectively help children learn. It is difficult for teachers effectively to facilitate mathematical learning if they do not understand how young children learn mathematics. This finding suggests important implications for teacher development and mathematics education.

The identified key elements of the teaching dilemmas described in Table 1 suggest that teachers particularly need to explore how young children learn to count and identify numerals, how children learn arithmetic processes and how the language demands of mathematics influence learning. These aspects should be included in professional development programs for teachers. Another important need for professional development suggested by the dilemmas relates to considering strategies for catering for the individual learning needs of students within mixed ability settings.

Table 1
The number of times each of the key elements were identified within the teaching dilemmas described by teachers.

Aspects of Mathematics Teaching	**Key Elements of Teaching Dilemmas**	**No. of Times Key Features Were Identified**
Understanding mathematical learning	1. Teaching students who have difficulty with counting;	19
	2. Need for deeper understanding of how young children learn mathematics;	16
	3. Teaching students who have difficulty with numeral recognition and writing numerals;	16
	4. Difficulties with teaching arithmetic process and algorithms;	13
	5. Teaching students who have difficulty coping with the English language demands of learning mathematics, particularly LOTE background students;	11
	6. Developing children's number sense and mathematical understanding;	10
	7. Teaching students who lack confidence in their ability to learn mathematics or who lack a positive attitude to learning mathematics;	3
Organising and planning for teaching	8. Catering for the mathematical learning needs of all students within mixed ability settings (including identifying suitable mathematics activities & teaching strategies);	15
	9. Classroom organisation, particularly grouping students effectively;	5
	10. Effectively assessing students;.	5
	11. Covering the mathematics curriculum in one year;	4
Parent Partnerships	12. Working with parents.	6

Conclusion

Teachers constantly face dilemmas when teaching young children mathematics. The process of describing dilemmas provides a stimulus for teachers to reflect on the effectiveness of their teaching. This in turn may lead teachers to decide that their practice may be problematic and in need of change.

The dilemmas described in this project provided a powerful prompt for discussion. Teachers were able to offer comments which helped to clarify the nature of the dilemmas, stimulated teachers to explore the situation giving rise to the dilemmas in more detail, or suggested possible ways to respond to the dilemmas, thus leading to more effective mathematical learning.

An analysis of the teaching dilemmas identified 12 key elements. The majority of these suggested the need for teachers to develop a deeper understanding of how young children learn mathematics. The 12 key elements may be used by those producing professional development programs to inform the content of courses, so that professional development programs are able to meet the needs of teachers, resulting in lasting teacher change and more powerful mathematical learning in the early years of schooling.

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Students' perspectives on using computers and graphing calculators during mathematical collaborative practice

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This paper presents the results of a study of students' perspectives of, and attitudes towards, the use of mathematically enabled technologies in a collaborative secondary mathematics classroom setting. Students' responses to a repeated questionnaire indicated the inclusion of technology in mathematics instruction to be generally beneficial to both learning and peer-peer interaction. A follow-up interview confirmed and elaborated on this position and provided insights into the limitations of different technologies in the promotion of collaborative learning.

The call for the inclusion of information technologies as a necessary adjunct to the teaching and learning of mathematics has been echoed in the literature and curriculum documents for close to two decades. Despite long term attention, the use of computers and other technologies in school mathematics classrooms has been restricted to date by economic, social and practical constraints (Kissane, Kemp and Bradley, 1995). None-the-less, significant changes in the affordability of these technologies, the development of new hypermedia technologies and the convergence and packaging of mathematically important technologies into "super-calculators" or hand-held "Personal Mathematical Assistants" (Kissane, 1995), as well as the ever increasing dependence of society on technological infrastructure means that "to expect that schools and teachers can continue to exist apart from serious technological support is hopelessly myopic" (Kaput & Thompson, 1994, p. 682). At the same time, the movement to embed the use of information technologies in classroom practice has been paralleled by a focus on the role of social interaction in teaching and learning, particularly that of peer collaborative practice as a mediator of mathematical understanding. As a consequence there is an emerging interest in the way technology can be used to mediate collaborative practice.

Because computers can be used to provide concrete representations of abstract ideas, and thus become a public and concrete focus for students' joint mathematical activity, collaborative peer interaction can be facilitated by computer environments (Kennewell, 1994; Dubinsky and Tall, 1991). In a study of 11th grade students enrolled in advanced courses of mathematics Lehtinen and Repo (1996) found evidence that the strategic use of a computer algebra program promoted social interaction between students and also between students and teachers. They argue that technologically rich environments provide opportunity for students to carry out activities that require them to externalise abstract thinking processes and to reflect on their own and their peer's thinking processes. Further, computer based learning environments enable students and teachers to discuss mathematical concepts before students have developed adequate topic specific language to talk about a new mathematical idea.

There is evidence to suggest the type and style of interaction are affected by the choice of software (Hoyles, Sutherland and Healy, 1991) and nature of the task (Geiger and Goos, 1996), although much of the work to date has been based on software designed for use within collaborative contexts. By contrast some researchers have observed collaborative behaviours between students in environments built around software that is generic in nature (Geiger and Goos, 1996) or designed to explore specific mathematical ideas and concepts (diSessa, 1995).

Similar results have been reported on the use of hand held mathematics specific technological devices, ie calculators. Groves (1996) claims that the introduction of calculators into school mathematics classrooms encouraged teachers to talk more with their students and provided students with opportunities to share and discuss each others ideas. Further, graphing calculators have been reported to encourage students to talk about problems, find solutions without the assistance of the teacher, and then show each other how to solve problems (Kennedy, 1994).

Penglase and Arnold (1996), in a critical review of recent research in the use of graphing calculators in mathematics education, found that students' attitudes towards the study of mathematics generally became more favourable as a result of regular use of graphing calculators. A firm belief in the helpfulness of graphing calculators in learning mathematics was also evident in a range of studies, the only reservation being expressed related to a fear of "de-skilling" or loss of competency in doing certain forms of mathematics without the support of a graphing calculator.

This study grew out of the observation of the author's own secondary classroom and classroom practice and is part of a larger study examining the role of mathematically enabled technologies in facilitating students' exploration of mathematical ideas, in mediating teacher-student and student-student interactions, and in promoting metacognitive activity. Although the knowledge base in this field is developing, most studies to date have been experimental in nature (eg Lehtinen and Repo, 1996; Hoyles, Sutherland and Healy, 1991) or focused only on one particular type of technology (eg Dubinsky and Tall, 1991; Kissane 1995). The research reported in this paper examines student perspectives on the role of technology in their own learning of mathematics and their perceptions of its impact on student-student interaction or collaborative community of practice (Goos, Galbraith and Renshaw, 1994) in natural classroom settings while using a range of mathematically enabled technologies.

Context

This study is set within a coeducational secondary school and involved 12 students (3 females and 9 males) enrolled in a specialist mathematics course over 14 months through Year 11 and into their first month of Year 12. The delivery of the course was consistent with the Queensland Mathematics C syllabus guidelines (Board of Senior Secondary School Studies, 1992) but integrated the use of computer or graphing calculator technology wherever appropriate and practicable. The syllabus provides encouragement for teachers to make use of technology for teaching and learning mathematics but does not mandate the use of graphing calculators or computers. The decision to make use of technology is school based, being dependent on available resources and expertise within individual schools. An outline of the course topics and the relevant technologies appears in appendix 1.

Students were provided with a graphing calculator, either a Texas Instruments TI-83 or TI-92, everyday in class but were rarely allowed to take them home. Computer facilities were available during classtime in a computer lab when needed and were also generally available for use at lunchtime. All students had access to a computer at home with spreadsheeting and function graphing software installed.

Both graphing calculator and computer technologies were used in a variety of settings and for a range of mathematical activity. Technologies were used as *Tools* (Taylor 1981) to perform mathematical activities they would otherwise have conducted in some other way or to perform tasks that would have been beyond their capabilities without the assistance of computer or calculator technologies (eg calculations involving operations on large matrices); and as *Catalysts* (Willis and Kissane 1989) as a means of provoking mathematical explorations and discussion or to invoke the use of problem solving skills (eg the search for conditions associated with population stability in an exploration of the logistic equation).

Graphing calculators were available during all assessment components of the course.

Method

The purpose of the study was to gain insight into students' perspectives on how they learn and do mathematics in the context of a technologically rich environment and to map out if and how their attitudes change over time. In order to capture a snapshot of students' attitudes to technology and its role in learning mathematics a questionnaire was administered during the third term of Year 11 after the *Introduction to the Theory of Chaos and Fractal Geometry* unit (see appendix 1). The questionnaire was completed at this time, as by this stage, students had been engaged in a range of technology based experiences. These included exposure to both graphing calculator and computer technology as well as activities that were procedural, that is, instances where it was far more efficient to make use of technology rather than pen and paper techniques (eg calculations involving large matrices), and investigative, where technology is used to augment students' problem solving skills in open-ended, problem oriented situations. The questionnaire was then readministered in the first term of Year 12 after students had gained further experience with different technologies and modes of usage in order to make judgements about the stability of students' attitudes and as a way of detecting changes in these attitudes over time. Trends, patterns and differences were then used as stimuli to structure discussion in a whole class interview conducted two weeks after the completion of the questionnaire for the second time.

The questionnaire was based on Likert style items (Sections 1-4) as well as a number of open ended response items (Section 5) and is adapted from questionnaires developed by Galbraith and Haines (in press) and Goos (1995). It focuses on five issues:

Section 1 - attitudes toward the use of technology in learning mathematics
Section 2 - preferences for working together, or alone, when using technology
Section 3 - how technology is used while learning mathematics
Section 4 - how technology is used at home versus school
Section 5 - open ended responses to questions about use of technology to learn mathematics and preferences for using computer or graphing calculator technology

Students were asked to indicate strong agreement (5), agreement (4), neutrality (3), disagreement (2) or strong disagreement (1) for each item in Sections 1, 2 and 3, and frequency as always (5), often (4) sometimes(3), seldom (2), or never (1) in Section 4. Mean scores were calculated for each Likert item on both questionnaires; a higher score thus indicating a more positive response to an item.

This paper is based on the questionnaire and interview data alone and was collected in parallel with classroom observation, video and audio taping which form the broader base of multiple methods used in the larger research project in which this study is situated.

Results

Decaying Extremities

Student responses indicated that they had become less extreme in their opinions with fewer scores of 5 (61 vs 119) and 1 (23 vs 44) recorded on the second response to the questionnaire in relation to the first response. Exploration of this observation during the whole group interview indicated that students had greater experience with the technologies under discussion and in general were more confident with its use. This meant that students were less likely to be intimidated, or impressed, by the novelty of the technology alone. They were also more aware of the strengths and weaknesses of these technologies and so less emphatic about their views on related issues.

> Fran: *Since the first time we've had more experiences and so are less definite about our answers.*

Attitudes

Responses indicated stable and overwhelming support for the use of technology to support the study of mathematics. The mean scores on items from Section 1 ranged between 4.08 and 4.69 on the first response to the questionnaire and between 3.92 and 4.42 on the second. Students enjoyed the use of technology, were confident when using technology to explore new problems and in fixing mistakes, and were prepared to persist with its use even when the solution to a problem was not immediately apparent.

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These attitudes were attributed, during the general interview, to the options provided by technology to do problems in different ways in a shorter time span, the facility to verify hunches and conjectures "on the fly" without needing to stop and lose a train of thought, and the opportunity to study a visual representation of a problem especially in relation to work on functions. The use of technology was also viewed in a positive light because of the variety it introduced into the regular classroom practice of using pad and pen.

Size Matters and Letting Go of the Apron Strings

Section 2 (Table 1) of the questionnaire was designed to elicit responses which indicated students' dispositions toward working with others or alone while using technology. This section represented the greatest change in response trends between the first and second completion of the questionnaire with a change of 0.5 in the mean for three out of the five items. While generally the means for these items indicated a strong positive attitude towards working together while using technology, the decrease in the mean in three of the five items indicated that enthusiasm for working together had been moderated even if relatively mildly.

Table 1

		Mean 1	Mean 2
1	I prefer to work with technology on my own when studying mathematics	2.92	3.42
2	I prefer to work with others when using technology because I feel I need help if something goes wrong	3.77	2.92
3	I prefer to work with others when using technology because I like to discuss what I see on the screen	4.23	3.75
4	I don't like others to see the work I do with technology in case they criticise what I've done	1.54	1.83
5	When I use technology to study mathematics I really feel I need to share with others what I find	3.38	3.33

When presented with this analysis during the whole class interview students firmly emphasised they favoured a collaborative working environment.

Colin: *I prefer working with others.*

The contrast between this position and the trend in the data, was explained by students in two ways.

Firstly, the role of experience was cited as an influence. Students felt they needed less assistance as they gained experience in the use of a technology. Since all participants had gained in both experience and confidence, due to further exposure to mathematically enabled technologies between administrations of the questionnaire, then they were less likely to require assistance from others. Collaborative activity, therefore, was more likely to be associated with the mathematics of a problem rather than any technology related issues.

The second issue is related to screen size, students indicating that it was far more difficult to share ideas, confer and focus a discussion around an image on a graphing calculator screen in comparison to a computer screen.

Fran: *With calculators it's hard to confer, but it's different with computers. I like to work together.*

Sean: *Calculators are more personal.*

Fran: *With calculators it's too hard to show each other; the computer screen is easier for more than one person to see. "Technology" is too broad a term* (referring to the use of the word in the questionnaire which doesn't differentiate between calculators or computers in this section), *it* (referring to a student's response) *depends on the type of technology.*

Students further commented that they did not feel a great need to share discoveries and so tended to interact with peers primarily when "stuck" on a problem.

The Calculator as a Personal Mathematical Assistant and Collaborator

Students revealed a sophisticated understanding of how they used technology to learn and do mathematics. The Likert items in Section 3 (Table 2) again indicated students were comfortable with using technology.

Table 2

		Mean 1	Mean 2
1	I prefer to just learn the mathematics and find the need to learn technology as well a burden	1.46	1.67
2	Good students don't need the assistance of technology to understand mathematics	1.85	2
3	Technology is only there to check what you do with pen and paper	1.38	1.67
4	Technology allows me to explore my own ideas about mathematics as well as those discussed in class	4.23	4
5	I am sometimes forced to use new mathematics when exploring the use of technological tools	3.85	3.67
6	By looking after messy calculations technology makes it easier to learn essential ideas	4.62	4.25
7	I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly	2.77	3.25
8	I tend to use technology to do calculating basic tasks but not much else	1.69	2.08
9	I find technology particularly useful when exploring unfamiliar problems	4.69	4.08
10	Technology allows me to learn mathematics more easily because I can work through a greater number of examples more quickly	4.38	3.75
11	Technology helps me to link knowledge, eg the shapes of graphs and their equations	4.46	4.25
12	I can often solve problems using technology in the classroom but when thinking about the same mathematics later I feel I don't really understand it	2.62	2.67

The general interview supported questionnaire data in relation to students' enthusiasm for working with the assistance of technology. Students expressed the view that the advantages offered by technology were principally related to the efficiency and power available for exploring a situation:

Fran: *So you don't have to use pen and paper* (alone)*;it's easy to change what you have typed in, it helps with investigating.*

Question 7 referred to the order in which a mathematical idea or concept should be presented for learning when technology was available. Questionnaire data indicated an increase in support for learning how to use technology after students had studied a mathematical topic. As interpreted by Jack:

Jack: *Better to understand first before using a calculator.*

Students related this change to technology based activities used to introduce units of work. Some activities, as to be expected, were subject to experimental variation which some students found conflicted with their view of mathematics as an exact science:

Jessie: *Theory is always right!*

Fran: *I get confused with "explore" activities using technology. It just doesn't work for me.*

Thus some students placed little value in practical activities and preferred being taught theory from the beginning.

A drop in support for using technology to explore unfamiliar problems (question 9) was also observed. This change was put down to an unease about how well an activity "worked":

Jack: *I've seen more failures.*

Exposure to activities which were subject to experimental error seemed to have marginally eroded students' confidence in using technology to investigate an unfamiliar situation. It should be noted, however, that despite this trend students' comments were still very positive about the use of technology in unfamiliar contexts (means of 4.69 and 4.08) which indicates that students were moving to a more practical realisation of the strengths and weaknesses of technology rather than viewing it as a cure all, that is, as a helpful if fallible collaborator or assistant, just as their peers in class.

Section 5 of the questionnaire was designed to compare and contrast commonalities and differences between school and home use of technology. What this revealed was that there was very little home use related to school activities other than writing up assignments. This section, combined with the general interview, did provide, however, insights into a developing capacity among the students to make discriminating use of technology.

Table 3

2	I use technology at school:	Mean 1	Mean 2
a	when the teacher tells me	4.69	4.42
b	when I get stuck on a problem	3.92	3.92
c	to look at a problem in a different way, eg a picture or a table	3.85	3.83
d	as a way of discussing a problem with others	3.92	3.42
e	when I feel pen and paper isn't helping	4.15	4
f	as a first resort when looking at a mathematical problem	3.23	2.92

The trend in this portion of the questionnaire is one of strong support for the use of technology in both sets of responses to the questionnaire except for 2(f) where the feeling appears to be neutral. When students were asked for an explanation for this difference during the general interview they made it clear they understood it is often best to think about a problem first rather than reach for a calculator as a matter of course:

Fran: *Only use technology if you think it will help, look at the problem first.*
Jessie: *You need to decide what's more effective* (ie to use technology or not).

Students had thus shown they were developing a mature and flexible understanding of, and approach to, the use of technology. They were aware that technology could be used in a variety of ways and were developing the capacity to make decisions abut when and how it could or should be used.

Discussion

Students' responses to the questionnaires revealed strong support for the use of mathematically enabled technologies in the mathematics classroom. Further, there was a strong belief that the use of these technologies provided valuable support for the learning of mathematics and the investigation and exploration of mathematical situations set in unfamiliar contexts. This stands in agreement with other studies which have investigated attitudes about learning in computer (eg Lehtinen and Repo, 1996) and graphing calculator (Penglase and Arnold 1996) enhanced environments.

The questionnaires and follow up interview indicated students also felt very positively about using technological tools while collaborating with each other. This was more likely to occur while using a computer because the size of the screen made it easier to make public ideas being explored by members of a group. This is a finding consistent with Lehtinen and Repo's (1996) assessment of the reasons for apparent facilitation of positive peer to peer interaction during sessions of computer assisted algebra programs.

Responses also indicated that the nature of the task was a crucial influence in the mediation of collaborative interaction. Just as Geiger and Goos (1996) found that tasks that were problem orientated were more supportive of this type of interaction than procedural activities, students made it clear that they were most likely to feel a need to interact if there was a genuine problem to be solved given they felt comfortable with the operation of the technology itself. This finding also serves to emphasise the importance of the sensitive and sensible interpretation of the questionnaire data. Insight into the perceived changes over time related to students' attitudes to working collaboratively with technology required elaboration by students during the general interview session to gain a more fully informed interpretation of events consistent with the students' overall positive view of this type of interaction.

The longitudinal nature of this study has provided evidence that tangible benefits to the learning and doing of mathematics may be accessible and actualised through the careful implementation of technologically supported classroom environments. It has demonstrated that students feel positively about the use of technology in mathematics classrooms and has documented their positive belief that technology can help them learn, understand, and solve problems in mathematics. These findings should be tempered, however, with a caution that what students believe themselves to be doing may not be supported by the actuality of a situation. This heralds the importance of the need for further and more detailed observation of students working with mathematically enabled technologies in natural classroom settings. This is one of the agendas to be pursued in follow up studies which are components of a larger research project.

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Appendix 1

Year 11 Subject Matter	Technology
Structures and Patterns - recognition of patterns in well known structures including Pascal's Triangle and Fibonacci sequence; applications of patterns; life-related applications	Computers using a dynamic graphing package Cabri-Geometre in assignment work - investigative
Introduction to Groups - closure, associativity, identity, inverse	
Real and Complex Number Systems - structure of the real number system	
Matrices and Applications -matrix operations; applications of matrices	Graphing Calculators used for operations on matrices. Principally used as a calculating device - procedural
Vectors and Applications - for vectors as structures which are used for the storage of data; operations on vectors; simple life-related applications of vectors	Computers using a dynamic graphing package Cabri-Geometre to illustrate the visual representation of vectors - instructional
Matrices and Applications - determinant of a matrix; singular and non-singular matrices; solution of systems of homogeneous and nonhomogeneous linear equations using matrices; applications of matrices	Graphing Calculators used for operations on matrices. Principally used as a calculating device - procedural
Vectors and Applications - for vectors describing situations involving magnitude and direction - two dimensional vectors and their algebraic and geometric representation; operations on vectors; unit vectors; calculation of the angle between two vectors; resolution of vectors into components acting at right angles to each other; applications of vectors	
Introduction to the Theory of Chaos and Fractal Geometry - basic notions of recursive functions and iterative processes; exponential growth and decay; basic notions of Fractal Geometry; Koch curves	Computer spreadsheets, graphing packages associated with spreadsheets - investigative
Real and Complex Number Systems - structure and representation of complex numbers including algebraic definitions and interpretations, Argand diagrams and interpretations, polar form, conjugates	Graphing calculators using complex number facility for the calculation of operation related problems - procedural
Dynamics - Newton's Laws of Motion in vector form: straight line motion in a horizontal plane with constant force; vertical motion under gravity without air resistance; projectile motion without air resistance	Computers using video capture and editing facilities and Learning in Motion software for the investigation of video facilities - investigative
Year 12 Subject Matter	
Real and Complex Number Systems - de Moivre's Theorem; simple, purely mathematical applications of complex numbers	Graphing calculators using complex number facility for the calculation of operation related problems - procedural

"I don't know if I'm doing it right or I'm doing it wrong!" Unresolved uncertainty in the collaborative learning of mathematics

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This paper reports on a study which investigated patterns of collaborative metacognitive activity in senior secondary school classrooms. Although peers working together on mathematical tasks may enjoy the metacognitive benefits of being able to monitor and regulate each other's thinking, collaboration does not guarantee that they will achieve a mathematically productive outcome. Analysis of a videotaped lesson transcript illustrates how metacognitive uncertainty, itself a trigger for collaboration, remained unresolved when students did not have the means of validating their solution.

The last decade has been marked by the emergence of new mathematics curriculum and policy documents which place increased emphasis on problem solving, mathematical reasoning, and communication, and promote peer interaction and discussion as a means of helping students to develop their understanding of mathematics (Australian Education Council, 1991; National Council for Teachers of Mathematics, 1989). However, our theoretical understanding of problem solving processes, and how students' mathematical thinking is shaped by their interaction with peers, is far from complete (e.g. Lester, 1994; Schoenfeld, 1992).

One aspect of mathematical thinking which deserves continued research attention concerns the role of metacognitive processes, that is, how students monitor and regulate their thinking while working on mathematical tasks. The importance of metacognition is now widely acknowledged, and many studies have investigated the metacognitive strategies which secondary school students use in problem solving. However, these studies have tended to focus on students working individually, in experimental settings, on tasks prescribed by the researcher (e.g. Fitzpatrick, 1994; Randhawa, 1994). Despite increasing research interest in the social and cultural aspects of mathematics learning (e.g. Brown et al., 1993), few studies have sought to examine the characteristics of collaborative metacognitive activity occurring when students work together in natural classroom settings, or the conditions under which such interaction leads to successful or unsuccessful problem solving outcomes.

The research discussed in this paper is part of a larger study which investigated patterns of classroom social interactions associated with metacognitive activity, and assumptions about teaching and learning mathematics underlying teachers' and students' actions. A major aim of the research was to suggest mechanisms through which peer interaction mediates metacognitive activity, and results reported previously have demonstrated that jointly transacted monitoring and regulation can help students to overcome the obstacles in their progress towards a successful solution (Goos & Geiger, 1995; Goos, 1997). Nevertheless, it would be misleading to claim that peer collaboration always achieves a mathematically productive outcome. The purpose of this paper is to identify circumstances in which collaboration may be metacognitively fruitless. One instance of metacognitive failure is examined in detail in order to illustrate some of the conditions responsible for students' lack of success.

Metacognitive Success and Failure

Frameworks for analysing task-oriented mathematical thinking typically identify phases or episodes representing distinctive types of problem solving behaviour, and describe the ideal characteristics of each episode (e.g. Artzt & Armour-Thomas, 1992; Schoenfeld, 1992). While these frameworks acknowledge the central role of metacognitive processes in keeping the solution process on track, they do not consider in detail the types of monitoring and regulatory activities that would be appropriate and expected at each stage of the solution. In particular, previous research in this area has not distinguished between the routine monitoring which serves merely to confirm that all is well, and the more controlled monitoring and regulatory processes triggered when students become aware of specific difficulties. It is helpful to think of these triggers as metacognitive "red flags", which signal the need for a pause or some backtracking while remedial action is taken. "Red flags" could be raised when students realise that they are making no progress, notice a calculation error, or recognise that their answer violates the problem conditions or does not make sense. Appropriate responses to each of these warning signals are shown in Figure 1.

"Red Flag"	**Congruent Response**
Lack of progress	Reassess appropriateness of the solution strategy. Decide whether to persist or abandon the strategy, identify useful information to be salvaged.
Detection of an error	Check and correct calculations.
Anomalous result	Check calculations. Reassess the solution strategy.

Figure 1. Metacognitive "red flags" and congruent responses

A further problem with existing analytical frameworks, and with research on metacognitive processes in mathematics generally, is the lack of explicit attention given to characterising different forms of metacognitive failure, other than to note that students did not exploit useful information, or that checking behaviour was absent. When students work individually or collaboratively on mathematics tasks, one could imagine that several different metacognitive scenarios might arise, as illustrated in Figure 2. While metacognitive *success* will occur if students recognise a "red flag" and take appropriate action to deal with the difficulty (or recognise that nothing is wrong and continue on the same solution path), less successful outcomes are likely in at least three other circumstances. First, students can be guilty of metacognitive *blindness* if they fail to notice that something is amiss, for example, by persisting with the wrong strategy or overlooking a calculation error. Second, students might commit metacognitive *vandalism* by taking inappropriate action to deal with an impasse, for example, by changing the problem to enable them to apply knowledge already available to them. Finally, the "red flag" itself may be spurious and represent a metacognitive *mirage* if students "see" difficulties which do not exist, and mistakenly abandon a useful strategy, amend calculations which are not in error, or reject correct answers.

A less obvious form of mirage materialises when students are unsure about what they "see" and, unable to make any judgment about the correctness of their strategy or answer, remain lost in the desert of uncertainty. The means by which such uncertainty is resolved have been studied by Clarke and his colleagues (e.g. Clarke & Helme, 1997), who have conducted an extensive study of mathematics learning as it occurs in legitimate classroom settings. Clarke and Helme (1997) argue that learning in social settings, such as classrooms, proceeds through the negotiation of meaning, and that the social goal of negotiation is to resolve uncertainty. They identify four forms which the process of resolution may take, basing the categorisation on the dominant authority to which students appeal in seeking resolution:

- prior experience;
- empirical evidence;
- a knowledgeable person; or
- a text.

It seems reasonable to conclude that students' inability to make metacognitive judgments may be due to lack of access to one or more of these forms of authority. Such a situation is illustrated in the remainder of the paper, which examines one case of unresolved uncertainty in a Year 11 mathematics classroom.

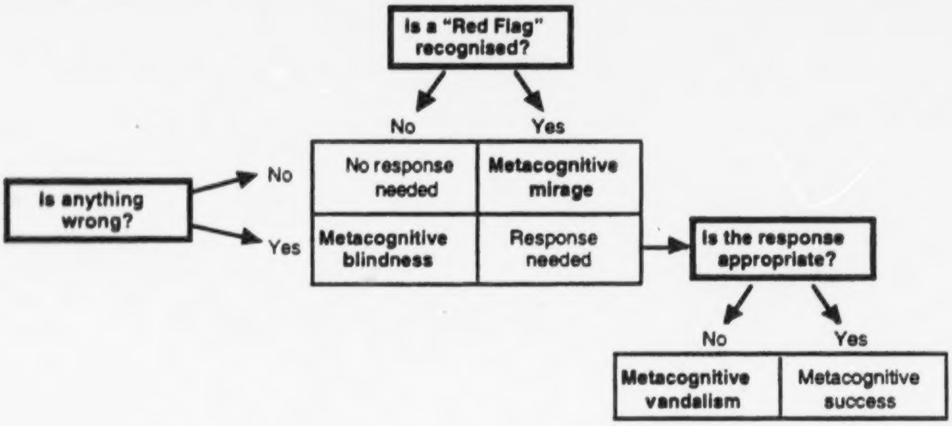

Figure 2. Metacognitive success and failure

The Classroom Study: Data Gathering and Analysis Methods

The study was carried out over a period of three years, and involved eight secondary school teachers and their Year 11 or Year 12 mathematics classes. Methods used to investigate students' individual and collaborative metacognitive activity included questionnaires, interviews, and classroom observation supplemented by audiotaping and videotaping. Target students were chosen for videotaping and interview on the basis of their metacognitive sophistication and preference for working collaboratively with peers, as judged from preliminary observation and responses to questionnaires (see Goos, 1995, for details of questionnaires). One lesson was observed each week, and target students were audiotaped and videotaped as they worked together and discussed their ideas in class. Portions of the videotapes were later transcribed for analysis. Semi-structured, individual interviews were carried out to probe and follow up some of the issues raised in the open ended metacognitive self-knowledge questionnaire. All interviews were audiotaped and transcribed.

Classroom videotape transcripts were first parsed into macroscopic episodes which categorised the group's problem solving behaviour as Reading, Understanding, Analysis, Exploration, Planning, Implementation, Verification, or Transition (episode characteristics are as described in Artzt and Armour-Thomas, 1992). A finer grained analysis of conversational turns was then carried out to identify metacognitive activity. Two types of metacognitive acts were coded: proposing a *New Idea* (recognising potentially useful information, mentioning an alternative approach); or making an *Assessment* of a particular aspect of the solution (such as one's understanding of the problem, the appropriateness or execution of a strategy, or the accuracy or sense of results) (Goos & Geiger, 1995). Analysis of the interview transcripts focussed on individual students' ability to recognise and respond to metacognitive "red flags".

Unresolved Uncertainty in Solving Combinations Problems

This paper draws on videotape and interview data from one Year 11 mathematics class in the third year of the study. The videotape transcript comes from a lesson in the early stages of a unit of work on combinatorics. Knowing that he would be unavoidably absent for this lesson, the teacher had set a series of problems which would give the students their first opportunity to apply their newly gained knowledge of combinations. These problems were contained in a teacher-prepared handout which also included explanations and worked examples, and served as the students' sole text for the topic. Four target students—Dylan, Alex, Sean, and Rhys—figure in the transcript. (Pseudonyms are used to preserve anonymity.) Figure 3 shows the first problem on which they worked (Question 19 in the problem set), together with model solutions.

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1 9 How many selections of five cards can be made from a pack of 52 playing cards so that there are:
a at least three aces? **b** three hearts? **c** at least one heart?

Solutions

a Possible hands could contain either three or four aces.

$$\therefore \text{ number of selections } = \left({}^{4}C_3 \times {}^{48}C_2\right) + \left({}^{4}C_4 \times {}^{48}C_1\right)$$
$$= (4 \times 1128) + (1 \times 48)$$
$$= 4512 + 48$$
$$= 4560$$

b The hand must contain three hearts and two non-hearts.

$$\therefore \text{ number of selections } = \left({}^{13}C_3 \times {}^{39}C_2\right)$$
$$= 286 \times 741$$
$$= 211926$$

c The hand may contain either one, two, three, four, or five hearts. (A simpler method is to find the number of hands with no hearts and subtract this from the total number of five card hands.)

$$\therefore \text{ number of selections } = \left({}^{13}C_1 \times {}^{39}C_4\right) + \left({}^{13}C_2 \times {}^{39}C_3\right) + \left({}^{13}C_3 \times {}^{39}C_2\right) + \left({}^{13}C_4 \times {}^{39}C_1\right) + {}^{13}C_5$$
$$= (13 \times 82\,251) + (78 \times 9139) + (286 \times 741) + (715 \times 39) + 1287$$
$$= 2\,023\,203$$

Figure 3. Combinations problems

Episode Analysis

Analysis of the transcript is limited to the time in which the students jointly focussed on Question 19, before their solution paths diverged and social interactions became more fragmented and difficult to track. The following conventions were adopted in transcribing students' interactions: (a) conversational turns (referred to as Moves) are numbered sequentially; (b) students are identified via their initials; (c) non-verbal information from the videotape is included in parentheses; (d) the symbol (...) indicates that part of the transcript has been omitted; and (e) annotations indicating metacognitive acts are recorded in italics.

Episode 1—Reading/Understanding: After reading the stem to Question 19 and making the observation that there were $^{52}C_5$ hands in total, Dylan immediately recognised that this would give too large a number for part (a) of the question, which imposed the constraint of having at least three aces in the hand.

1. D: (...) So it's fifty-two C five. *(New Idea)* (No response from other students. They look at him, smiling, expressions of disbelief on their faces.) Sounds a lot, doesn't it. *(Assessment—strategy appropriateness)*

Nevertheless, all three boys used the $^{n}C_{r}$ buttons on their calculators to gain a feel for the problem by confirming that $^{52}C_5$ is indeed a large number (2 598 960).

6. D: (...) Anyway let's see what fifty-two C five is ... (uses calculator)
7. All: (in unison) Two million five hundred and ninety-eight thousand, nine hundred and sixty!

Episode 2—Analysis: Although they had identified the relevant information in the problem, the students struggled to formulate a strategy for taking account of the specified selection of at least three aces. Eventually, Alex and Dylan proposed that $^{52}C_2$ might represent the number of five card hands *without* three aces, foreshadowing an approach based on mutually exclusive operations and the addition principle. (Note that they still had not come to grips with the "at least" condition.)

15. A: (doubtfully) Yeah, but how do you work out these three aces?
16. D: No, you've got five cards, so it's only fifty-two, ah ... fifty-two C— *(New Idea)*
17. A: Ohh! Do C two, that's how many *won't* have— *(New Idea)*
18. D: Yeah, and you *got* to have—
19. A: (simultaneously) —a certain three cards.

Despite their initial enthusiasm for this strategy, it soon became apparent that the boys had no way of knowing whether or not they were on the right track.

24. D: Aah I don't know if I'm doing it right or I'm doing it wrong! *(Assessment—strategy appropriateness)*

Episode 3—Exploration: The students' uncertainty was manifest in the way they appealed to each other for assistance.

25. S: (to Alex) So what have you done for the first one?
26. A: (Unsure) I don't know. *(Assessment—understanding)* You go fifty-two ... (to Dylan) So how are you doing (a)? Dylan? Dylan? (Dylan is talking off-task to Rhys.) How are you doing (a)?
27. D: I don't know. *(Assessment—understanding)*
28. A: Are you doing fifty-two, C, two—
29. D: —two—
30. A: —and then subtract it?

Before long, the boys abandoned part (a) of the problem and acknowledged that they were stuck on Question 19 as a whole. Not yet willing to give up completely, they considered two potentially useful strategies for dealing with impasses such as the one they faced—working backwards from the answer (Moves 41 and 42), and looking at a similar problem (Move 45). Unfortunately, they were unable to take advantage of either strategy, since the teacher-prepared handout did not provide answers to the problems, and they overlooked a worked example in the text which might have provided some clues. While Alex continued to hunt for a helpful example in the text, Dylan moved on to Question 19 (b), and began hesitantly to reason out a strategy which would lead him to the correct answer (Move 44).

40. D: So how do you do it?
41. S: If we had an answer—an answer sheet— *(New Idea)*
42. D: Yeah, we could figure it out. *(New Idea)*
43. A: You could always think about it without the C rule. And go like, OK, for hearts you've got, however many choices, and, the next choice you've got however many choices, the next choice you've got ... *(New Idea)*
44. D: You got a quar—ter ... (hesitation, draws out this word) A quarter of fifty. *(New Idea)*
45. A: (Not listening to Dylan) Think about it the long way. Hey, is there an example somewhere? *(New Idea)* (Checks quickly through handout, overlooks Example 12.)

Episode 4—Implementation: In this segment of the transcript there is evidence that Dylan was beginning to develop a general understanding of how the choices of cards can be constrained. In the case of Question 19 (b), if a five card hand is to contain three hearts, then the hearts are selected from only one suit (a quarter of fifty-two cards), not the full pack.

46. D: What's a quarter of fifty? Twenty ... No, that's half of fifty.
47. S: Quarter of fifty, or fifty-two?
48. D: Quarter of fifty, because there's two Jacks (meaning Jokers). Oh hang on. Is it fifty-two including, Jokers? Or fifty-four?
49. A: (Still flicking through handout, searching for example.) No, fifty-four.
50. D: Fifty-four. Then it's a quarter of fifty-four.
51. A: (Unable to find an example) We could probably do it if we thought about the long way—
52. R: (to Dylan) That's something and a half. Twenty-five and a half.
53. D: (Uses his calculator; looks at Rhys) Thirteen. (Rhys and Dylan laugh.)
54. D: (writes) This is thirteen out of fifty-two ... is ... hearts. So what would you go? Would you go, thirteen ... C ... [inaudible]. *(New Idea)*

Transition (Moves 55-62): Having successfully, if somewhat laboriously, calculated the number of hearts in the pack, Dylan now became absorbed with completing Question 19 (b), and he worked in silence while his friends considered their next move.

55. S: So are you still trying to work out something for—?
56. A: No, I'm just going to leave that for now. And wait until *he* comes up with—

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57. S: Leave Question 19 altogether?
58. A: Yeah, I don't know how to do it. *(Assessment—understanding)*

Episode 5—Implementation: The start of this episode is heralded by Dylan's triumphant announcement that he had worked out how to attack Question 19 (b). By this time, however, Alex and Sean were busily working on Question 20, and they did not acknowledge his breakthrough.

63. D: (to himself) So should we go ...? I know, I've figured it out! I've figured it out! *(Assessment—Understanding)* (Pause) Multiply that by ... what's the [inaudible]? It's thirteen take fifty-two. *(New Idea)*
64. R: Thirteen *take* fifty-two? *(Assessment—strategy execution)*
65. D: Sorry! Fifty-two take thirteen. Thirty-nine, yeah. (Quietly, to himself) Thirty-nine C two.
66. A: (Reading Question 20) How many committees of five ...?
67. D: (to himself, using calculator) Two hundred and eighty-six times ... seven hundred and forty-one! (Sounds surprised)
68. R: Is that for (a) or (b)?
69. D: That's for (b)! I think (a)'s wrong actually, but anyway ... *(Assessment—accuracy of result)* (Long pause, writing. Goes on to Question 19c.) C ... one ... C four ... is thirteen times ... eight thousand two hundred, no, eighty-two thousand two hundred ... (Long pause, writing. Responds to inaudible question, from student off camera.) Well we don't have any answers, so we don't even know if we're right. *(Assessment—accuracy of result)* (Continues working) Thirteen C two ... (now doing Question 19c)

Although Dylan did not verbalise all his working, it is clear that he was pursuing the correct approach to solving parts (b) and (c) (see Moves 67 and 69, and Figure 3).

The Sequel

The episode parsing analysis concludes here, because the original group of students now broke up to work on different questions. However, later exchanges between Dylan and Rhys provide further evidence that Dylan had reasoned out the correct strategy for dealing with combinations problems like Question 19. Rhys had lagged behind the other three boys, and only belatedly sought to compare his own methods with theirs.

122. R: Dylan, what did you get for 19 (c)? (Reads Dylan's working) Thirteen C one times thirty-nine C four ... (Sean and Alex talking at the same time. Rhys seems to be disagreeing with Dylan). That's what I reckon ... Shouldn't it be fifty-one C four? Because it says "at least one heart"; it doesn't matter if you have [inaudible]—
123. D: But there's thirteen hearts to choose from—
124. R: Yeah, but then the rest of the hearts could be chosen in the next, thing, so only one of them's there [inaudible]. There's thirty-nine to choose from, plus another twelve, which is fifty-one.
125. D: [inaudible]
126. R: (looking at Dylan working) Woh, man, our answers are so way different now.
127. D: You're stung with my answers, aren't you!

For Question 19 (c), Rhys argued that the condition of "at least one heart" in a hand of five cards is satisfied by selecting four cards from the fifty-one cards remaining once the required heart is removed. Although Dylan did not yield to his friend (see Move 123), he appeared to be unsettled by the difference in their answers, and the lesson ended with all students still unsure whether they had found the correct way to approach these problems.

Metacognitive Function of the Dialogue

The numbers and types of metacognitive acts were recorded for each student, and for the dialogue as a whole. Of the 69 Moves in the portion of the transcript subjected to analysis, 22 were coded as having a metacognitive function. Dylan and Alex were the main contributors, sharing ten of the eleven New Ideas (six for Dylan, four for Alex) and all eleven Assessments (seven for Dylan, four for Alex). However, this quantitative representation does not tell the full story, since it obscures an important quality of the students' metacognitive activity while working on the combinations problems—their inability to make valid judgments about their strategies and answers. For example, three of the four Assessments of understanding revealed that the Alex and Dylan *did not understand* how to approach Question 19 (Moves 26, 27, 58). Similarly, Dylan was *unsure* of the appropriateness of the strategy he implemented for Question 19 (a) (Move 24), and he expressed his *doubts* as to the accuracy of his answers (Move 69).

Another finding which deserves comment is the absence of any Assessments of the *sense* of results. The sheer size of the answers to combinations problems often astonishes students, and seems to contradict their expectations of what a reasonable answer would be. Lack of prior experience with combinations calculations may have contributed to the boys' difficulties in deciding whether or not their answers were acceptable.

Individual Interviews

Individual interviews were conducted with all the students who appeared in the transcript (coincidentally, on the same day as the lesson from which the transcript was obtained). Relevant questions, the metacognitive "red flag" they targeted, and student responses are summarised in Figure 4. Clearly, the students had access to a wide range of metacognitive strategies which could have helped them to deal with the type of impasse they faced in solving the combinations problems (see Question 1). Nevertheless, their dependence on the quality of their answers in making judgments about solution strategies (see Questions 2 and 3) is significant in this context, where the answers they obtained were difficult to assess for accuracy and sense.

Interview Question	Response Category	Examples of Student Responses
1. What do you do when you get stuck on a problem? *Lack of progress*	Assess appropriateness of strategy	Keep going back to the beginning, go through your reasoning again. (Alex)
	Change the strategy	See if there's another way I can do it. (Dylan)
	Identify new information	Try real examples that I know are going to work so that I know what the answer will be. (Rhys)
	Seek help from teacher or peers	I'll talk to my friends. If they're stuck as well we go and see Mr G (teacher). (Dylan)
2. How can you tell you've solved a problem correctly? *Anomalous result*	Assess result for accuracy and sense	You can check it another way or work backwards. (Alex) Just whether it makes sense. (Dylan)
3. How do you decide whether to change your approach to a problem? *Lack of progress* *Error detection* *Anomalous result*	Assess result for accuracy and sense	You can look and see if the answer makes sense so far. (Alex) You just think it seems a bit strange, or it's too easy. (Dylan) If you're getting real weird numbers. (Rhys)

Figure 4. Responses to interview questions

Discussion

This paper has been concerned with metacognitive failure, and the circumstances under which peer collaboration—often beneficial in helping students to clarify, justify and evaluate ideas—fails to achieve a mathematically successful outcome. It was argued that students can be misled by spurious metacognitive "red flags", which falsely warn that something is amiss, or raise doubts about the validity of the solution method. Such was the case in the lesson discussed here, when students found it difficult to judge whether their strategies were appropriate or their answers made sense, even when the solutions produced were correct.

Complementary data from individual interviews and the transcript of collaborative problem solving showed that, collectively, the students possessed a useful metacognitive

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repertoire which they attempted to bring to bear on the problems they had been set. Their lack of success was measured, not in terms of failure to solve the problems, but by their inability to resolve the uncertainty which had brought them together in the first place. Dylan did correctly solve Questions 19 (b) and (c), but the status of his solution remained doubtful in his eyes because it could not be validated by appealing to the authorities usually available to him—the teacher, a text, and his peers. In these circumstances, collaboration was metacognitively fruitless because the students did not have access to the means of resolving their uncertainty (Clarke & Helme, 1997). To begin with, they lacked *prior experience* with strategies for counting combinations, and they had no *empirical basis* for evaluating the accuracy or sense of answers which seemed improbably large. Neither were *knowledgeable others*, in the form of the teacher or peers, available or able to provide help. Finally, potentially useful worked examples in the *text* were sought out but overlooked.

Long term observation of the students participating in this study has provided evidence of the metacognitive benefits of collaborative interaction. Nevertheless, peers are only one several sources of authority to which students may appeal when making judgments about the validity of their solution strategies and results. This paper has highlighted the need to examine peer interaction in the context in which it occurs, that is, the classroom community in all its complexity, if progress is to be made in understanding the implications of collaboration for students' mathematical thinking.

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Visual reasoning and teaching styles in mathematics classrooms

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Four mathematics teachers with different preferences for the use of visual strategies in solving problems are compared in their use and support of visual methods in their classrooms. Presmeg (1985) found that no teacher who displayed a strong preference for non-visual strategies in problem solving made substantial use of visual methods in their classroom. In this study, the teacher who least preferred to use visual strategies to solve problems displayed a strong use of visual methods in the classroom.

Introduction

Visual reasoning involves understanding a problem or concept in terms of a diagram or a visual image. Zimmerman and Cunningham (1991) provide a workable definition that emphasises both the physical and mental aspects of the visualisation process: 'Mathematical visualisation is the process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding' (p.3).

The use of visual images to assist in the understanding of mathematical concepts has become an area of renewed interest in mathematics education research. Some of this interest may stem from an acknowledgement that much pedagogical use has always been made of diagrams in mathematical discussions. There have been moves in recent years for an acceptance of visual methods beyond that of a pedagogical tool with Dreyfus (1994) arguing that 'the status of visualisation in mathematics education can and should be upgraded from that of a helpful learning aid to that of a fully recognised tool for mathematical reasoning and proof' (p.107). More recently, this promotion of the use of visual forms of mathematical reasoning has coincided with developments in the use of computer graphics and has led some to challenge the conventional presentation of mathematics. Tall (1991) notes that the concept of a limit appears to be 'an unsatisfactory cognitive starting point for the study of calculus' (p.110) and promotes the use of a graph-plotting program that can magnify at will small sections of graphs to develop the idea of the 'local straightness' of a curve.

Yet traps exist for the unwary in the use of visual methods. Presmeg (1986) notes three problems: '(1) The one-case concreteness of an image or diagram may tie thought to irrelevant details, or may introduce false data; . . .(2) An image of a standard figure may induce inflexible thinking which prevents the recognition of a concept in a non-standard diagram; . . .(3) An uncontrollable image may persist, thereby preventing the opening up of more fruitful avenues of thought ' (p.44). While some of the reluctance to accept visual methods may stem from the traps inherent in their use, Eisenberg and Dreyfus (1991) note a reluctance on the part of students to reason visually in calculus courses 'even if they are explicitly and forcefully pushed towards visual processing' (p.29). In a separate article, Dreyfus (1994) postulates that student reluctance may be the result of teachers' attitudes towards visual arguments and their use of them solely as introductory, accessory or auxiliary arguments. In other work, Presmeg (1985 and 1986) found a weak statistical relationship between a teacher's preferred personal use of visual methods to solve problems and the extent to which the teacher uses visual presentations when teaching mathematics. Presmeg (1985) defined 'the extent to which that person prefers to use visual methods when attempting mathematical problems which may be solved by both visual and non-visual methods' (p.42) as the *mathematical visuality* of the person. She defines a mathematics teacher's *teaching visuality* as 'the extent to which that teacher uses visual presentations when teaching mathematics' (p.42). While Presmeg came to the overall conclusion that 'irrespective of whether or not they prefer to solve mathematical problems by visual methods, teachers may or may not use visual presentations in the classroom' (1985, p.269), she also observed that the removal of one teacher from a problems by visual methods, teachers may or may not use visual presentations in the classroom' (1985, p.269), she also observed that the removal of one teacher from a sample of thirteen teachers led to a significant statistical relationship being detected between mathematical visuality and teaching visuality (1985, p.144-145). In addition, 'no teacher of low mathematical visuality was found to be a visual teacher' (1985, p.269).

These reports of the possible connection between teacher attitudes towards visual methods and the support of such methods in the classroom have been the starting point for the present piece of research. This paper reports on a study of Canterbury secondary mathematics teachers and uses the work of Presmeg as a basis for analysing the teachers' personal use of visual methods in problem solving and the use of visual methods in the classroom.

Methods

The research proceeded in two stages. The first stage involved quantifying the mathematical visuality of teachers and using this to select four teachers of differing mathematical visuality for further case study. The second stage involved observing these selected teachers, interviewing them and drawing conclusions about their teaching visuality.

Sample

For the first stage a 'Problem Solving Questionnaire' was posted to 35 secondary schools in the greater Christchurch region. Responses were received from 22 teachers in 17 schools. For the second stage of the research, four teachers were identified as case studies using the results of the questionnaire. Two of the teachers displayed high mathematical visuality while the other two displayed low mathematical visuality.

Measuring mathematical visuality

The mathematical visuality of teachers was gauged by the use of a 'Problem Solving Questionnaire'. This consisted of eighteen problems that could each be solved either by visual or non-visual methods. Aside from a couple of minor variations in wording, the problems were the same as those used by Presmeg (1985) in her analysis of the mathematical visuality of teachers.

The responses to the Problem Solving Questionnaire were analysed and each question was assessed on the extent of the visual reasoning that had been used in the solution, ranging as scores of 2, 1 or 0. A score of 2 was given to a solution that used a visual method as an essential part of the solution. A score of 0 was given to a solution that used no visual method. Scores of 1 were rare and were given for a few solutions where there were some ambiguities in the methods used. These scores were totalled for each teacher to give a measure of the mathematical visuality of the teacher.

Measuring teaching visuality

The teaching visuality of the teachers was developed from the work of Presmeg (1985). In her thesis, Presmeg (1985, p.126-133) measured teaching visuality according to the relative occurrence or absence of the following classroom aspects: a classroom atmosphere that is controlled but relaxed and unhurried; a pictorial presentation of a topic; use of spatial arrangements in algebra; use of gesture (hand waving); use of instructions to form mental images; use of mobile models or images; instruction to pupils to use arm, finger or other body movements; use of concrete materials; use of colour; use and encouragement of intuition; use of pattern-seeking methods; delaying the use of symbolism; creating deliberate cognitive conflict; and accepting and demonstrating alternative methods. Further to this, Presmeg (1985, p.193-194) went on to observe teachers in some detail and itemised a further 45 behaviours in the classroom, most of which were noted in the visual teachers and absent in the non-visual teachers. So as to make the task of observation manageable, the most significant behaviours that were characteristic of visual teachers were included in the observation schedule for this case study: feels rushed because of a lack of time; uses humour; makes use of pupil efforts; uses 'real world' examples; stresses general methodological principles; shows awareness of other methods of solution; expresses feelings towards mathematics; uses language evocative of imagery; and values a visual presentation.

An observation schedule was constructed and trailed with two teachers who had not been selected as case studies for the second stage of the research. A final format was arrived at in which each behaviour was ticked if it occurred and room was left to add a comment to this to assist in the writing up of the observation.

Procedure

Copies of the 'Problem Solving Questionnaire' were posted to 35 secondary schools in the greater Christchurch region. Teachers were told that the questionnaire was part of a research project into problem solving strategies. They were not informed that the purpose of the research was to investigate the use of visual methods. In this way the responses would not be biased towards the display of visual methods in the solution of the problems. Teachers were also asked at this stage to indicate their willingness or otherwise to participate further by being observed in their classroom.

The responses were analysed and each question was assessed for the extent of the visual reasoning that had been used in the solution. Of the teachers that were available for the next stage of the research, two with the highest visual reasoning scores and two with the lowest scores were selected as case studies to be observed in their classrooms. The teachers with high visual scores will be referred to as Teacher H_1 and Teacher H_2 respectively and the teachers with low visual scores will correspondingly be referred to as Teacher L_1 and Teacher L_2.

The case study teachers were then observed teaching in their classrooms. Each teacher was observed for two lessons with year 10 students (15 years old), approximately three weeks apart. One teacher, Teacher H_2, was observed additionally with a year 9 class. Each lesson lasted 50 to 60 minutes. In the first lesson, each teacher was observed in a normal classroom setting teaching a class and topic of their choosing. For the second lesson, the teachers were asked to incorporate two of the problems from the questionnaire that both Teacher H_1 and Teacher H_2 had solved by visual methods, while Teacher L_1 and Teacher L_2 had solved by non-visual methods. It was hoped that the inclusion of this common material would also reduce some of the variables from the first set of observations.

A week after the second observation each teacher was interviewed. They were presented with their original solutions to the problems in the questionnaire and asked to explain their particular solutions to the two questions used in the second lesson. This led to other questions also being discussed. The teachers were also asked about how they went about constructing the approach they took to the second lesson that involved the use of these two problems. A conversation of about 30 minutes duration evolved, at the end of which the full purpose of the research was revealed to the teachers and summaries of the lesson observations were given to them.

Analysis of Responses to the Problem Solving Questionnaire

The responses to the questionnaire were analysed as detailed before with scores of 2, 1 or 0 used to indicate the extent of visual reasoning used in each solution. Out of a total possible score of 36 from the eighteen questions, the scores ranged from a low of 6 to a high of 17. The median was 11, the mean 10.8 and the standard deviation 2.3. Presmeg (1985, p.122) reported for her sample of 30 teachers a range from 3 to 26 with a median of 12. From this initial survey sample, six teachers indicated they were not available for any further aspects of the research. These included two teachers with the lowest visual scores (6 and 8) and one teacher with a high visual score (14). From the teachers that remained, two teachers with the highest visual scores (17 for Teacher H_1 and 13 for Teacher H_2) and two with the lowest visual scores (8 for Teacher L_1 and 9 for Teacher L_2) were then selected as case studies to be observed in their classrooms.

The four case study teachers were all female. This came about through accident, not design. Of the 22 responses received to the questionnaire, 13 were from male teachers

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and 9 from female teachers. The male with the highest visual score and the male with the lowest visual score both were unavailable for further aspects of the research.

There were only two questions which Teacher H_1 and Teacher H_2 solved by a visual method while, at the same time, Teacher L_1 and Teacher L_2 solved with no evidence of a visual method. These were questions 10 and 13:

10. If you place a round of cheese on a pan of a scale and three-quarters of a round of cheese and a $\frac{3}{4}$ kg weight on the other, the pans balance. How much does a cheese weigh?
13. The distance that a tourist travelled by train is 150 kilometres longer than the course he travelled by steamer, and 750 kilometres more than his journey on foot. Determine the length of his entire trip if it is known that the distance he covered on foot was one third of the distance he covered by steamer.

These two questions discriminated most clearly between the high and low mathematical visuality of the teachers. For this reason, these two questions were selected as the questions to be used by all four teachers with their classes during the second lesson observation.

The Classroom Observations

A qualitative approach was used in the analysis of the observational data from the classroom lessons. During each observation, case study notes were taken according to the headings available in the observation schedule. These notes were then used to write a description of each lesson. The lessons were then viewed together and themes that recurred were extracted under suitable headings. Only those headings for which behaviours were observed have been used in this summary. In addition, Presmeg combined five of these categories under the heading *Teaching without rules* : use and encouragement of intuition; use of pattern-seeking methods; delaying the use of symbolism; creating deliberate cognitive conflict; and accepting and demonstrating alternative methods. The summaries below use this heading to simplify the comparison of the lessons.

Topics of Lessons

In the first lesson, each teacher chose their own topic for the content of the lesson. Teacher L_1 shaped a lesson around line symmetry, Teacher L_2 used enlargement, Teacher H_1 worked in applications of relations and their graphs, and Teacher H_2 used statistics with the year 10 class and solutions of linear equations with the year 9 class.

For the second lesson the teachers were asked to incorporate questions 10 and 13 from the questionnaire in any way they liked. All the teachers chose to devote the whole of the second lesson to problem solving.

General Classroom Atmosphere

The classroom atmosphere with Teacher L_1,Teacher L_2 and Teacher H_1 was controlled but relaxed throughout both lessons. Each lesson progressed at a pace that was comfortable for the students - not at all hurried. There was a friendly liveliness between both Teacher L_1 and Teacher L_2 and their classes. The rapport between Teacher H_1 and her pupils is best described as a gentle warmth. The teacher expressed some affection for the class in her manner and her language and the class appeared to have reciprocal feelings towards the teacher.

During the first lesson, Teacher H_2 adopted a measured, controlled delivery with the year 9 class. This appeared necessary to keep one or two bouncy individuals in the classroom focused on the task in hand. In the second lesson the arranging of the students into groups became an opportunity for much off-task behaviour. The atmosphere in the room appeared to have deteriorated since the previous observation.

With her year 10 class, Teacher H_2 adopted a low-key but firm approach in the first lesson. The class had divided itself into several factions which had placed themselves within the room as far away from each other as possible. In the second lesson the atmosphere was more friendly. An invitation to reorganise and work in different groups caused no disruption. Indeed the class remained on task throughout the period and tackled the problems with some commitment.

Use of diagrams

Only one instance was noted during the first lessons of the drawing of an inessential diagram by the teacher. This occurred when Teacher H_2 started the solution of equations with the year 9 class. A suggestive diagram was introduced when the word 'equation' was written on the board and then underlined with a stylised triangular balance. Students were then asked to suggest why this had been done.

For the second set of lessons diagrams were much in evidence in all the classrooms. Their use was inessential as the problems could be solved without the use of diagrams. Yet the teachers were all encouraging of their use. It is particularly worth noting that when Teacher L_2 provided her class with an answer to question 13, it began with a diagrammatic representation of the information in the problem to assist in the solution of the problem. This was *not* the method she had employed in her response to the questionnaire.

Use of Gesture

In the first lesson with Teacher L_1, students were required to make shapes which had at least one line of symmetry. One student during the exercise asked if that meant 'you can fold it over in half, can't you?'. This visual interpretation was confirmed by the teacher who then gestured with her arms sweeping out an arc until her hands met in the middle. Talk by the students of 'reflection' and 'symmetry' in two separate incidents evoked from the teacher a similar hand waving response. In the second lesson she frequently used gestures while explaining aspects of the questions to individual students. On one occasion she emphatically cupped her hands firstly to the right and then to the left to help illustrate the content of the two pans on the balances. Another question introduced into the lesson involved the use of an 'old fashioned toaster' in which two slices of bread have one side toasted first. They are then removed, turned over and the other side is toasted. While explaining the action of the 'old fashioned toaster' she seemed to let her hands almost represent pieces of toast as her held them face down to indicate that one side only was toasted first and then she flipped her hands in a half-circle to illustrate the toast then being turned over.

Teacher H_1 used gesture in several forms. It was a mixture of the inessential 'dramatic' move - as in opening the arms slightly and hunching the shoulders when asking of a student 'how far below the surface?' - and some spatially significant gestures - as when pointing down when saying 'down'. Her hand would sometimes trace in the air when describing a linear relationship. Individual encounters with pupils at their desks were often accompanied with gestures. In the second lesson, while explaining the use of balances to some students, the teacher would interlock her hands and sway her elbows up and down to indicate the up-and-down motion of a balancing arm with the pivot being at the interlocking hands. While talking of a weight being put on to a balance, the teacher would motion her hand downwards as if she were dropping something.

Neither Teacher L_2 not Teacher H_2 made significant use of gesture in their lessons.

Use of Concrete Materials

Only Teacher L_1 required students to use concrete materials in the first lesson. The main activity of the lesson required the students to cut out and use three shapes - a square, two squares joined, and four squares joined in the shape of a T - to make as many shapes as possible which had at least one line of symmetry. She also encouraged the use of concrete materials to help solve the problems in the second lesson by providing counters and using such statements as 'Yes, the counters are there, you can use them', or 'Just come and grab a handful - how ever many you think you will use.' Students displayed some dexterity in using and manufacturing materials to assist in the solving of problems.

The only other incident involving the use of concrete materials occurred, almost by accident, in the second lesson with Teacher H_1. She distributed giant jellybeans, four per group, as an encouragement to the students to engage in the tasks. Some groups used the jellybeans as concrete materials to help them calculate the answer to question 10.

Teaching without rules

All the teachers encouraged students to use their intuition. This was particularly evident during the second lesson for each teacher where students provided the answers for all the problems with a minimum of assistance from the teachers. As an example, when one student asked Teacher L_1 'How many counters do we need?' to solve one of the problems, she replied 'That's up to you to work it out'. When another student wanted to know if they had the right answer, the teacher refrained from giving it and reflected the problem back to the student with the question 'Does it work? Does it fit?' The general methodological principles of problem solving were frequently stressed, with lists of strategies being written on the board as students used them. Alternative methods were encouraged by all teachers with such phrases as 'Did anyone do it any other way?' or 'Is there more than one answer?' In this respect the efforts of Teacher L_2 are worth a comment. For her 'warm up' question, three solutions provided by the students were written on the board. All were done by trial and error by the students. It is worth noting that Teacher L_2 stayed with the students' attempts to solve the problem by this method for some time, thereby delaying the use of symbolism before, almost as a last resort, prompting the students with the remark: 'We've been trying trial and error. It hasn't totally worked, has it? Can we use algebra?'

Similar instances occurred for all teachers during the first lesson. Teacher H_1 used a problem involving the number of chirps a cricket makes and the temperature at the time. She was not only aware of many methods of solution to the problem, but expected the students to supply as many alternatives as they could. When some students had difficulty with using a formula solution to the problem the teacher consoled them with the observation that 'you can get into a bit of a mess of algebra' trying to use the method and steered them into the use of pattern-seeking methods. She was not bound to formula methods in solving the problems.

Use of Humour

With Teacher L_1 obvious incidences of humour took the form of playful banter between the teacher and students. The teacher smiled freely while helping students on a one-to-one basis while walking around the classroom. This pattern continued through both lessons.

There was a friendly, low-key rapport between the Teacher L_2 and her pupils. While there is a mildly mocking tone to the humour, it was well intended and well received by the pupils. Friendly teasing both between students and between teacher and students was a feature during the lessons. Even the teacher's admonition to an inattentive student towards the end of the second lesson, with a quiet aside of 'Are you going to join us?' was done in good fun.

Teacher H_1 generated humour by suggesting the unexpected, particularly in the first lesson. An example of the relationship between the number of chirps the cricket makes and the temperature at the time was reworked by the teacher in her suggestion that the data may have come from a science experiment in which a Bunsen burner was placed under the cricket to increase the temperature so that the scientist could note any changes in the number of chirps.

For Teacher H_2 the occurrence of humour was subtle during the first lesson with year 9. During the second lesson humour was not particularly evident. In the year 10 class this pattern was reversed.

Use of Pupil Effort

All the teachers made significant use of pupil efforts in the structure of both of their lessons. The efforts of Teacher L_1 and Teacher L_2 illustrate a typical pattern. In the first lesson, Teacher L_1 used pupil contributions to develop an understanding of line symmetry. On being asked for a meaning of symmetry, one student started an explanation that involved some hand waving and the teacher instructed the student to come up to the board to draw a figure if that was going to help her explanation. For

Teacher L_2 pupil contributions very much determined the pace, content and direction of the first lesson. Responses to the request to 'think of three facts about enlargement you could tell someone who didn't know anything about it' were written on the board using the pupils' words. The teacher was very interested in having the pupils find out for themselves the consequences of varying scale factors and centres of enlargement. At each stage responses from the floor were encouraged to describe the consequences of each of these manoeuvres. Pupil involvement was also evident in the assessment of their efforts. Both peer assessment and self assessment were used.

In the second lesson all teachers used solutions provided by the students, often asking students to come to the board to explain their reasoning.

Use of 'Real World' Examples

In all the second lessons observed, virtually all of the problems that were given to the students were set in a real world context. For the first set of lessons, the different topics chosen made it less likely that this would occur. As it turned out only Teacher L_1 made no use of real world examples.

Discussion and Conclusions

No attempt is made here to quantify the teaching visuality of the participants in the fashion of Presmeg (1985). Instead trends are identified and the possible reasons for them discussed. At this point several of the episodes in the interviews with the teachers become useful in explaining the motivation behind some of the actions of the teachers.

There were several aspects of teaching visuality that the four teachers appeared to have in common. All the teachers, regardless of their mathematical visuality, were supportive of visual interpretations to the problems in the second lesson, encouraging students to draw diagrams. They all made much use of pupil effort during the lessons, often encouraging them to use their intuition and accepting, almost expecting, several methods of solution to problems.

This common behaviour appeared to stem from a reflective approach that they had with respect to their teaching. In support of a visual method, Teacher L_2 noted that her experience led her to believe that students didn't like an algebraic method, so in the class she maintained that she 'went from what the kids did. . .I tended to try and picture it so they can see it more clearly than they can see it when it's in algebra.' On the other hand, Teacher H_2 was able to reflect on what worked for her and use it in her classroom: 'A diagram to me helps to summarise and make sure that I understand the problem and because I find it easy that way I encourage them to summarise it that way.'

This reflective approach and an exposure to alternative methods in teacher training courses seems to account for the consistent use of concrete material by Teacher L_1. In both lessons she planned for students to be engaged in some physical activity that used materials as an aid to the students' understanding of the lesson. When asked in the interview why she encouraged students to use manipulatives to help solve the problems, Teacher L_1 replied: 'Because I think it's a good problem solving strategy and I think it's probably something that I think's quite powerful but . . . it's only something that I've really been exposed to in recent times through courses and things like that. People have sort of given you a problem and shown how you can solve it using manipulatives and I think, yeah, it's so powerful ...I've missed out on that in earlier times . . .'

While there was much that was common in the teaching visuality of the teachers, there were differences. These did not appear to relate to the mathematical visuality of the teachers. It had already been noted that Teacher L_1, the lowest scoring mathematical visualiser, was the principal user of concrete materials. Additionally, the use of gesture was as likely to be observed with teachers of high mathematical visuality as it was with the teachers of low mathematical visuality. Teacher L_1 and Teacher H_1 were the most consistent in their use of gesture in the classroom, displaying some form of gesturing in both lessons. Both Teacher L_2 and Teacher H_2 were relatively reserved in their use of gesture.

The existence of a weak statistical relationship that Presmeg (1985) found in her sample appears unlikely with this sample of teachers. All four teachers displayed common characteristics of teaching visuality, while aspects that were not common were as likely to be observed with the teacher of low mathematical visuality as with the teacher of high visuality. While Presmeg (1985) observed that in her case study sample of thirteen teachers 'no teacher of low mathematical visuality was found to be a visual teacher' (p.269), in this research Teacher L_1 appears to fit the category of a teacher with low mathematical visuality who has high teaching visuality. With respect to the Problem Solving Questionnaire she confirmed her low mathematical visuality when she explained her solution strategy as: 'Most of the time my initial reaction is to try and go to an algebraic expression . . . you see, I've got my higher level algebraic skills. . .' In her teaching however she does much to encourage the use of visual aspects, being the most consistent of the four teachers in the use of concrete materials, while also employing spatially significant gestures in her communication with students. Diagrammatic approaches to solutions are accepted and encouraged even if they are not the teacher's personally preferred strategy. Her teaching is not 'rule bound' as she accepts alternative methods from students and encourages them to use their intuition.

Of interest is the attitude towards the use of visual methods that teachers displayed in the interviews. Teacher H_1 appeared particularly to see the drawing of a diagram as a developmental stage that people grow through 'like when you are starting with playing with manipulatives and then you can abstract it to drawings of them.' When noting that she still approached her problems from a diagram rather than immediately from an algebraic approach she responded: 'Maybe I've got another level I can go.' Later, in explaining her approach to the solution of a problem she said: 'I've got a method: my method is draw a diagram, put the numbers on the diagram, find the equation and solve it. That's my method. . . Obviously some of your higher-up people are missing the 'draw a diagram' stage.' The implication from this teacher with high mathematical visuality and high teaching visuality appears to be that capable mathematicians don't need diagrams, an attitude that Dreyfus (1994) contends may lead to a reluctance in students to continue to use visual arguments. Teacher attitude towards the use of visual methods in mathematics is a possible field of further research.

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Flexible/inflexible: Clare and Mandy's story

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Research has shown that mental computation is a valid computational method which contributes to mathematical thinking as a whole (e.g., Sowder, 1990). This paper reports on a pilot study of young children's understanding of mental computation, and compares the mental architecture of two mental computers, one flexible and one inflexible. Further questions which have been raised as a result of this pilot study will also be discussed.

Recent research has suggested that mental computation should play a more prominent role in number strands of mathematics curricula (e.g., Cobb & Merkel, 1989; McIntosh, 1996; Reys & Barger, 1994; Sowder, 1990; Willis, 1990). Reasons for its inclusion are: mental computation enables children to learn how numbers work, make decisions about procedures, and create strategies (e.g., Reys, 1985; Sowder, 1990); mental computation promotes greater understanding of the structure of number and its properties (Reys, 1984); and it can be used as a "*vehicle* for promoting thinking, conjecturing, and generalizing based on conceptual understanding" (Reys & Barger, 1994, p. 31). In effect, mental computation promotes number sense (National Council of Teachers of Mathematics, 1989; Sowder, 1990). Further, mental computation has an utilitarian purpose (Clarke & Kelly, 1989). In fact, Willis (1992) suggested that mental computation should be the main form of computation, with written computation to serve as memory support.

Researchers (Kamii, Lewis, & Jones, 1991; Reys, Reys, Nohda, & Emori, 1995) have recommended that children should be free to formulate their own mental strategies, as understanding of algorithms is improved if children construct strategies in line with their own natural ways of thinking. McIntosh (1996) supported this and suggested that teaching mental strategies the same as formal pen and paper strategies have been taught in the past is not the solution to the present lack of attention given to mental computation.

Other researchers (Cooper, Heirdsfield, & Irons, 1996a, 1996b; Heirdsfield & Cooper, 1996) have reported the effects of pen and paper instruction on children's spontaneous mental strategies. Before instruction, children exhibited a variety of efficient strategies; whereas, after instruction, children tended to employ a mental strategy which appeared to reflect the pen and paper algorithm. The researchers also argued for a hierarchy of strategies, with *separation* strategies being least efficient, then *aggregation* and finally *wholistic* being the most efficient (Heirdsfield & Cooper, 1997). From this research, questions arise as to why some students are more accurate and flexible with more efficient strategies than others, and how their expertise relates to other number topics.

Connections have been drawn between mental computation and other factors, including number sense (McIntosh, 1996; McIntosh, Reys, & Reys, 1992; Reys, 1984; Sowder, 1990, 1992, 1994; Trafton, 1992), numeration and place value (McIntosh, 1996; Reys, 1985; Sowder, 1992), computational estimation (Heirdsfield, 1996; Reys, Bestgen, Rybolt, & Wyatt, 1982; Sowder & Wheeler, 1989), and number fact knowledge (Hope & Sherrill, 1987; Sowder & Wheeler, 1989). The results of year 4 children's mental computation, computational estimation, and number fact knowledge (Heirdsfield, 1996) indicated that children who were accurate and flexible in mental computation possessed advanced number fact skills (i.e., they were able to access basic facts using recall, or were able to employ advanced derived facts strategies). Further, these children were also proficient in computational estimation.

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Hope (1987), Hope and Sherrill (1987), Reys (1985), and Sowder (1994) have identified characteristics of proficient mental computers. Skilled mental computers use a variety of strategies in different situations (depending on numbers and context), because they are disposed to making sense of mathematics (Sowder, 1994). Therefore, they must be aware of a variety of strategies and have the confidence to use them. There is also evidence of reflection and regulation. Hope (1987) and Dowker (1990) reported children and adults choosing strategies based on their knowledge of number and operations, and choosing appropriate strategies to deal with the problems.

The reasons that some children are unable to use better strategies than the pen and paper algorithms in different situations, vary. The study of good mental computers may go beyond cognition and metacognition, to affects and beliefs (Sowder, 1994).

In summary, research on mental computation and number has proposed connections among mental computation and number sense, particularly number facts, computational estimation, numeration, and properties of number and operation; social and affective issues including attributions, self efficacy, and social context (e.g., classroom and home); and metacognitive processes.

This paper reports on two children's responses in a research project designed to address issues such as: Why are some children better mental computers than others? What are some contributing factors?

Method

Subjects

Two children, Clare and Mandy, were selected from a population of 16 year 3 children from one classroom in an inner city Brisbane school, as a result of testing for accuracy and flexibility in mental computation. Clare was accurate and flexible, while Mandy was accurate, but inflexible, using a single strategy consistently.

Instruments

Mandy and Clare were given a series of indepth interviews. Tasks were given for numeration, operations, mental computation, number fact knowledge, computational estimation. Questions were asked self efficacy, beliefs, and metacognition. The children were also required to complete the Student Preference Survey (SPS) (McIntosh, 1996). In order to get a feel for classroom and home contexts, the children were encouraged to indulge in general conversation, and the teacher was invited to respond to initial and general inferences.

Procedure

Mandy and Clare were withdrawn from their classroom on a one to one basis, and interviewed in a quiet room. The interviews were videotaped, and each interview session lasted for no more than 30 minutes at a time. Because of the variety of aspects covered, each child received three interview sessions.

Analysis

Mental computation, computational estimation, and number fact responses were analysed for strategy choice, flexibility, accuracy, understanding of number and numeration, and metacognition. Number and operations tasks were analysed for understanding of associativity and inverses, and relationships (e.g., 69-43=26, ∴69-44=25). Analysis of students' responses to numeration tasks were based on Ross's five levels (1986). Analysis across individuals' interviews was undertaken to investigate connections with mental computation, for instance, whether understanding of noncanonical partitioning of numbers was used for mental computation.

Results

Clare's story

The strategies that Clare employed for mental computation revealed numeration understanding, knowledge of the effects of operations, and facility with number facts. These findings were supported by evidence from interviews which specifically addressed these aspects.

Clare appeared confident in computation, and stated she liked mathematics, because she found it easy and is therefore good at it, that is, she attributed her success to ability. When asked how she knew she was correct, Clare replied, "I just think I'm right. I am usually right." Thus, Clare attributed success to internal, stable factors. Clare possibly attributed failure (if any) to "very foolish mistakes", an unstable internal factor (lack of immediate effort) (Weiner, 1985).

Further, Clare *needed* to achieve, and only felt confident attempting questions if she believed she *could* succeed. After being unsuccessful at calculating 265-99 in the selection interview, she went home and asked her father how to calculate such examples. She was happy to attempt a similar question (234-99) in the next interview, because she now knew how to calculate it. However, she did not know why it worked ("That's what Dad told me to do."). Her confidence was also exhibited by her stating that her subtraction method (of levelling) "annoys Miss A...", but she was determined to continue to use it. However, she *did* realise that method was too complex for 3 digit examples. In the follow up mental computation interviews, when asked to think of another solution method, she saw no reason to think of a different method, except for the fun of it (appease the interviewer?). However, once she reasoned that some of her second methods were better than her first methods, she thought it was quite a good idea to indulge me. At times, hints had to be given, e.g., "what is 19 near?". Other times, no hints were given, for example, after solving 80-49 by 80-40= 40, take another 10, 10-9=1, 31, Clare then turned 49 into 50 and proceeded, 80-50+1. Clare's confidence in her ability and her reluctance (at first) to try a different method was reflected across all her classroom work. She had a strong preference for her own methods. Her later acceptance of alternative methods and even preference for these came as a shock to her teacher ("out of character for Clare"). It is suggested that she had nothing to prove to the interviewer by remaining adamant about the suitability or otherwise of her strategies. In discussions with her teacher, it was suggested that Clare was expected to succeed, but also to enjoy school and learning. Her parents did not consider it essential for Clare to be a high achiever, although she was.

Clare stated that she believed she would be able to solve the mental computation questions, and she could. This was reflected in her responses to the Student Preference Survey (SPS) (McIntosh, 1996). For all Clare's confidence, when asked to solve subtraction problems, she replied, "I don't particularly want to. I don't like doing take away in my head." This was despite the fact that she could. This attitude towards subtraction was reflected in her response in the SPS, where she responded positively to calculating mentally for only the simple subtraction problems.

Clare's ability to manipulate operations was not consistent. In the number and operations interviews, she was not always sure whether to add or subtract one when taking away one more or one less (e.g., 73-45=28, 74-46=?). Thus, although her father had shown her a method based on this principle, there was not solid understanding. The similar concept for addition (e.g., 234+99=333, because 234+100=334, and take 1, so 333), however, posed no problem for her.

Clare's number facts were fast and accurate. In the number facts test, she used recall and *derived facts strategies* (DFS), predominantly. One of the DFS, a levelling strategy (e.g., 17-9: take 7 out of 9 and out of 17, so 10-2=8), was similar to a subtraction strategy used in the mental computation interviews (e.g., 52-19: take 2 out 9 = 7, so 10-7=3, 40-10=30, 33). Clare stated she had not been taught this strategy, but had worked it out for herself. This offers support for children who employ DFS understand relationships between numbers, and are able to use this understanding of number properties in mental computation. Her agility with number facts was an advantage in the mental computation interviews, as working memory was available for efficiently solving more complex problems. Immediately before the indepth mental computation interviews, the children were presented with the number facts test, in which Clare calculated 15-8 by levelling. She was then able to recall this fact for the same question in the mental computation interview, that is, she had learnt from the experience.

The children's teacher was amazed that Clare had formulated the levelling strategy. She stated that she had used similar strategies when modelling addition tasks, but did not expect children to be able to use them either for addition or subtraction. It appeared that Clare had the capacity to build up a rich, interconnected network of knowledge, and access this knowledge, readily.

Clare defined estimation as a "type of guessing", a definition in common with other children in her class. She stated that she only estimated when given classroom estimation tasks that were treated as *rounding* only. However, Clare did not employ *rounding* in the interview. Rather, she used other strategies more appropriate to the situations, for instance, *truncation* and *wholistic*. Because Clare's mental computation was so good, she attempted to calculate accurately. This type of response has been reported elsewhere for proficient mental calculators (Heirdsfield, 1996; LeFevre, Greenham, & Waheed, 1993). It was decided to present Clare with additional 3 digit estimation questions that were too difficult for exact calculation. Clare's responses reflected an understanding of magnitude of number, place value, and the effect of operations. One example of a successfully completed task was: "Your friend has $152 and spends $144 on a cassette recorder. You have $156 and spend $142 on another cassette recorder. Who has more money left?" Response: "I do, because I started with more and spent less."

The numeration tasks revealed Clare's understanding of both canonical and noncanonical representations of number (Ross, 1986). She was particularly flexible with different representations of such numbers as 560 (5x100 + 6x10 + 0x1; 56x10 + 0x1; 500x1 + 6x10; 55x10 + 10x1; 5x100 + 3x10 + 30x1) and 209 (2x100 + 0x10 + 9x1; 20x10 + 9x1; 209x1; 19x10 + 19x1). Although MAB (Multibase Arithmetic Blocks) were available, Clare did not use them. However, there were times the interviewer had to encourage her to elicit more combinations, and she appeared to delight in the challenge.

Sowder (1994) suggested "(f)lexibility is but a manifestation of self-monitoring". However, Clare admitted that she generally employed the first method "that pops into my head"; therefore, there were times she chose an arguably less efficient mental strategy. Nevertheless, later in the interviews, such statements as, "why didn't I think of that in the first place?" indicated she began to consider strategy choice more carefully. During the course of interviews, Clare verbalised her thoughts, for instance, "No, that can't be right", "I'm lost now", "I'm usually right", "This one's difficult", "This one's easier", "I like this one, because it has something to do with 99", and "Seventy-five is easier to use than 76, so I'll use 75". These statements revealed the existence of metacognitive processes and beliefs. Clare had access to a variety of strategies, but stated that she rarely consciously chose the most appropriate strategy for the number context. However, when encouraged to think of other strategies, she made judgements regarding the suitability of the strategies. Clare was confident in experimenting with different strategies. She seemed to disregard what was taught in the classroom, rarely using the taught algorithm to solve the problems mentally. In fact, Clare revealed that she often used her levelling strategy for subtraction to solve written exercises.

Mandy's story

The other child, reported here, was Mandy who was also accurate in mental computation, but consistently employed a mental image of the pen and paper algorithm (i.e., she imagined the numbers one under the other, as if using pen and paper. The individual numbers were first separated into place values and then operated on by moving right to left). Mandy attributed her success to effort and practice. To check her work, she said she would check her answers by working through the examples the same way, and then wait for feedback from the teacher. Thus, she attributed success to internal, stable factors; however, although Mandy appeared to attribute failure also to an internal factor, she relied on teacher feedback (external factor) (Weiner, 1985).

Mandy stated she would be able to complete the tasks mentally, and could, although she was not particularly confident with subtraction. On the other hand, Mandy had to be deliberately encouraged to use strategies other than "calculating operations" (the term she used for pen and paper strategies). Mandy was successful at completing such tasks as: 257-100=157, so what does 257-99=? (after a good deal of thought), but she stated that she still preferred "using operations". The similar concept for addition (e.g., 234+99=333, because 234+100=334, and take 1, so 333) posed no problem for her. However, Mandy could not and would not apply the concept for the mental computation tasks. In discussions with Mandy's teacher, it was revealed that Mandy had high expectations for accuracy and speed when completing tasks. This could explain her using the same "automatic" procedure for solutions, and maintaining confidence in this procedure. Also, expectations from home could be summarised as "succeed at any cost". Mandy was expected to work hard, do as the teacher told her, and succeed (i.e., score well in tests).

Mandy had employed a mental image of the pen and paper algorithm in the selection interviews, and stated several times that she preferred that method and found it easier, as she was "used to it". Through prompting, Mandy developed a left to right aggregation strategy (e.g., 63-29: 63-20=53, 43; 43-9=34), and started to use it later in the interviews, because she said she wanted to practise the new way which may be easier for mental calculations; although she stated she still preferred the "old way". In fact, when employing alternative strategies, she still imagined the numbers one under the other, as though setting the examples out on paper.

Mandy used counting and some recall in both the number facts test and the mental computation interviews. She was also slower than Clare. This was despite Mandy's expectation for speed and accuracy. However, she did not have problems with memory overload when computing mentally. It is interesting to note that, in the mental computation interview, she recalculated the answers to the number facts (using similar strategies to those employed in the number facts test), although she had already done so, not 5 minutes before. She had not remembered the number facts solutions, nor had she made any links between what she had done previously and the task at hand.

Mandy was generally unsuccessful at the estimation tasks, and she could only

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relate estimation to measurement.

In the numeration tasks, Mandy was slow at representing numbers in different ways. She had to be prompted with such questions as, "What about some ones?", and needed the support of MAB for many examples. Even with MAB, she did not show a solid understanding of grouping and regrouping, as she constantly checked and recounted her manipulations. An alternative explanation for her constantly recounting blocks could be that her need for absolute certainty overshadowed her understanding of number. However, it appears curious that she would have to count and recount tens to ones, if she truly understood regrouping.

Concluding comments and further questions

Although both children possessed a high degree of accuracy in mental computation, Clare who exhibited flexibility contrasted in many ways to Mandy who employed a mental image of the pen and paper algorithm. Some of these differences are now summarised (Table 1) and further issues discussed.

Table 1

Comparison of Clare and Mandy

Aspect for comparison	Clare	Mandy
Accuracy	100% accuracy	100% accuracy
Flexibility	Used full hierarchy of strategies.	Used mental image of pen and paper algorithm.
Access to alternatives	Accessed valid and efficient strategies.	Had difficulty, but with scaffolding, developed an aggregation strategy.
Attribution	Attributed success to ability	Attributed success to effort.
Metacognition	Some evidence of conscious choices, reflection and evaluation.	Little evidence, except when conscious decision made to practise new mental strategy.
Self efficacy	Confident in her ability to solve tasks, and she could; although found subtraction more difficult. Confident in her own strategies.	Confident in her ability to solve tasks, and she could; although found subtraction more difficult. Confident in teacher taught strategies.
Numeration	Canonical and non canonical understanding, without concrete aids. Flexible with different representations.	Poor understanding, relied on concrete representations.
Number and operation	Not consistent, better understanding with addition.	Showed some understanding in number and operation interview, but did not apply in mental computation tasks.
Computational estimation	Proficient. Exhibited good number sense.	Poor.
Number facts	Accuracy and speed evident. Employed DFS and recall.	Accuracy, but less speed. Employed recall and count.

It is interesting to note that the aspects of number and operation, and numeration which were investigated in relation to mental computation, are not taught in school. Mandy would not have these understandings, because she would not have been directed to do so by the teacher. On the other hand, Clare had access to a variety of strategies which required various understanding. What she had been taught in the classroom did not prevent her from exploring other possibilities. This was also evident in the numeration interview, where Clare exhibited understanding, beyond what would be presented in a year

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3 classroom. It is theorised that a more complete understanding of numeration is necessary for accurate and flexible mental computation, but little or no understanding may be necessary for the use of the pen and paper algorithm.

Clare appeared to have a well connected network of knowledge of number, and was able to apply this knowledge across the different tasks, for example, number facts were used in the mental computation interviews, numeration understanding (multiplicative nature of number system, noncanonical number representations) was applied in mental computation, computational estimation indicated a feel for number which was also reflected in mental computation, number and operation (although not well advanced) was at times reflected in mental computation. None of this evident in Mandy.

Because Clare possessed well structured and connected knowledge, her long term memory was more easily accessed for strategic knowledge, resulting in less load on working memory (the mechanism responsible for both temporary storage and processing of information). Mental computation requires concurrent processing and storage of information, that is, it is cognitively demanding. Mandy avoided this cognitive demand problem by relying on an automatic strategy. Further, she appeared to have well developed temporary storage. Mandy could remember partial sums and differences, even though she worked consistently right to left. However, she admitted this method was taxing on her memory. Clare appeared to be superior in processing of information. She may not have made conscious decisions regarding strategies, but there is evidence of something more than habitual patterns. The model hypothesised by Baddeley (1986) and Logie (1995) suggested that the central executive plays a role in reasoning, and is involved in the allocation of attention and automatic retrieval. It is attention, though that is involved in higher level cognitive tasks. The central executive interplays with the knowledge base, and it is this interplay that results in learning and acquisition of new knowledge. This certainly was evident with Clare. She attended to the task, accessed her knowledge base for strategies (and at times facts), processed the information, and much of this information was then stored in long term memory for future use, for instance, number fact recall or strategy use, when applicable. In fact, Clare would have overridden any habitual response (e.g., pen and paper algorithm) when it was necessary to employ more efficient strategies.

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We really put our minds to it: Cognitive engagement in the mathematics classroom

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This paper reports the analysis of videotape and interview data from four year 8 mathematics lessons from the perspective of student cognitive engagement. The study attempted to contribute to our understanding of cognitive engagement by locating empirical evidence for its occurrence within the classroom. On the basis of the data we have examined, cognitive engagement can be consistently recognised by specific linguistic and behavioural indicators and appears to be promoted by particular aspects of the classroom situation, the task, and the individual.

Introduction

This paper comprises an analysis of videotape and interview data from four year 8 mathematics lessons from the perspective of student engagement. Our focus on the nature, role and significance of engagement stems from a consistent valuing of engagement within the literature on learning that has not been accompanied, in our view, by a satisfactory empirical demonstration of the role played by engagement in the learning process. The term "engagement" usually refers to the extent to which a student is involved with the content, or some aspect of the content of a learning activity, in a manner requiring deliberate and considered thought on the part of the student (Ainley, 1998). The "quality" or "level" of this involvement is generally believed to have a profound effect on learning outcomes, in that students who "really put their minds to it" are much more likely to learn successfully than students whose engagement with the subject matter is low.

Pintrich (1989) supported a "multivariate contextual model of student learning" and stressed the importance of "exploring the pattern of relationships among the various cognitive, metacognitive, and motivational components", particularly emphasising "the dynamic interplay between motivation and cognition" (Pintrich, 1989, p. 153). It is within this interplay that we suggest "engagement" is located. In his discussion of theories of self-regulated learning Zimmerman (1990) also stressed the interdependence of learning and motivation. Self-regulated learning can be identified with one aspect of engagement, in which the learner's engagement is a matter of personal inclination and volitional control. "High perceived control" was a postulated (and confirmed) correlate of student engagement in the work of Skinner, Wellborn and Connell (1990).

Ainley (1993) has associated student beliefs and goals with different "styles of engagement" and argued for a multidimensional character of student engagement. In Ainley's analysis, styles of engagement were linked to student achievement. Engagement itself was associated by Ainley with "students' general orientation to learning" and with "student construction of the task" with the proviso that any relationship between engagement and achievement may also be a function of features of the specific learning domain (Ainley, 1993, p. 404). These associations suggest possible indicators by which engagement might be recognised within classroom videotape and interview data such as that collected in this study.

Our research into engagement has focused on what we have called "cognitive engagement". In focussing our attention on cognition, we do not intend to ignore the motivational and metacognitive aspects of engagement. In fact, we would argue that the term "cognitive engagement" has been most appropriately employed in cases where there is evidence of significant metacognitive activity, as in the case of self-regulated learning. The term "cognitive engagement" appears to have sufficient currency in the literature for some authors to see no need to define the construct, either theoretically or operationally. For example, Pintrich and De Groot (1990) make repeated reference to cognitive

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For example, Pintrich and De Groot (1990) make repeated reference to cognitive engagement, in a paper dealing with motivation and self-regulated learning, without definition. Nolen (1995) similarly employs the construct in relation to self-efficacy, but without definition. Meece, Blumenfeld and Hoyle (1988) have also used the term "cognitive engagement" in associating different "goal orientations" with student engagement patterns, and identified cognitive engagement with "students' reported use of metacognitive and self-regulation strategies" (p. 515). We have avoided sole reliance on student self-reports of cognitive behaviour by seeking evidence of cognitive engagement from two main sources: videotapes of classroom activity and student interviews. From this evidence, we hope to construct a viable model of classroom learning in which student cognitive engagement is usefully situated.

Many early studies of student engagement appear to have operationalised the construct in terms of time-on-task (for example, Peterson & Fennema, 1985; Hart, 1989), although the usefulness of this measure has been challenged over a significant period of time (for example, Peterson & Swing, 1982). We argue that cognitive engagement is qualitatively different from time-on-task or student participation. The fine-grained nature of our data collection has provided us with examples of occasions when a high level of student "percentage time-on-task" was not accompanied by a high level of student cognitive engagement, as we would define the construct. Ball (1990) employs the term "meaningful engagement", which suggests that more is intended than mere participation. However, in Ball's usage, meaningful engagement appears to be more closely related to what students do than to what they think. Similarly, Fullarton (1996) distinguished students' cognitive engagement, behavioural engagement, and emotional engagement, but did so in terms more aligned with "student involvement" than what we would term "student engagement."

This paper is primarily the report of an investigation of cognitive engagement. However, we assert that the behavioural and affective correlates of cognitive engagement need to be considered if the construct is to be meaningfully understood. Thus we have found it helpful to think of engagement in terms of three dimensions: cognition (what are students thinking?), behaviour (what are they doing?) and affect (what are they feeling?) (Baird, 1998). Once engagement and its relationship to classroom learning is better understood, we may then be able to use strategies to enhance engagement in the classroom, and possibly the consequent quality of student learning.

Operationalising engagement

We define cognitive engagement as the deliberate task-specific thinking that a student undertakes while participating in a classroom activity. But what forms does this thinking take? The purpose of this analysis was to identify instances of such activity using classroom videotape data as a primary source. We have focussed on linguistic indicators of cognition as our primary data source, such as questions, comments and explanations. The reflective comments students made in interview provided a second source of data.

There are many behavioural correlates of cognitive engagement. These include for example active participation, persistence, effort, and non-verbal indicators of attending such as eye contact, body orientation and certain gestures. Emotive responses such as expressions of enthusiasm or satisfaction can also be associated with cognitive engagement. Caution has to be taken however in making inferences solely on the basis of observed behaviour. For example, a student in a science class in the present study, who observed her behaviour in video-stimulated recall, spontaneously commented that her own behaviours could be easily misinterpreted by the teacher as lack of attention, when in fact she was giving serious consideration to what the teacher was saying.

Influences on cognitive engagement

Cognitive engagement operates within a complex web of influences, and the present study intends to take these into account. These influences can be broadly conceptualised as individual, task-related and classroom-related. Individual students bring to the learning situation many characteristics which influence their engagement, such as prior knowledge, cognitive and metacognitive skills, perceptions, expectations, interests, needs, values and goals. Self-regulated learners, for instance, (Zimmerman, 1990) actively seek out opportunities to learn and systematically use metacognitive, motivational and behavioural strategies to achieve desired learning outcomes. Within any classroom, a 'classroom culture' emerges creating conditions that either constrain or promote particular teaching and learning strategies and particular styles of interaction with the teacher and other students (Edwards & Mercer, 1987). Current analyses of data collected in the Classroom Learning Project suggest that teaching style and the nature of peer interactions have a powerful influence on student cognition and metacognition (Lerman,1998; Holton & Thomas, 1998; Baird, 1998). Classroom learning tasks and activities provide the vehicle for a student's cognitive engagement. Recent research into task characteristics such as complexity (Williams & Clarke, 1997), challenge (Csikszentmihalyi & Csikszentmihalyi, 1992), familiarity (Helme, 1994), intrinsic interest (Ainley, 1998), and personal meaningfulness (cf. Clarke, 1996) suggests a possible relationship between such characteristics and cognitive engagement.

Methodology

The current data base for the project consists of videotape records of 55 secondary maths and science lessons obtained using two cameras. One camera was directed at the teacher, while the other camera was focused on a group of about four students. The teacher was recorded through a radio microphone, and a single microphone was used to record the students. The two video images were mixed on-site to produce a composite picture in which the students occupy most of the screen with the teacher image superimposed in a corner of the screen. This combined image was recorded onto video-8 tape using a compact video recorder attached to a laptop computer. The researcher, seated at the rear of the classroom, was able to listen simultaneously to target students and the teacher and record field notes onto a word processing document. The field notes were "time-tagged" to corresponding events in the video record using *CVideo* software (Roschelle, 1992). The researcher was therefore able to document impressions of classroom episodes and learning events as well as to provide reference markers for the subsequent interviewing of students immediately after the lesson.

The videotape record was used in the interview to stimulate student recall of classroom events. The *CVideo* software enabled the researcher to locate within the field notes reference to actions of the student which seemed to be of significance either to the researcher or to the student. Having found a particular item in the word document, the software could be used to find very quickly the corresponding moment on the video record. This was then played back and discussed. Thus students were able to reconstruct their motivations, thoughts and actions, prompted by the video record of the classroom events. The audio record of the interview provided a third source of data. Interviews with teachers were conducted at a later time. Each teacher viewed the videotaped lesson and paused the tape whenever they wished to comment on events in the classroom which showed something important to them about teaching or learning. The audio record of the teacher's commentary thus provided a fourth source of data. Eight "integrated data sets" were generated from four mathematics and four science lessons. The four year eight mathematics classes (and associated teacher and student interview records) were analysed in this study for evidence of cognitive engagement. For more details of the origins, rationale and practicalities of data collection see Clarke (in press) and Clarke and Helme (1997).

Results and discussion

We identified a number of student behaviours in the videotaped lessons which we believe can be associated with cognitive engagement. These are listed in Table 1. The observed behaviours occurred in clusters which could be associated with distinct classroom situations. It is worth noting that the five situations in Table 1 can be divided into those situations in which the teacher is present and those in which the teacher is absent. The clustering of these behaviours and their consistent association with qualitatively different social situations provided retrospective support for their validity as indicators of the single phenomenon of cognitive engagement.

Table 1. *Indicators of cognitive engagement*

Classroom situation	Behaviour
Individuals working in parallel	Verbalising thinking; self-monitoring; concentration (resisting distractions and/or interruptions); gestures (interpreted as externalising thought processes); seeking information and feedback.
Collaborative small group activity	Questioning; completing peer utterances; exchanging ideas, directions, explanations, or information; justifying an argument; particular gestures.
Small group interactions with teacher	Answering teacher's questions; giving information; explaining procedures and reasoning; questions addressed to teacher; reflective self-questioning.
Whole class interactions with teacher	Asking and answering questions; making evaluative comments; contributing ideas; completing teacher utterances
Interactions between teacher and individual student	There were no episodes of sustained interaction in the data set currently available which could be subjected to analysis

From the interview records we were able to identify a number of additional possible indicators of cognitive engagement. These included student claims to have made a genuine attempt to learn something, resolve uncertainty or to have learned something in the lesson; student discussion, communication or recall of details of lesson content; and the claim to have been engaged during the lesson (e.g. "we really put or minds to it"). An example from two of the classroom situations listed above will be discussed in detail below. Numbers in brackets refer to line numbers in transcript excerpts. Unidentified students are referred to as S1, S2 and so on.

Individuals working in parallel

K and L were working alongside each other, trying to calculate how many blood cells they have lost in their lifetime up until today:

1. T: I want you to tell me how many blood cells to the *day* you have lost.
2. K: [to L] Does that mean how many up to now?
[Overlapping talk between T and L for next three lines]
3. T: So you've got to multiply by the years, by the months—
4. L: And the months, and the days.
5. T: By the days. OK?
6. L: Well!
7. T: That's when you've finished that.
8. K: Seven hundred and thirty million a day, no, per year, times fourteen years. Shit.
9. S1: In your lifetime?
10. T: Yes, how many blood cells have you lost in a lifetime. If it's two million a day.
11. S2: I'm not dead yet. We've got to do so many months, so many days. Oh oh.
12. T: I'll go round and check the rest [T goes around room and checks work].
13. L: [counts on fingers, talks to self] May, June, July, August, September, October. That's six. And how many days have we had in October? [looks at diary] The nineteenth.
14. K: [working aloud] You times that by fourteen, equals one point oh two two to the power of ten. Oh yeah, I understand that. One oh two two. One, two, three,

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four, five, six, seven, eight, nine, ten. Would that be right? [looks towards L] How many days in a month approximately?

15. L: [working aloud] Times nineteen. [looks towards K]
16. K: [to L] Would you do approximately thirty days for the month? It would be thirty point five days. No it'd be twenty-nine point nine days or something.
17. L: Hang on. [goes back to own work] Times, times [inaudible].
18. K: Would it be this? Do you reckon, L___? Um, L___, can I borrow your calculator? L___.
19. L: Yeah [inaudible, working aloud].
20. K: [talking as she works, L does not appear to be listening] That equals one point oh, oh, oh, no, one point oh two two to the power of ten, make it—
21. L: [to herself] That's wrong.
22. K: [to herself]—ten billion, two hundred and twenty million. [Looks toward L] Ten billion, two hundred and twenty million. Is that right? [no response, L is bent over work] Don't worry.
23. L: I hate you Mrs B___! [possible emotive response related to frustration of challenging task]

Indicators of cognitive engagement: L thought aloud as she worked (13, 15,17, 19), and her intense concentration was evidenced by her resistance to K's interruptions (17, 19). She actively used available resources to help her resolve uncertainty, such as repeating the teacher's instructions in order to clarify the task requirements (4), and referring to her diary to work out the number of days (13). She made use of gesture to externalise her thought processes (13), and showed evidence of self-monitoring (21). The emotive dimension of her engagement was apparent in line 23, in which she expressed a certain amount of frustration about the task. The quality of K's engagement was different from that of L. Apart from one instance of self-monitoring (8), she tended to rely on L for basic information (14), to clarify the task requirements (2), and to give her feedback about her ideas (16), and her progress with the task (18, 22).

L's interview record contained a number of insights into the influence of task factors on the quality of her cognitive engagement. It suggested that task novelty, context and personal meaning acted to heighten L's participation and cognitive engagement. However the link between cognitive engagement and successful learning has yet to be made in a convincing and empirically-grounded fashion. Although the data available to us cannot confirm that cognitive engagement facilitated learning in this particular situation, both the video record and the teacher's interview clearly indicate that L was a highly successful student. As evidence of this, the teacher considered the problem above to be quite challenging, "It's *quite a tricky task*", and her opinion of L's competence was confirmed by the fact that she addressed L personally when she initially set the task. Pending further analysis, we suggest that L's consistently high cognitive engagement contributed significantly to her success and to the quality of learning suggested by the teacher's description of L as *"like a little mini teacher in the room"*.

Collaborative small group activity

F and M had a set of graphs from which to select the one that best fitted the v-t graph of a ball thrown into the air. They had also been asked to do a sketch of the graph. It was an extremely challenging task, because of the risk of confusing the trajectory of the ball with the graph of its velocity (which in their final selection the girls appear to have done).

1. M: [To F] Ready, what do we do with the ball thrown into the air?
2. F: This [points to material on table].
3. M: Which one?
4. F: I reckon [selects an alternative by tapping her page with pencil].
5. M: Uh uh, I don't reckon. It would go up—it wouldn't go up fast, and it would come down real fast, so.
6. F: But it doesn't come down real fast.

7. M: It does [nods head].
8. F: [shakes head]
9. M: It comes down faster than it goes up.
10. F: No. What happens, is it goes really fast [with pencil traces what appears to be trajectory of ball thrown into air] and then it slows down once it gets to the top, and comes — it comes up and then slows down [repeats first gesture], and stops when it turns around. And then it comes up again [pencil moving up] it goes down quite fast and then it slows down when it gets to the bottom [sketching second half of parabola] because it has to—
11. M: [looking at book] Yep, this one. Because it doesn't, but it doesn't, that's like it goes up and then [speaking slowly] kind of moves real [speaking slowly, gesture with hand], you can see how it. But that one just kind of goes like that.
12. F: But it doesn't go Oo-op [moves pencil upwards from desk in straight diagonal line] slow down suddenly, it sort of gradually slows. [another student briefly interrupts about another matter] I reckon—
13. M: D.
14. F: D. 'Cause it doesn't really just go uomp [moves ruler up from desk in steep curve with a jerk] and then slow down straightaway [moves ruler slowly in arc] and then—
15. M: All right, do D. [makes side comment about pen]
16. F: Then we have to draw it.

Indicators of cognitive engagement: F and M employed various resources to help them with the task. Both students used gestures to create a 'slow motion' version of the path of the ball and to help them work out how the velocity changed during its flight (10, 11, 12, 14). Gesture here seems also to have enabled them to create a 'shared representation' (Clarke, 1996) which they could both 'see' and modify until they reached agreement. M used her voice to enhance this process: as she described the changes in the ball's velocity, her speech slowed down and speeded up accordingly (11). The episode was primarily characterised by a "negotiative event" (Clarke and Helme, 1997), in which the students initially disagreed and used argument and explanation to eventually reach consensus (4-15). There is evidence of monitoring of task requirements in line 16, in which F reminded M that they had to draw the graph.

The data available to us does not enable us to demonstrate a connection between cognitive engagement and learning outcomes. We would argue though that the challenging nature of the task pushed these students to make full and creative use of their cognitive resources to help them make progress on what was for them a very very difficult problem.

We really put our minds to it: The case of H

H participated actively in a group discussion, prompted by the task: "Name some rates that affect your life". Her contribution of child maintenance rates was clearly about something quite personally meaningful.

I: So can you tell me what made it, what was good about it, what worked for you?
H Uh, I think, um, it was just that we all really put in together, we really put our minds to it, thought about it.

H's statement strongly suggests that cognitive engagement is intrinsically satisfying and motivating. This was the only occasion in over 100 interviews relating to 55 lessons in which any student made a spontaneous positive claim about their engagement. Both the classroom dialogue and the above interview excerpt suggested significant cognitive engagement on the part of Student H. Our conjecture is that it was the novelty of the task and its connectedness with H's personal experience that promoted the high level of cognitive engagement. The following interview excerpt appears to support this interpretation.

H: 'Cause we hadn't done anything like that before, in maths.
I: Uh huh.
H In maths, yeah.
I: Uh huh, so what at home do you know, what was that, what were you thinking about then?
H: Well, water rates, 'cause I remember my brother did an assignment on that, um, child maintenance rates because um, my father has to pay them for my brother and I, um.
I Right.
H: All, all the rates that we pay, bank rates, home loans, we just bought a new home and stuff like that. So yeah.

Conclusions

Cognitive engagement is valued by the education community and widely held to facilitate learning. Most studies which have investigated this link tend to rely on measures of cognitive engagement such as time-on-task and student self-reports. The former are unsatisfactory because of their superficiality and limited construct validity. The latter are limited by an individual's ability to accurately recall their actions and reconstruct their thought processes and motivations. The present study attempted to contribute to our understanding of cognitive engagement by locating empirical evidence for its occurrence within the classroom, and by exploring the relationship between forms of cognitive engagement and different classroom situations and activities.

On the basis of the data we have examined, we are prepared to assert that cognitive engagement, as we have defined it, is observable in classroom situations and can be consistently recognised by specific linguistic and behavioural indicators. In this paper we examined cognitive engagement in two distinct classroom situations, individuals working in parallel and collaborative small group activity, and observed that different patterns of cognitive engagement appeared to characterise each type of activity. Preliminary analysis of teacher-student interactions suggests significant differences in the patterns of cognitive engagement from those reported here. We have also found evidence to support the view that task characteristics influence cognitive engagement, as do individual-task interactive factors such as the personal meaning which the task holds for the student. The emotive aspects of engagement were also evident in our data. It appears to us that some students possess particular skills which act to support their cognitive engagement. These skills may be metacognitive in character. This issue will be the subject of further investigation.

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Maori mathematics education: The challenge of providing immersion programmes for preservice Maori primary teachers

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With the rapid growth of Maori language schooling in New Zealand in recent years, and the development of a Maori mathematics curriculum, there has been considerable demand for Maori primary teachers who are both fluent in Maori and competent in teaching mathematics. Trying to meet this demand has proved a real challenge. This paper outlines and discusses a number of factors faced by Maori mathematics teacher educators in preparing preservice Maori teachers for the primary classroom.

Introduction

In between 1973 - 1978 the number of native speakers of Maori in New Zealand had declined to approximately 70,000 (out of a Maori population of 300,000) (Benton, 1981 pp 15). So critical had the number become that Maori was deemed to be a dying language. At the same time, the level of education among Maori had also been declining since the early 1900s. Since language contributes significantly to identity, and hence hope for the future, several measures have been initiated by Maori to try to reverse the language decline and improve Maori educational achievement. These include the development of Kohanga Reo (language 'nests') for preschool Maori children, Kura Kaupapa (Maori immersion) schools, and a Maori mathematics curriculum (Ministry of Education, 1996). Already the Maori language renaissance has been successful in developing the self-esteem, self-worth and performance of children in these schools, but while the development of language fluency is admirable the development of sound mathematics concepts and processes among such children is highly debatable - as I explain further below.

The Maori mathematics curriculum, known as *Pangarau - I roto I Te Marautanga o Aotearoa*, is modelled closely on *Mathematics in the New Zealand Curriculum* (Ministry of education, 1992). However, the document is written in Maori and attempts to embrace Maori contexts and learning experiences that will suit Maori. This provides some hope, given McMurchy-Pilkington's (1997) finding that Maori children are capable of mathematical thinking and reasoning outside the formal European schooling context. This curriculum document is used by teachers in Kura Kaupapa Maori schools, and in Bilingual Units and immersion programmes within mainstream schools. Today, 13% of Maori primary school children attend such programmes, and the figure is growing.

With respect to mathematics, there is a real sense of urgency to improve the achievement of Maori children. The recent TIMMS Report (Garden,R. 1997) revealed that at the eight to nine-year-old level the performance of New Zealand children was in the bottom third of the international sample. It appears that Maori and Pacific Islands children in New Zealand have contributed disproportionately to this result. Gehrke, for instance, has calculated that if these children were excluded from the New Zealand sample then New Zealand's mean score would have risen to about halfway in the table of country results (Biddulph, Gehrke, Taylor and Carr, 1997). The effect of Maori children's relatively poor performance in mathematics, of course, is that they are eventually denied entry to many of the high status professions and occupations which require a high standard of mathematics achievement as a prerequisite for entry. This, as Ohia has pointed out, is not acceptable to Maori. "In mathematics, Maori people desire to achieve the same outcomes as non-Maori learners and to the same standards" (Ohia, 1993 p 113). New Zealand as a country is also the loser when the mathematics potential of Maori children is not developed to the fullest extent possible.

A key factor in developing the mathematics potential of Maori children is an adequate supply of Maori teachers who are fluent in Maori and competent in teaching mathematics. Meeting the demand for such teachers falls on the shoulders of teacher education institutions such as the Auckland College of Education, and the School of Education at the University of Waikato. And meeting the demand has proved a considerable challenge for Maori mathematics teacher educators. Some hint of the difficulties is revealed by Nathan and McMurchy-Pilkington at Auckland.

> *...we have a responsibility to re-introduce the students to mathematics in a way that challenges their belief about the subject, challenges their mathematical "being" and allows them to develop an alternative view of mathematics, a view that includes a human face to mathematics and its development.* (McMurchy-Pilkington, 1997, p.3)

They found that changing Maori student teachers' perceptions of mathematics was crucial to helping them feel more confident about teaching mathematics.

This paper identifies a number of factors that mathematics teacher educators need to address in their efforts to meet the demand for a growing number of quality Maori primary teachers of mathematics. Many of these issues are not new. Several years ago Ohia realised that,

> *A focussed teacher - training package will need to be developed to increase the facility of mathematics teachers to speak and write the Maori language . In this way, a true connection will be made between mathematics and the Maori language and people. (Ohia, 1993, p.111).*

Ohia added,

> *Maori developers are no longer talking about just adding Maori concepts to mathematics, but reconstructing the mathematics programme to align it with the Maori language, culture and qualities to achieve the same outcomes and standards, (Ohia, 1993, p.111).*

At the same time Fairhall stressed that the learning of mathematics in Maori "depends upon Maori people; there are no others" (Fairhall, 1993, p.122).

Source of data

Data for this paper were collected through systematic observation of and discussion with a group of 23 Maori student teachers in their first and second year Rumaki (Maori immersion) mathematics education programmes which I taught in the second semester of 1997 and the first semester of 1998. Field notes of the observations and discussions were kept. Prior to that I had been teaching in a bilingual unit in an intermediate school and so was keenly aware of what student teachers needed by way of fluency in Maori and mathematics understanding if they were to become effective teachers of Maori children in Kura Kaupapa schools, or in bilingual or immersion programmes in mainstream schools.

Issues

My observations indicate that Maori student teachers involved in the Rumaki programme usually work collaboratively in a genuine atmosphere of helping each other learn. There is little notion of competitiveness, and they often act within "tikanga" (customs) of Maori culture. Perhaps this is because they remain together for the majority of their teacher education courses, or because many are mature adults who bring to their studies a range of life experiences and wisdom. Whatever the reason, many do not have fond recollections of their own schooling, are convinced that Maori children today deserve better, and hence are fairly highly motivated. These are the positives which provide strength to the Rumaki programme, but there are also factors which provide a real challenge.

1. Significant variation in the quality of student teachers

Because of an acute shortage of Maori speaking teachers and Maori role models in schools, there has been vigorous recruitment of Maori into primary teacher education programmes. However, given the limited number of Maori in New Zealand who are fluent in the language, together with the relative lack of achievement of many Maori in mathematics, the Maori applicants who have been persuaded to consider primary teaching constitute a very diverse range, in several respects.

Language: Only one in five of the Rumaki group I have taught were fluent in Maori when they arrived on campus. Those who entered with little or no Maori fluency will be expected to teach 100% in Maori when they enter the teaching profession at the end of their three or four year teacher education programme. If they don't then they could find that some of their pupils are more fluent speakers of Maori than themselves. To assist in this regard, the 80% without fluency are required to take Maori language courses where they have 'options' in their diploma or degree programme.

The real issue for mathematics teacher educators is how to help such student teachers develop the understanding needed to teach mathematics in Maori at the same time as many of them are trying to learn the language. There is no simple answer to this, although it may help if the mathematics education courses are taken later in the teacher education programme rather than in the first two years as at present. This would enable the students to concentrate on learning the Maori language in their earlier years in the programme. On the other hand, there is some expectation in schools that the Maori student teachers will have some competence in teaching mathematics when they go out into schools on teaching practice. There is a dilemma here that needs further consideration.

Mathematics understanding: Many of the Maori student teachers bring to their courses negative feelings about mathematics, and a lack of understanding of the major concepts and processes that primary children can be helped to develop. It is extremely difficult to help them overcome such feelings and gain the understanding needed in the one and a half courses allocated in the Rumaki programme at Waikato. There is certainly a need to change their perceptions about mathematics education, as Nathan and McMurchy-Pilkington found. Unfortunately, although student teacher motivation is high, there is not the space in the Waikato programme, as it is presently arranged, for them to undertake additional mathematics education courses. Effectively they are being asked to completely change their perspective on mathematics education, upgrade their own mathematics concepts and processes, learn the Maori terms for these, and develop the ability to teach the mathematics, perhaps critically, within the context of living Maori culture. It is a big ask. My fear is that through lack of preparation time, our Maori student teachers will graduate without feeling secure about teaching the Maori mathematics curriculum in the way intended.

Study experience: Often it is years since many of the mature Maori students have done any serious study and in such a tight programme where they have to do considerable reading and study to promote their professional development in Maori mathematics they find it quite difficult. Many of the Rumaki students appear to be starting way behind their counterparts in the regular mathematics courses. The result is that, as their mathematics teacher educator, I have had to provide them with much more help and support than I do the students in my mainstream mathematics education classes. This poses another dilemma: such help and support require time, which is also in short supply. In other words, providing mathematics education courses for Maori students in the Rumaki programme requires greater resourcing than mainstream courses.

2. Community expectations

As if the Maori students in the Rumaki programme didn't have enough to contend with, some are under considerable pressure from family and community expectations that they do well. This is a burden that they just have to bear as often family and community, especially small rural communities, are supporting them financially and otherwise through the years of their teacher education programme so that they will return and work within the community. Although community expectations provide a motivating force, when they combine with the pressures of the

Rumaki teacher education programme itself they can begin to have a negative impact and interfere with the students' ability to process course work. It would be much better if the programme were flexible enough to allow some students to take courses at a pace more suited to their developing professional competence.

3. Delivering mathematics education courses within a Maori framework

I have found it quite a challenge to model a Maori mathematics education perspective for Rumaki students, especially those who are far from fluent in Maori and who tend at first to translate Pakeha (English) concepts and ways of thinking into Maori words. For example, I need to give further consideration to promoting team teaching, a supportive, non-competitive whanau environment, contexts relevant to Maori that incorporate tikanga (customs) and are suitable for the student teachers and children, and to achieving a balance between traditional and modern contexts.

In conjunction with this, I am also very conscious of the need to help the students overcome their very real anxiety about and fear of mathematics. Helping them to develop a more positive view of mathematics can take a very large part of course time. Developing in our students a Maori mathematics perspective, fluency in the associated language, and a positive feeling about learning and teaching in an integrated, holistic way is something which I recognise as desirable but not easy to achieve. With respect to this, I am reminded of Barton's (1993) view that Maori mathematics education may incorporate the functions of present mathematics but not necessarily resemble it. Similarly, the delivery of Rumaki preservice teacher education programmes in mathematics may well need to be different from mainstream mathematics education programmes.

Final considerations

The challenge of providing appropriate mathematics teacher education immersion courses is both time consuming and energy sapping. The main preoccupation at present is with trying to get the Maori student teachers up to a minimum level of competence. What is needed of course is a longer term strategy to encourage the best teachers to consider undertaking post graduate qualifications (degree courses for those who have completed a diploma only, honours courses for those working on a degree, and masters and doctoral studies for those with a degree) so that they can take leadership positions within Maori education generally and Maori mathematics education in particular. I am aware that at present Maori with higher qualifications tend to be enticed away from schools to work in universities and government agencies, but we need to look beyond this shorter term position.

It is also time to begin to address the mathematics achievement of the 87% of Maori children who are taught in mainstream classes. What can be done for these children is problematical because little is known at the moment about the difficulties they experience learning mathematics, or the difficulties teachers experience trying to help them learn. This is an area where there is a need for research that will identify such difficulties and explore effective ways of overcoming them. It may be, for instance, that strong links have to be drawn between the home life of Maori children and their learning in mathematics. Whatever the findings, such research could then inform mathematics courses for preservice and practising teachers.

As part of this push for development, there seems to be a need to raise the expectation of all mathematics educators, Maori parents and the children themselves that Maori children can succeed in mathematics.

Likewise, further development of mathematics education courses for Maori teachers, such as the student teachers in the Waikato Rumaki programme, could also benefit from relevant research. A systematic, indepth research approach is now needed to investigate these issues, and ways in which they are being or might be addressed. Research of this nature could obtain a variety of data. For example, data on who the trainees are, their level of fluency and their views of mathematics need to be sought. Lecturers and tutors of the rumaki programmes need to be identified.

Data about the level of fluency and mathematical experience of lecturers and tutors could also be monitored.

Data could be obtained in a variety of ways. For instance, both qualitative and quantitative means could be used to track an intake of student teachers, initially during their first year, but then longitudinally over the period of their degree course. A series of hui/meetings with some of the main preservice immersion training providers could also be held to discuss, debate and further clarify the issues.

Within such a research programme, the nature of Maori research and approaches that are acceptable within a Maori format would need to be adhered to. Such approaches may vary from region to region, but it is possible that they could be combined with common western approaches to research. Acceptable research methodology is something that itself needs to be investigated.

Research of the kind outlined has the potential to ensure effective delivery of Rumaki preservice mathematics education programmes, which in turn would benefit Maori children in Rumaki schools.

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Towards an explanation of curriculum control

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The Mathematics curriculum has undergone extensive change over the past few years and much attention has been directed towards coping with the demands of this change. However curriculum has social implications and it is important to examine the intentions of curriculum innovation, which, in turn, leads us to consider the issue of control of the curriculum. This paper draws on the work of Max Weber as it attempts to fashion an explanatory model that will provide a conceptual framework for the consideration of curriculum change within the context of curriculum control.

The industrial revolution signalled a change in the social forms of modern society. It also signalled a change in the educational aspirations of the population at large. Mass public schooling in early eighteenth century England had its origins in the form of charity, or pauper, schools that blossomed alongside the workhouse movement. By the end of the eighteenth century the role of the charity school had been transformed.

> Socially and pedagogically, such charity schools were a transitional form of educational life. On the one hand they were an integral part of the domestic or craft economy of the 16th and 17th centuries, yet, on the other hand, they were also a response to the spread of wage labour (and its *alter ego*, unemployment). From the first perspective, charity schools pre-date the factory system. They were a surrogate of family life, not an alternative mode of existence. By the end of the 18th century, however, the second perspective dominated. Workhouse charity schools could no longer cope, educationally or financially, with the increasing numbers of indigent poor children who populated areas of urbanization and industrialization (Hamilton, 1980, 284).

Urgent solutions to this crisis situation were required. The customary response to the growth in size of charity schools was to form new schools. The rapid increase in school numbers towards the end of the eighteenth century, coupled with the financial burden this imposed, precluded this solution. Lancaster adapted educational administration practice from France and reorganised existing schools into a hierarchy of smaller administrative units, of 'anything up to a hundred pupils', known as 'classes'. Each class was supervised by a monitor. The basis for much of the inspiration for these new ideas rested in the arrangements that governed the relationship between workers and supervisors within the factory system (Hamilton, 1980). The application of the scientific method to the task of manufacturing resulted in rationalised modes of mass production. It is not surprising that the application of a similar approach was envisaged as a solution to the problems caused by the extension of mass schooling. ***The class, like the factory, could be systematised, made rational and controlled in ways similar to those used by science and technology in organising material life*** (Popkewitz, 1984, 22).

The emergence of the notion of class went hand in hand with the notion of sequential courses of study, and, thus, to the displacement of more traditional approaches to education, where the 'elect', predominantly those with the ability to pay, were involved in the pursuit of a self-directed course of studies with the assistance of a personal tutor. In schooling, rationalisation meant the dominant forms of individualised domestic production were replaced by the simultaneous instruction of batch mass production. Although this transition took several decades to emerge, the classroom as we have it today saw its birth in the application of the scientific method to the challenges of mass-schooling (Popkewitz, 1984).

The emergence of the classroom, and subsequently of curriculum, serves to highlight the close association between the educational ideas of the time and prevailing social conditions. Education has a close relationship with the wider organisation of culture and of society and this translates into a concern with the nature of the content of schooling (Williams, 1971). The determination of the nature of school knowledge becomes an issue of social control.

> Those in positions of power will attempt to define what is to be taken as knowledge, how accessible to different groups any knowledge is, and what are the accepted relationships between different knowledge areas, and between those who have access to them and make them available (Bernstein, 1971, 52).

Curriculum is pivotal not only in terms of defining the nature of the knowledge to be transmitted but also in deciding those questions that relate to the nature of a society and of its culture. Awareness of the nature of the knowledge to be learnt, as well as that of the groups that determine it, is crucial to an understanding of the nature of the curriculum.

Development of a Model

The emergence of modern Western education has taken place within the context of a capitalist society. Pertinent to an understanding of the emergence of capitalism is the work of Weber which reflects on the characteristics of capitalism and on its particular connection with Western societies (Runciman & Matthews, 1978). What factors contributed to the emergence of capitalism in Western society but not in other societies? In Weber's terms, modern Occidental society is distinguished by the rational capitalistic organisation of (formally) free labour. In using the term rational Weber is using it in much the same sense that modern business and industry uses it. He refers to rational organisation as the process whereby the firm or company is organised, or reorganised, in such a way that every arrangement is made to serve a central goal. For a business enterprise this would normally be the maximisation of profit. Not that 'capitalist' undertakings in the pursuit of profit are in themselves rare. Individuals and enterprises, with a keen eye on the acquisition of capital, have existed in all civilised societies throughout history. Our history and our literature carries records of merchants, wholesale and retail, foreign and local; money-lenders of all kinds, and banks, which began to appear in the sixteenth century. The speculator, the entrepreneur, the capitalistic adventurer have existed all over the world. With the exception of trade and banking the basic opportunities sought by these individuals has been in purely irrational speculation or in acquisition by violence or by 'booty', sometimes in actual war but particularly by fiscal plunder of subject-peoples over a long period. Even to this day examples of such activities are in evidence. However, *[s]uch economic activities are worlds apart from the methodical management of a large-scale corporation, in which success depends upon professional competence and an everyday steadiness in the conduct of affairs that is incompatible with the indispensability of any individual and the sporadic character of very risky transactions* (Bendix, 1962, 306). Capitalism, as it exists in the West, has its basis in the employment of free labour, and is structured in such a way that its pursuit of capital is conducted in a systematic and regular manner. Rational, industrial organisation, attuned to regular market opportunities for profit, rather than to political or irrationally speculative opportunities, is a characteristic of capitalism.

It is, however, not the only characteristic. Another factor, closely related to the above, was the introduction of rational book-keeping, invented in the sixteenth century by the Italian mathematician Luca Pacioli (Boyer, 1991). The introduction of book-keeping was closely associated with the separation of business concerns from family concerns. Book-keeping provided the technology to allow business to expand beyond the immediate supervision of the family enterprise. The invention of book-keeping introduced the possibility of legal accountability without reliance on family members. Corporate property became legally separated from personal property. A new environment for the pursuit of business interests was created.

The industrial revolution heralded an unprecedented expansion of scientific activity as the developing capitalism demanded technologies that would assist it to expand. Book-keeping notwithstanding, control over widely dispersed transactions would have been ineffective without adequate communications, transportation, and effective legal sanctions. The invention of book-keeping represents the depersonalisation and, in a sense the automation, of the accounting process. This same liberalisation was eagerly sought in the manufacturing environment. Rapid expansion was not possible if the work of individuals was essential to the production process. For this to occur the work of individuals had to be depersonalised; in other words, any person with appropriate training could take their place on the production room floor. The individual, even though highly trained, became merely another cog in the production process. The expansion of manufacturing industry necessitated the development of depersonalised, or automated, machinery.

A form of social organisation was also required that would provide for the necessary conduct of a society that was to allow the regulation of economic, and other, affairs beyond the intense supervision of the family firm. Essentially, mechanisms of control needed to be established within the society, and with this control the necessary sanctions that would ensure the control was effective. In this regard, Weber identified as most important the rational structures of law and of administration. For modern rational capitalism needed not only the technical means of production, but also a mechanism for control in the form of a calculable legal system and an administration based on formal rules. It required a regulated, depersonalised context within which it could operate, and it required trained and specialised personnel to administer regulations. Training and specialisation was particularly important for the position of trained official, the position essential for the smooth running and proper functioning of the administrative system.

Of course the official, even the specialised official, is a very old constituent of societies. However no country and no age has ever experienced, as has modern Western society, the absolute and complete dependence of its whole existence, of the political, technical and economic conditions of its life, on specially trained officials (Bendix, 1962). An important characteristic of the official, unlike in previous societies, is the depersonalisation of the role. Selection of the official is dependent on merit, rather than on the whim of the employer or contingencies of colour, race or creed. What becomes important in this context is the work of the official rather than the particular person who occupies that position. To this end training becomes important. Thus, the most important functions of the everyday life of society have come to be in the hands of technically, commercially and above all professionally trained government officials. Trained governmental public officials are an essential concomitant of Western society.

How has society resolved the potential challenge that the formation of these groups pose? To this end modern society has developed social structures that ameliorate the power of individuals and specific groups within the society - while power rests with the political structure, authority within society has been embedded in the institutional structures of bureaucracy and the professions. The one denies individuals the opportunity to attain exceptional power through the control of the administrative apparatus, while the other diverts emphasis from political prestige to acquisition of status. Both require further examination.

Bureaucracy is possibly one of the most-used and most-abused terms in modern political discourse. It has come to characterise the administration of large organisations, especially governmental ones. The fact that the term continues to enjoy widespread currency is, in itself, confirmation that bureaucracy is an important concept in discussion of administrative organisation (Spann, 1971).

Discussion of bureaucracy has its intellectual roots in the work of Weber (Albrow, 1970). Weber's work was concerned with the sociology of organisations and the study of political struggle - essentially the struggle of an organisation to prevail. Weber distinguished three main types of organisation: ad hoc groups based on personal ties; more permanent organisations (referred to as patrimonial) which acquire property and raise personal loyalties into a long-term world view; and impersonal bureaucracies which organise property around an abstract set of positions and treat individuals as temporary actors filling permanent slots (Collins, 1975).

What is important for all types of organisations is the ability of the organisation to survive. In this sense, Weber saw rationality emerging as the basis for the organisation of modern society, since rationality, through its close intellectual association with the growth of scientific knowledge and the ensuing specialisation of human activities, led to rearrangements of the work practices of the organisation that could better guarantee its survival. Survival in previous times had depended on capricious loyalty to particular individuals, whereas rational-legal organisation was founded in the rationalisation and abstraction of work processes. The survival of the organisation was no longer dependent on the whim, competence or charisma of individuals. However, the rationalisation of the nature of work within an organisation translates into a rationalisation of administrative functions - all areas of the organisation are subject to the same principles. This rationalisation of administrative functions unerringly leads to bureaucracy as the foundation of administration (King, 1980).

The claim to technical superiority of bureaucratic administration rests on its orientation to impersonal rules that enhance its reliability and hence the calculability of its operation; of the expertise of its officials; and of the development of a system of authority relationships that is practically indestructible. For a society administered by a bureaucracy, the conduct of public affairs depends upon expert training, functional specialisation, and coordination of a bureaucratic administration with its uninterrupted performance of the manifold tasks that are regularly assigned to the modern state. The bureaucratic form of administration is both permanent and indispensable; because not only do the myriad of employees have a vested interest in the continuance of such an organisation, but also because the complexity and specialisation of the tasks performed by the organisation put them outside the scope and grasp of any other administrative arrangement (Bendix, 1962). Once established bureaucracy is there to stay.

Bureaucratic administration is essentially administration by experts.

> Equal eligibility for administrative appointments means in fact equal eligibility of all who meet the stipulated educational requirements. Educational diplomas have replaced privilege as the basis of administrative recruitment, just as scientific education and technical expertise have replaced the cultivation of the mind through classical literature and the cultivation of manners through competitive games among social equals. The expert, not the cultivated man, is the educational ideal of a bureaucratic age (Bendix, 1962, 430).

The superiority of bureaucracy as a form of administration would appear to be assured. However, this was not what Weber was intimating when he referred to the inevitability of bureaucracy. What he was concerned with was bureaucracy as a particularly efficient form of organisation for *political purposes* because it kept the organisation together. The survival of the organisation was again at the forefront of his concerns. The development of bureaucracy does not stop the struggle for control, it transfers it to a new arena which creates certain advantages for the bureaucrats. Thus, a tendency towards bureaucratisation, as control, is an inevitable trend in all areas of direct government concern (Albrow, 1970). How much more so in the area of education which comprises such a major part of government political responsibility?

Whereas the functioning of the administrative responsibility of society is dependent on the possession of expertise, the institutionalisation of expertise within society has been associated with the development of professionalism. Professions play an important role in the regulation of modern society in two ways. Firstly, they provide a vehicle which the middle class can utilise in their efforts to achieve realisation of aspirations for social recognition. The tendency for all middle class occupational groupings to aspire to professional recognition is in itself a confirmation of this role.

Secondly, professions provide an opportunity for the middle class to become involved in the political process, thereby encouraging them to identify with the actions of government and in the process helping to dissipate any political dissatisfactions. Professions allow governments to cede major responsibilities for certain areas within society that are necessary to that society's proper functioning. In so doing governments rely on the social institution of professionalism to ensure that this responsibility is capably and efficiently discharged. Medicine, law, and engineering are readily recognised professional occupations, but many other areas, to a greater or lesser extent, are regarded as professional - the clergy, academia, and teaching are a few of many possible examples.

Professionalism emerges from the institutionalisation of expertise by modern societies. Thus, it relates to the work that a profession performs (Abbott, 1988). However true professionalism is only realised within a societal context. Professions ultimately depend upon the power of the state, since it is through the state that the legal functions that underpin the monopoly of a profession are attained. By acts such as establishing accreditation boards and recognising professional associations the state confers on one group of protagonists the societal legitimation that is essential to enable a profession to claim jurisdiction and to attain social status (Larson, 1977).

However, the state has a predilection towards bureaucracy. Thus when the interests of the state and of the professions coalesce it is the professions which are under greatest pressure, for not only is their legitimacy guaranteed by the state, and thus they are potentially vulnerable, but also the state has the advantage of the bureaucratic apparatus in the struggle to ensure that its will is imposed.

The making of curriculum is portrayed as professional activity. Responsibility for curriculum development has thus been entrusted to professional groupings (Goodson, 1988). However, in recent times the state has assumed an increasingly proactive role in the propagation of curriculum. Increasing conflict with the professional organisations would appear inevitable.

A Brief Case Study of Curriculum Change

The Victorian senior mathematics curriculum provides a particularly interesting case for exploration. Victoria began as a colonial state and has evolved to a modern democratic society. All this has occurred during the past two hundred years and has been substantially documented. Clearly substantial change has occurred throughout this period and thus many of the explanations of change proposed by Weber would be relevant. Weber, however, paid only scant attention to education, so the Victorian case provides an opportunity to examine the ideas of Weber, particularly as they relate to bureaucracy and professionalism..

The emergence of public education spanned a period stretching from the eighteen thirties to the nineteen sixties. Victoria began as a colonial society, and, not surprisingly, many of the influences of colonialism were evident in the social institutions, including education, of the emerging society.

Secondary education, in particular, owed its origins to its European forebears, and, as a fledgling colonial government struggled with challenges of civilisation and democracy, in the face of a harsh and formidable environment, both physical and political, it was left to private secondary schools and the university to construct the edifice which was to constrain the growth of public secondary education throughout the early twentieth century. Responsibility for secondary curriculum devolved to the traditional authority of the university professoriate, and an alliance between the university and the private registered schools ensured that this responsibility remained intact until well into this century (McCallum, 1990). Thus, secondary mathematics curriculum was constructed as an academic curriculum, and students and teachers alike were forced to cope with a curriculum that was primarily designed as preparation for university studies.

By the nineteen sixties classroom mathematics teachers had begun to express public opposition to a curriculum that was in no way appropriate for their students. Their voices were joined by others across the educational spectrum as educationists united to mount a challenge for control of the curriculum. Education had begun to emerge as an area of professional concern.

By the turn of this century education had become a major enterprise and the Victorian government initiated moves to regulate its provision. As the demand for

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education grew throughout this century so did government involvement, but so also did the numbers of those with a professional interest in education. The elevation of education to professional status within the university, combined with an emerging discipline of psychology and an embryonic philosophy of education, provided the basis for the acceptance of education as a legitimate discipline. Professional recognition, however, had to await the work of Piaget, whose reconceptualisation of the learning process provided the intellectual infrastructure that enabled educationists to propose a new conception of the meaning and purpose of education. Thereafter, professional educators, mathematics educators among them, embraced the challenges posed by this new conception of education which treated the child as an active participant in its own learning process. This conceptual transformation entailed the redefinition of mathematics education, which effectively laid the foundations of the discipline of mathematics education. A new professional grouping was constructed, and from deep within the educational fraternity, a successful challenge was mounted to assume control of the secondary curriculum.

The expansion of educational activity certainly had an impact on the practitioners in the classroom, but it also had an impact on government. Over recent years government has displayed a renewed interest in the processes of schooling. In particular, an increased focus on the role of education has been seen as one way of addressing wide ranging problems within society at large, such as youth unemployment, changed work patterns, and a declining economy. Education is a large budget item, and governments sought greater accountability for the increased economic outlays. To a large extent this manifested itself in greater government intervention in the outcomes of schooling (Horwood, 1997).

In the end, government chose the path of direct intervention - an inquiry into the purposes of the postcompulsory years of schooling within Victoria led to a reconceptualisation of the purposes of education and to the propagation of a new curriculum. The growth of administrative function normally goes hand in hand with government intervention and education proved no exception. Successive state governments oversaw the transfer of responsibility for secondary curriculum to the welcoming arms of government bureaucracy, and, in the time-worn traditions of bureaucracy, responsibility for curriculum expanded to embrace all years of schooling.

Conclusion

It is in the nature of the state that it appropriates all areas of primary concern, both as a protective mechanism and as a proactive one (Foucault, 1982). Education was one area of particular importance to which attention was directed. Control of primary education became a government responsibility in the nineteenth century, but the traditions of secondary education were much more resilient. In part this was due to the lack of popular demand for higher education, but it was also due to the entrenched authority of the professoriate within the education system and to a tradition of education that valued the cultivation of old world values above the needs of an emerging society. To early colonial society education provided a means to prestige and privilege so that prestige and privilege became enmeshed with the purposes of education. In this way the system of authority became interwoven with the social system in which the organisation operated, thus strengthening the resistance of this authority to challenges from others within the society.

This traditional authority prevailed until well into the twentieth century despite the fact that voices of radicalism and of democracy were pressing for the direct participation of government in the control of curriculum. In essence the demand for the state to assume control of education is a demand for a rational-legal authority as the basis for curriculum. For the process of impersonalisation that accompanies this transition is viewed as a means of ensuring that all can share equally in the privileges accorded by the state. The early attempts to wrest control of curriculum from traditional authority met with quite substantial resistance as entrenched interests within the society rallied to repulse the initial attempts to broaden the charter of secondary education.

However, the state has several means at its disposal by which it can attempt to exercise control over the essential processes within the society. One such means is legislation, and in creating an examination board in the early years of this century the state effectively laid the foundations for its eventual triumph in the area of control of curriculum. However this victory was not to come easily. For the first half of the twentieth century an alliance between the university and the registered school system managed to maintain control of curriculum and hence to mould the process of secondary education that was to evolve throughout those years. The only major threats to this system arose from the onset of mass education - in other words, the democratic demand for equal education for all, and the increased demands for specialisation, and hence professionalisation, within the society.

Curriculum was initially accepted as a proper jurisdiction for the university professoriate. The only model that was provided for secondary education was a traditional one imported with the early settlers. Professors were experts in matters of knowledge, and hence, of education. However the growth of democracy provided a challenge to traditional authority. This challenge took the form of professionalisation, since professions constituted an expression of democratic authority - a single figure of authority was replaced by the authority of a group.

Professionalisation, on the one hand, led to mathematicians eschewing direct control of the secondary curriculum, and on the other, to educationists claiming curriculum as their jurisdiction. The nature of the education profession is, however, somewhat diverse. It embraces a range of careers from teachers through to academics. The differentiation of the education profession entails that the determination of curriculum is, even within this profession, a highly contested arena. This allows the possibility that control of curriculum could be the preserve of a privileged subgroup of the education profession. Such a result would not be tolerable in the face of demands for the democratisation of curriculum. The implicit logic that lies at the heart of democracy eschews the notion that one particular person, or group, should be privileged at the expense of the others. Thus support is readily won for the institutionalisation of curriculum control. However, government control entails bureaucratic control. Professionals acquiesce in their own demise.

The situation in Victoria illustrates that the path to democratisation of a state inextricably involves the state on the path towards control of curriculum. An inevitable victory is not necessarily assured, except so far as to say that the enormous resources involved in the provision of a comprehensive educational system are generally available only to the state. However curriculum extends well beyond the provision of physical resources. Curriculum is central to the definition of knowledge and the accessibility of that knowledge to different groups within the society. The inevitable bureaucratisation that accompanies state control poses a threat to certain groups within society because it is the nature of bureaucracy that it cannot make allowance for the treatment of individual cases. Implicit in bureaucratic control is a particular notion of knowledge and particular interpretations of its accessibility. Thus, at core, bureaucratic control threatens the very democratic basis on which this control is predicated.

The current situation in Victoria clearly reflects the influence of bureaucratic control on the nature of mathematics curriculum. A curriculum has been propagated throughout years P-12 that treats content as non-problematic and in which attention has been focused on assessment and moderation. The use of external moderation procedures are, in the first instance, ostensibly designed to guarantee the equitability of the assessment procedures, but they have the added effect of providing a means for constructing teaching in terms of quantifiable outcomes and external objectives. One consequence of this approach is to diminish the role of the teacher in the definition of curriculum, and hence in the determination of the outcomes of schooling. Clearly the nature and purpose of education is being redefined but it remains an open question as to whether this redefinition is the proper responsibility of mathematics education administrators.

Bureaucratic control of education is not the sole preserve of democracy. Other political systems develop bureaucratic structures to control education. Some, in particular, are much more insistent about government control of curriculum than is compatible with a democratic rationale for a state controlled bureaucracy to oversee curriculum. This points to the fact that at the heart of control is a purpose. One such purpose might be the preservation of the institution. *However, for democracy one purpose is to ensure that the outcomes of education are accessible to all in equal measure and regardless of socio-economic situation.* Thus we need to be ever vigilant that the control arrangements that we have in place are such that all can continually aspire to that ideal. Bureaucracy is, in essence, an administrative form and thus the bureaucratisation of curriculum entails a redefinition of curriculum and, consequently, of teaching. Such redefinitions are currently clearly in evidence with respect to mathematics education within Victoria. Resistance to the bureaucratic control of curriculum can be expected to emerge from practitioners in the schools, and from others who feel marginalised by the definition of knowledge implicit in bureaucratic control, as these redefinitions force teachers and others to make accommodations that are unacceptable. The struggle for control of the mathematics curriculum continues.

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How sixth grade students explain connections between common and decimal fractions

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Sixth grade students' responses to a task which involved matching common fractions and decimal fractions were examined. Of interest was their understanding of relationships between these symbolic forms, evidenced in explanations offered in an interview setting, and the role of the follow-up question in helping them express their thinking. A most favored explanation involved relating decimal and common fractions to a unit of 100.

Translating between common fractions and decimal fractions has long been accepted as a difficult task for students. According to National Assessment data (Carpenter et al., 1981), 13-year-olds have only about a 50 per cent success rate in converting between even the simplest decimals and fractions—those in which the decimals are powers of ten. Hiebert and Wearne (1986) obtained similar results when they studied approximately 700 fourth- to ninth-graders. When they asked students to write decimals to match pictorial representations of 3/10, 4/100 and 1/5, less than half of even the ninth-grade students studied responded correctly to any of these items. Hiebert and Wearne (1986) attributed these findings to an absence of connections between conceptual and procedural knowledge. They noted:

> In general, students appear to work at a syntactic level that remains isolated from semantics. The two systems of common and decimal fractions are related through manipulation of the written symbols directly rather than through the intermediacy of conceptual reference (p. 209).

Mathematics education for many children is an experience which works to suppress their native and intuitive mathematical concepts. At the age of five or six children commence school with lots of informal and intuitive mathematical concepts and skills. But the schooling process places great value on *the* single answer for evaluation. Much mathematics instruction is authority-based with little space for children to develop intellectual autonomy. Authority-based instruction emphasises the memorisation of rules and procedures at the expense of sharing explanations and meanings. By the time students reach the upper elementary and junior secondary grades, the emphasis from the primary grades on conceptual understanding has been set aside and students become lost in a maze of rules and symbols. Teachers are often appalled when students derive answers that don't make sense either to themselves or their teachers, or are inconsistent with conceptual knowledge previously demonstrated. Teachers assume that since a conceptual base was introduced at some point, students will reference that knowledge as they work through problems. But this doesn't seem to happen.

As students progress through school, symbols used to represent mathematical ideas begin to take on lives of their own, often disconnected to any meaning. Many students no longer expect mathematics to make sense. This is especially evident in the case of fractions and decimals. No one would argue that procedural knowledge needs to be supported by a strong conceptual base. It is just as important though, to have students reference that conceptual base as they work. This leads us to question both the quality of the connections they are making as well as conditions that would enable them to draw upon those connections. In this study we were interested in better understanding: (1) how students understand the connections between common and decimal fraction notational forms, and (2) effectiveness of the follow-up question in an individual interview for enabling students to make such connections.

Method

Forty-nine sixth grade students from four metropolitan Melbourne schools were individually interviewed in November and December of 1995. Twenty three students attended two different private schools, eleven from one school; 12 from the other. Twenty six students attended two government funded schools, fifteen from one and eleven from the other. Classroom teachers were asked to select children whom they considered to be of average or above average ability in mathematics. As such, the sample is representative only of children judged to be successful in primary school mathematics.

These children had almost completed seven years of elementary mathematics education. As an indication of possible curriculum taught, the Victorian Government Department of School Education recommends that children at Level 4 (corresponding approximately to Grades 5 and 6) should be able to

- compare and order decimal fractionds with unequal numbers of places (for example, 3.05, 3.001, 3.4, 3.12);
- count in decimal fractional amounts;
- rename fractions in different forms, including percentages (for example, 250% is the same as 2 1/2); and
- use understood written methods to add and subtract decimal fractions with equal number of places.

Six fractions and decimals tasks were given, as part of a more comprehensive set of 24 tasks representative of the major curriculum topics in mathematics for that age group. The focus of this study is on the sixth task only. A brief description of the six fractions and decimals tasks follows:

Task 1. A flashcard of the fraction 2/3 was presented. The student was asked to draw a diagram to illustrate the fraction.

Task 2. Twelve picture cards were presented to the student. Two-thirds of the cards were to be found.

Task 3. Four flashcards, each showing two fractions, were presented. The student was invited to select pairs whose fractions were equivalent. The pairs were 2/5 and 4/5, 2/5 and 2/8; 4/5 and 8/10; 3/5 and 3/10.

Task 4. A 10x10 grid was presented, together with a flashcard containing the fraction 0.4. The student was asked "If this 10x10 grid was one whole square shade as much of the big square as needed to show this number".

Task 5. Three flashcards each containing the numbers 0.37, 0.123, and 0.62 were presented, and the student asked to identify the largest number.

Task 6. Eight flashcards are presented in random order. The student was asked to match the cards that have the same value. The cards showed these numbers: 1/2, 0.5, 1/4, 0.25, 1/10, 0.1, 2.5, 0.4.

For Task 6 the interviewers were expected to ask follow-up questions in order to better understand the thinking responsible for choices made. The interview protocol contained the general follow-up question "How did you know which cards have the same value?" However the interviewers were free to use their own discretion about what to ask, depending on the students' responses. Often interviewers focused their questions on particular pairs of cards that were chosen by the students.

Results

We will first present results of the kinds of connections students made between the pairs of fractions they selected. Then we will discuss the role of the follow-up question in helping students make connections that might otherwise have remained untapped.

Overall, 34 students (69%) correctly matched all four pairs of cards in their first attempt. Five students whose first response was incorrect changed their choices to correct pairings as a result of further questionning.

The most common way in which these students linked the common and decimal forms of notation was by means of what we will call a *100 scheme*. The 100 scheme can be characterised as a relational device whereby a common fraction or decimal fraction symbol is interpreted in relation to a unit of 100. Thirty two students (65%) were observed to offer justifications that involved reference to 100. Fraction pairs for which the 100 scheme occurred most frequently were 1/4 and 0.25, and 1/2 and 0.5. Justifications for the equivalence of 0.1 and 1/10 that involved references to adding a zero to the one in 0.1, might have either involved a figural correspondence of tens at the symbolic level, or may indeed have been a result of thinking of 0.1 as 10 of 100. We could only make a tentative conjecture based on the quality of explanations for other pairs of choices in this task.

Explanations for matching 0.1 and 1/10. Eight students simply asserted that 0.1 meant one tenth, or tenths, or one tenth of a whole, or the place after the decimal point is tenths. Three students explicitly referred to the digit one being in "the tenths column". As one student explained it: "you have columns, tenths, hundredths, thousandths, and then one (pointing to 0.1) which means one over ten". Two students explained that 0.1 is 10 out of 100. One student offered the argument that 0.1 plus 0.1 added together 10 times would make up a whole. Four explanations were tautological in nature, for example, "that's one tenth and that's one tenth", or, "one tenth is one part of the whole thing, and so is 0.1". Two explanations mentioned adding a zero to the one digit in 0.1: "if you added a zero it would be, like, ten".

Explanations for matching 0.5 and 1/2. The most common explanation involved conceiving 0.5 as half of 100 (10 instances). Typical responses: "one half of 100 is 50 and this equals point five oh", "if you add a zero there's 50 and 50 into 100 is two", and "if you put a zero on the end you get 50, 50 out of 100". There were four explanations in which 0.5 was considered half of 10, as for example, "half is half of anything, point five is half of 10", and "point five, that's out of 10". Five references were made to doubling one of the numbers to make one whole. Examples included "zero point five is five tenths if you doubled it, it would be ten tenths which equals one", "if I added another five that would be one whole", and "point five plus point five equals one". When the interviewer asked one student how he knew 0.5 was one half of a whole number, a geometric explanation was offered: "because it is right in the middle".

Explanations for matching 0.25 and 1/4. Eleven students used arguments that involved 100 and its factors of four and 25. Examples included: "four twenty-fives in one whole is a hundred; 25 is a quarter of a hundred", and "if you times 25 by four it equals a hundred". There were three explanations in which reference to per cent was used, as in "quarter is the same as 25 per cent", and "0.5 is 50 per cent and half of 50 is 25; that's quarter of a hundred". Other explanations also related one fourth as 25 hundredths to one half as 0.5 or 50 hundredths: "half is 50; one quarter is 25", and "0.25 is half of 0.5, and one quarter is half of a half". A similar argument was implicit in the explanation "because 25 plus 25 equals 50, and 50 plus 50 equals 100". Geometric arguments included "if you divide a block of chocolate into quarters color in 25 times 25 times four equals one whole, one hundred". The following arguments seemed to rest on the knowledge that four sub-units comprised the whole: "you've divided the whole by four even if its a decimal", "times it by four you get one", and "both are one quarter, there is four in one whole".

Explanations for matching 0.4 and 2/5. There were three broad categories of explanation used in justifying the matching of 0.4 and 2/5. Explanations based on 100 were observed in seven instances. One example: "two fifths out of 100, 20, there's five twenties, twentieths in 100 or five twos in 10, so um two times that would be four, that's forty". In this example we see the student coordinating unit systems based on 100 and 10.

Other examples: "one fifth of 100 is 20, so two fifths of 100 is 40", and "that (0.4) is 40, and this is 40 as well and this is two fifths and one fifth is 20 into 100, and so two fifths is 40". One explanation was couched in terms of per cent: "one fifth is 20 per cent so times it by two to get two fifths".

The second category of explanation was relationships where one fifth was thought of as two tenths: "two fifths (0.4) 'cause it goes two, four, six, eight, ten", "a fifth as a decimal would be 0.2 but if you times it by two you would have 0.4", and "one fifth would be two, two fifths would be four".

The third category was a more procedural explanation involving the idea that "whatever you do to the top you do to the bottom" or multiplying both numerator and denominator by two to give an equivalent fraction. Eight instances were observed, as for example, "four tenths, bring it down to two fifths", "times that by two halves, if you times it by one it would stay the same", and "times it (2/5) by two halves to give four tenths". One explanation combined both procedural and conceptual elements: "2/5 'cause if you divided 10 into five equal parts, it would be two, and then if you got two fifths, times it by two twos it would be tenths". One explanation involved an overview statement: "they are the same value except they are in different forms".

Follow-up questions. We will examine each of five cases where questions by the interviewer led to modifications to pairings initially selected.

Case 1. This student, Mark, initially made two correct pairs: 0.1 and 1/10, and 0.5 and 1/2, and two incorrect pairs: 2/5 and 0.25, and 0.4 and 1/4. The interviewer began by saying "explain to me why you chose 0.1 and 1/10?"

M: 'Cause that's (tapping the 0.1 card with pen), point one is one tenth. You sort of, you know that...
I: You know that...
M: Yeah.
I: Its just something you know.
M: Yeah.
Her second question: "What about a half and point five?"
M: Well point five is a half of, if there's a one there (pointing to the 0 in the units place) or if you added five onto that it would equal one whole, and half, half and half is one, so...they're the same.
I: Right, okay. Explain to me why you have chosen those two (places 0.4 and 1/4 in front of Mark).
M: They're wrong (reaches for 0.25 card and places it adjacent to 1/4 card).
I: They're wrong. Explain to me why.
M: Out of a hundred, there's, if you divide it equally into four parts, it'd be 25, 25 times four is a hundred, and that's asking for one quarter, one fourth, so and that's 25.

Mark's justifications for his first two choices were adequate though not particularly compelling. Many children found it difficult to express why 1/2 and 0.5 were the same value. Perhaps a reason for this difficulty is because knowledge of these fractions is so basic and fundamental. As ideas become familiar their supporting arguments seem to become less necessary. But clearly the reasons he offered to support his decision to change his third and fourth choices were founded on a solid conceptual basis. The unifying idea was the relationship between 100, 25 and four. The knowledge that he brought forward, which did not feature in the initial selection of pairs, was there nonetheless. This became evident when Mark was asked to explain his reasons for matching up the cards.

Case 2. Jim's initial response was the same as Mark's. He made the following matches to begin with: 0.4 and 1/4, 0.1 and 1/10, 0.25 and 2/5, and 0.5 and 1/2. The interviewer's first question was:

How did you decide that those paired up?

J: ‘Cause that’s one (pointing to 0.1 card) and that’s one tenth (pointing to 1/10 card), and that’s four (tapping 0.4 card) ‘cause its in tenths, and that’s one tenth (pointing to 0.1 card), and that’s four tenths (waving pen back and forth between the 0.4 and 1/4 cards), and that’s two fifths (pointing to 0.25 card) because its um, ‘cause it goes into a hundred, five times (pause), but it goes into it four times, but its, two fifths, and one half (pointing to 0.5 card) is point five, because that’s half of 10.

I: Okay so you just remember those do you (pointing to the 0.5 and 1/2 cards)?

J: Yes.

I: So because there’s a two and a five in the point two five (pointing to the 0.25 card) that’s the same as a two and a five in the fraction (pointing to the 2/5 card)?

J: Yes.

I: And that’s the same with the four and one quarter?

J: Actually... (picks up the 2/5 card and pauses for about three seconds)

I: You can change it if you like.

J: (Silence).

I: Tell me what you’re thinking?

J: (Picks up the 1/4 card and switches it with the 2/5 card) oh yeah, that would be there, and that would be there, because that’s one fourth of a hundred (pointing to the 1/4 card). That’s two fifths (pointing to 0.4 card), because it goes two, four, six, eight, ten (extending fingers on left hand in turn as he utters the number sequence). That’s two fifths, and that’s one fourth (pointing to 1/4 card) because it goes 25, 50, 75, 100.

I: So you are happy with that arrangement now?

J: Yes.

Jim’s incorrect choices seemed to be made on the basis of similarity of figural features of the cards. Perhaps emphasis by the interviewer underscoring these surface similarities, in the case of 0.25 and 2/5, provoked a review of the situation. Perhaps the conviction that 0.4 was four tenths did not transfer readily to the fraction 1/4. Perhaps the verbalisation of 0.25 (two fifths) being related to 100 because “it goes into it four times” set up a dissonance that needed to be resolved. Jim, like Mark, had the necessary concepts to support a sound choice. He was able to rectify his mistake by drawing on his own knowledge. He drew on number sequences to do this. He related two fifths and 0.4 to a number sequence corresponding to a count of fifths in a unit of 10. He related 1/4 and 0.25 by verbalising the sequence of counting by 25s to 100. There were four counts.

Case 3. Carla first selected 2/5 and 1/10 as a pair having the same value, but then decided against it. She then selected 0.5 and 1/2, and 0.1 and 1/10, and provided appropriate explanations. She did not match any other cards, so the interviewer asked: What would two fifths be, in tenths?

C: In tenths that would be...

I: How could you write that as tenths?

C: ...One, um, (pause) four zero, nought point four, oh yes (smiles)

I: Got one?

C: (Picks up 2/5 and 0.4 cards and places them aside).

I: Right, so that was four tenths and that was point four (pointing to cards), why are they the same?

C: Because, if I transferred that into a whole (pointing to 2/5), which I would double five to make a whole, and that means I have to double the two as well, to make four.

I: Uh-huh.

C: And so that’s marking that, sort of, out of ten. And so that would be four tenths, and that’s (pointing to 0.4) done out of ten as well.

I: Oh, okay, so that really is another name for point four is four tenths.

C: Yes.

I: Okay, alright I'll accept that one. Now look at the last one.
C: And I thought there might be a second one, I wasn't sure.
I: Okay, is there any way of working out whether, whether they are the same?
C: Yes, if there is—I'll just put them over here (moves remaining cards in front of her)—there is four 25s in one whole..., in one hundred.
I: Right, So I think you really mean, oh in one hundred, Okay, four 25s in one hundred...
C: Yes. And that's a quarter (pointing to 1/4 card), so that 25 is a quarter of a hundred
I: Right.
C: So that says a quarter, so they both mean the same thing.

Carla seemed to find the interviewer's leading question to be of assistance, since she selected the appropriate card, 0.4, to match 2/5. However her explanation was couched in procedural language akin to the idea of multiplying numerator and denominator by two. What was the whole to which Carla referred? She mentioned the term *whole* twice in this discussion. The other time was when she spoke of four 25s in one whole. In that case she seemed to mean a unit of one hundred. So it is possible that the first time she used this term she was thinking about a unit or whole consisting of 10. Her later reference to "marking that, sort of, out of ten" lends support to the conjecture. Carla was able to link the 1/4 and 0.25 cards by remarking that there were four 25s in 100. Here the 100 scheme was in evidence. Carla seemed to understand the symbol 1/4, and argued that the 0.25 was a quarter of 100, which in turn was a whole unit.

Case 4. Emily gave sound explanations for pairing 0.5 and 1/2, 0.4 and 2/5, and 0.25 and 1/4. She used the 100 scheme for the latter two pairs. For example, 2/5 was 40 out of 100, and 100 was divided by four to give 25, which related to 0.25. Emily did not think 0.1 and 1/10 indicated the same value, to begin with.

I: Now, are they the same?
E: No.
I: Okay, just look at that one for a minute (covers the 1/10 card with her hand). Point one. Can you write point one as a fraction?
E: Yes they are the same actually.
I: Oh, are they?
E: 'Cause you can write point one as a fraction, you write it as one over ten.

The interviewer deliberately focused Emily's attention on the 0.1 card by covering the 1/10 card with her hand. Her prior explanation for 0.5 does not shed much light on her interpretation of 0.1.

I: How did you decide that (0.5) was half of one whole? How do you know that is half of one whole?
E: Because that would usually have a one there (pointing to the zero), but that's point, sort of, minus, no its not minus, its um half of the big number, a whole number
I: So point five is half of a whole, is that what you're telling me?
E: One whole.

In this case it does not seem possible to conclude either way whether Emily did in fact have a meaningful understanding of 0.1, or had interpreted the interviewer's question to indicate that both cards did in fact have the same value.

Case 5. In this case we see the student, Brenda, make adjustments to the initial set of pairs, but there is very little information the interviewer is able to elicit to confirm the revised choices made. However the adjustments were all correct, indicating the possibility that there was relational knowledge which was not forthcoming in the conversation.

Brenda made the following pairs at the beginning: 0.5 and 2/5, 0.1 and 1/10, 0.25 and 1/2, and 0.4 and 1/4. The interviewer asked her why she chose 0.5 and 2/5:

I: Can you remember what you were thinking?
B: Oh, maybe it was (indistinct utterance).
I: Maybe you'd like to change it.
B: I think, ..., I dunno, 'cause that one equals half a whole (pointing to 0.5), and I thought that, that was (pointing to 2/5) one....
I: Okay, is there another card that says half a whole?
B: Oh, here, this one (picks up 1/2 card and places it next to 0.5).
I: Okay, are you happy with that now?
B: Yep.
I: What about this one (pointing to 0.1 and 1/10)? Why did you choose....?
B: Well that's one tenth (pointing to 1/10 card) and there's point one (pointing to 0.1 card).
I: So are they the same?
B: Yep.
I: Okay (points to next pair: 0.25 and 2/5).
B: And, I don't know about this (appears to be looking at 0.4 and 1/4 pair. There is a pause for about three seconds. Brenda picks up the 0.25 card and swaps it with the 0.4 card, so that 0.4 and 2/5 match up, and 0.25 and 1/4 match up). I think that,..., yeah, now they're swapped around.
I: Okay, now why do you think two fifths is the same as point four?
B: Well, 'cause two fifths,... equals, oh point four, I think. 'Cause a quarter equals oh point 25.
I: So you're quite happy with that?
B: Yeah (nodding head).

Discussion

Explanations sixth graders offered for matching up pairs of decimal and common fractions were sought in individual interviews. These explanations revealed a range of different relational connections upon which students' knowledge of fractions and decimals was based. This relational knowledge was closely tied to procedural strategies. A large number of explanations seemed procedural in nature, yet there was evidence of a meaning base present also.

By far the most common mental object used to explain the similarities between common and decimal forms was a 100 scheme. The 100 scheme was observed operating in explanations for all fraction pairs, with the pair 0.1 and 1/10 evoking this explanation the least. A possible explanation for the occurrences of this phenomenon may be found in an earlier task (Task 4), where a 10x10 grid was presented for the student to represent the fraction 0.4. Recent experience of responding to this task may have been remembered. Thus the 100 grid may have served as a context to which both common fraction and decimal fraction symbols were related.

The cases of students who were observed to change their pair choices suggests that opportunities to reflect on a choice, and in addition, to think about how to explain to another individual reasons for such a choice, can have positive outcomes. Students in this study were asked to provide explanations for their choice of pairs as a natural consequence of a clinical interview. Yackel and Cobb (1995) have noted that students who are encouraged to provide explanations to others for their mathematical actions create objects of reflection, both for themselves and for others. In other words they make public their thinking, which can then be tested, accepted, clarified, or queried.

As we have seen, sometimes it can surprise us to learn that a student can make choices based upon superficial task features such as similarities between clusters of digits in flashcard displays, even when they are later found to have quite impressive numerical relations and conceptual schemes for connecting different syntactic systems. However, even when opportunity to review and justify choices is afforded, it is not always possible for students to articulate their reasons for making those choices, as Brenda's case shows. She may well have made lucky guesses on the day, but it is important to consider the possibility that she knew more than she was able to explain. Concluding that a student does not know when in fact he or she does, is a serious problem in assessment and evaluation of student achievement, and indeed in the teaching of mathematics generally. The individual interview is able to reduce this kind of error significantly, but not eliminate it altogether. Opportunities to reflect and justify can sometimes bear little fruit. A number of students in this study who were asked to explain their incorrect choices did not or could not see anything amiss in their responses. It is these students who need further attention. The teacher needs to review with them basic meanings of common fractions and decimal fractions, and interrelationships between them. Physical models such as area grids, and number lines can be useful in this regard. Some experimental work (Case, 1997) that takes children's understanding of per cent as a starting point for instruction in fractions and decimals holds promise.

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Mathematics curriculum change in the northern territory: What do teachers really think?

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Northern Territory secondary teachers' awareness of curriculum change in mathematics and their thoughts on its impact upon teaching and learning are reported. A survey followed by semi-structured, in-depth interviews were used for data collection and analysis was done inductively. Teachers were aware of a number of changes and expressed many concerns about their impact, especially in relation to appropriate in-service opportunities. Discussion of findings in relation to geographical, educational and cultural features of the NT is included.

Background

The Northern Territory, where this study was conducted, is but one of many distinct educational contexts within Australia. During the last thirty-five years, the Northern Territory has experienced considerable change in relation to mathematics curricula and many of these changes have followed similar changes elsewhere in the country and in the world (Moon, 1986). Changes have generally been made without consideration of the unique educational environment of the Northern Territory. Although the education system in the Northern Territory holds much in common with systems in other parts of Australia, it also differs in some important respects. The Northern Territory's relatively large geographic size within Australia (about one sixth of the total land area) along with a relatively small population (170,000) makes access to educational resources difficult for a number of teachers and students. Further, the population size has made it inevitable that many primary teachers and most secondary teachers have received at least part of their education outside the Northern Territory.

In addition to recruitment of teachers from other states or countries, the Northern Territory has traditionally relied on other states for its syllabus documents. Even since self-Government (1979) the Northern Territory has continued to rely on other states, currently using the West Australian primary mathematics syllabus and the South Australian secondary school program. However, a significant fact that distinguishes the Territory from these other states is that about 30% of the population is Aboriginal. A high proportion of these people live in remote communities in which English is a second language. Provision for primary education in these communities is generally bilingual, although some students enrolled are non-Aboriginal. In schools in the towns and cities there are also ESL (English as a Second Language) students because the Northern Territory has many migrants, largely from Asian countries.

While the nature of curriculum change might vary somewhat from one country to another, or one region to another within a country, successful change is linked to the way in which changes are implemented. There is a complex interplay between curriculum change and the need for teachers to reconceptualise the content of teaching and related teaching and assessment processes. "Teachers are the essence of the innovation process and should play a major role in material development and curriculum planning, as well as being involved in the evaluation of the effectiveness of new materials and methods" (d'Ambrosio, 1991; p.84). In the Northern Territory, where curriculum change has generally occurred in response to developments elsewhere nationally and globally, where student and teacher needs differ to elsewhere in the country, and where many teachers received their training outside the Northern Territory, there are numerous questions that could be asked.

This paper reports on a component of a larger study that investigated Northern Territory teachers' and students' perceptions of their own mathematics education experiences in relation to curriculum changes (Jacob, 1997). Specifically, this paper reports on the following two research questions from the larger, more comprehensive study:

1. What past or present curriculum changes are NT secondary mathematics teachers aware of and what do they think about these changes?
2. How do secondary mathematics teachers rate the effectiveness of their pre-service and in-service professional education, particularly in relation to preparation for curriculum change?

The importance of addressing these questions lies in the fact that curriculum changes that have occurred in recent decades in the Northern Territory have never been formally evaluated as to the extent or actual nature of their outcomes.

Three areas of research were useful in formulating this study. The first concerns the ideas of change processes as they apply to education. Teacher experiences with change processes were a focus because others (eg. Elmore, 1995; Hargreaves, 1994) have shown curriculum renewal efforts are dependent upon teachers' attitudes towards the changes and capabilities in implementing them. The second area of research is directly related to the first and concerns the nature and role of teacher professional development (eg. Taylor, 1986; Hunter, 1985; Greenes, 1989). The third area that influenced development of this study was the call for reform in mathematics education that has been pervasive worldwide in the last decade (eg. National Council of Teachers of Mathematics, 1989; Australian Education Council, 1991, 1994). Recent changes in some states in secondary school mathematics programs reflect a growing recognition worldwide of a need to re-examine the content, teaching techniques and assessment practices within mathematics curricula. These are amongst the numerous external influences that have impacted upon mathematics curricula in the Northern Territory.

Method

The study was essentially an *ex post facto* approach (Cohen & Manion, 1980). It was a survey study, based on the use of questionnaires and subsequent in-depth interviews. The choice of research design was influenced by factors such as the historical breadth of the study and the desire to obtain data from a wide range of teachers, including those from remote regions and from a range of years of experience.

A self-completion questionnaire was developed to focus on teachers' mathematics education background, their knowledge of mathematics curricula changes, and their pre-service and in-service experiences. Some questions required response on a Likert-type five-point scale or a Yes/No choice, while others were an open-ended response format inviting the individual to comment more fully on experiences.

The questionnaire was distributed to all 162 secondary mathematics teachers in the Northern Territory. Fifty-three returned completed questionnaires, one third of which were from outside the Darwin area (where most of the NT population is located), and more than 80% of whom received their own education outside the Northern Territoty. Fourteen of these teachers then participated in the follow-up, in-depth, semi-structured interviews. The set of interview questions was not prepared until after examination of the questionnaire responses so that a series of themes developed from this examination could be further investigated in the interviews. Interviews lasted 60-90 minutes. Since the intent was to test the themes developed from a preliminary analysis of the questionnaire responses, it was perceived that the gist of the interviewee's contribution to the discussion, which could be recorded in note form, was of more importance than a word for word transcription. Consequently, a tape recorder was not used for the interviews. Very soon after the interviews had been completed, all the responses were recorded item by item in a consolidated point-form format. This allowed for comparison of responses from each subject for each item included in the interview (for full details of this process, see Jacob, 1997).

Examination of the data was based on an essentially interpretive approach

(Erickson, 1989) rather than quantitative methods. An interpretive approach for data analysis was chosen because it offered avenues to develop insights into the nature and development of descriptive findings. Thus, data collection and analysis proceeded in three stages: (1) obtaining information concerning mathematics education experiences related to curriculum change, (2) analysing the information to identify any relevant themes, and (3) testing these themes through further analyses of data, proceeding through a progressive crystallisation process to reach conclusions that could serve to inform future changes and future research.

Results and Discussion

Awareness of Curriculum Change

The questionnaire results indicated that most teachers were aware of a variety of aspects of a number of mathematics curriculum changes, the most prominent being: various aspects of 'new maths', increased use of investigations in mathematics teaching, increased use of calculators, introduction of the end of Year 10 examinations, and new courses in Years 11 and 12. The last of these two points are particular to the Northern Territory, although other states in Australia have also recently developed new Years 11 and 12 courses.

In reference to teachers' thoughts about curriculum changes, views were expressed that tended to support many of the changes, particularly those that resulted in students developing a greater understanding of mathematical concepts. The general picture created by questionnaire responses was that change in itself was often good. However, there were also a number of criticisms directed at changes. Highlighted in this regard were the short time lines associated with some changes, the disadvantaging of particular sub-groups of students (for example, the less able or more able students, or students with English as a second language), and lack of appropriate resources to support changes.

The five major areas of change that teachers and students expressed awareness of in the questionnaires were examined further in the interviews. A short discussion of each follows, with comments where appropriate from both the questionnaire and interviews used to support and elaborate more general summaries.

New maths: Most of the comments arising in relation to 'new maths' curriculum changes were negative in nature, and were mainly directed at three areas: (1) the fact that changes resulted in the inclusion of topics thought to be largely irrelevant for a majority of students (eg. set theory and calculations in binary and other number bases), (2) communication difficulties that resulted from the language changes introduced, and (3) the inadequacy of teacher preparation for what were perceived to be dramatic changes.

The teachers concerned with teaching higher level mathematics were most supportive of the changes introduced in the new maths. However, in general it seemed that teachers felt there had been unnecessarily sweeping changes that created barriers to implementing changes effectively (such as the language referred to earlier). They expressed beliefs that more gradually developed change processes might have been better received and might have been more effective in the longer term.

Mathematics investigations: Support for using investigations in teaching mathematics was not extensive. Some of the more experienced specialist teachers felt capable of determining when and how to use investigation approaches in their teaching, while less confident or less qualified teachers expressed uncertainties. This latter group perceived there to be a lack of appropriate professional development in using investigations, and they expressed doubts about students' capabilities to do them, making comments such as: ". . . some very weak students do not have the ability to draw the conclusions expected from them. More able students might be frustrated" (Teacher 12).

Issues raised about difficulties associated with investigations were in the contexts of possible learning disadvantages and the time-effectiveness of this approach. In relation to the first of these points, some teachers expressed concern that students might 'discover' an outcome that is later proven to be incorrect or incomplete, resulting in a sense of confusion or a loss of confidence. Some of the interviewees expressed concern that there were teachers whose competence in using investigation techniques was questionable.

Some of the questionnaire responses indicated a concern that investigation approaches were time consuming to an extent that was disadvantageous for those students wishing to pursue mathematical studies at higher levels. Also, quite a few of the replies indicated that teachers were aware that investigative methods are a valid option but not the only suitable one: "Being used too much. Investigative learning is not a good basis. SOME investigation is GOOD. Talk about it, write about it. But TOO much is a total waste of time. Should be used for a purpose" (Teacher 12). "Is a risk of over doing it. Value of investigation is to engender understanding - it is not an end in itself" (Teacher 39).

Increased use of calculators: Teachers were aware of the increased use of calculators at all levels of mathematics. They had many opinions as to the optimum stage at which calculators should be introduced, but there was a common theme of a need to have some basics, including paper and pencil skills. A prevalent theme of their responses was the concept of the calculator as a tool, along with an awareness of the fact that it is more appropriately used in some situations than in others.

Year 10 examination: The Year 10 examination was introduced into Northern Territory schools in 1989. Some teachers saw it merely as another assessment item (the result contributes a maximum 30% of the final Year 10 mark which is recorded on the Junior Secondary School Certificate or JSSC), while others saw it as quite unnecessary and disruptive. None of the teachers provided a wholeheartedly positive response to questions concerning whether this examination tests effectively the aims and objectives of the current junior secondary mathematics courses. Their main criticisms were directed at the fact that only a relatively small number of students leave formal education without any certification beyond the JSSC, and so the educational reasons for having an examination at this stage of school are unclear. Further, the examination results are not used in determining an appropriate choice for a student's Years 11 and 12 mathematics courses.

New courses in Years 11 and 12: Among the most recent changes in Northern Territory secondary schools has been the introduction of new courses for students in the post-Year 10 (ie. post-compulsory) stages of education. Teachers did not feel they had been sufficiently informed and prepared to implement the new courses. More experienced and qualified teachers appeared to have coped better with the changes, as they felt competent to use their own initiative to make any adjustments they felt would be advantageous for their students. They were still in most cases critical of the way in which the courses were introduced, indicating that "Clear information was not available soon enough nor enough in-servicing" (Teacher 23) and they were " . . . not given enough time to write courses" (Teacher 22). This latter comment was made by a teacher involved with distance education. It is of significance in the context of the Northern Territory because distance education is an essential and significant component of the Northern Territory education system. Since resources and support mechanisms for distance education students must often be developed from scratch, it is not surprising that teachers responsible for these developments felt extreme time pressures when new courses were introduced. Clearly the inclusion of distance education within the Northern Territory education system warrants special consideration when curriculum changes are introduced.

Additional Thoughts about Curriculum Change

When the questionnaire and interview data were analysed, it became clear that as teachers identified areas of curriculum change of which they were aware, they had a range of concerns supplementary to those related to these areas. These included concerns about: the rationale for changes, the ways changes were introduced, the impact of some of the changes, and the impact upon their own self as a professional.

Rationale for and implementation of changes: The rationale underlying change was an issue raised in many of the questionnaire responses. Some responses indicated a high level of cynicism towards curriculum change. Others indicated acceptance of the need for change, while also being critical of some ways it had been introduced. For example, the following comment from a teacher reflects both positive and negative reactions to changes:

> Has been a lot of change and it comes faster. We have to have tools to cope with this increasing speed. Ask 'How is it affecting you?' Have been disappointed by invalid change (eg. Year 10 exam). Who uses a JSSC? Minority of change has resulted in growth. If suitably prepared and professionally oriented, change is no threat. (Teacher 12)

In relation to these concerns, the following main points arose from examination of questionnaire data:

- pilot schemes or trials were not used or were not extensive;
- adequate preparation to teach for the changes was not offered to teachers;
- experiences of other States who had introduced changes were ignored so mistakes were repeated;
- changes occurred after an interval too short to permit valid evaluation of the previous change.

Examination of whether change is meeting the presumed purpose of being beneficial to all students was a theme of the interview questions and resulted in comments such as: "Year 10 exam was externally imposed and not beneficial. Change for change's sake. Now more analysis. More trialing needed. Imposing change is ineffective. Change people's values etc first" (Teacher 27). "Needs maybe to be more selective so that the disadvantaged benefit without removing advantage from the rest. Change should be gradual. Difficult to teach something which you feel uncomfortable with" (Teacher 48).

Change was generally seen as being imposed from outside, in the absence in many cases of prior discussion, trialing or professional development, as is reflected in the following comments: "Developments have been well researched (not Year 10 exam) but method of introduction has not been very professional or supportive" (Teacher 33). "Well meaning but sometimes lacking in planning, training and resource allocation" (Teacher 2).

Impact of prescribed teaching and assessment strategies: The teachers' comments tended to support the notion (particularly in relation to investigations) that prescribing a teaching strategy can be counter-productive and could disadvantage some students. It seemed clear that those teachers who were adequately prepared mathematically and pedagogically felt they carried out their job in a highly professional manner. This included using whatever strategies they perceived to be the most suitable for the particular situation and the students involved. This seems to imply that provision of professional development for those teachers less confident or less prepared mathematically would be more appropriate than the prescription of teaching strategies which might not even be within an individual's current repertoire. They would then be able to select a teaching/learning strategy appropriate for the student(s), the situation, and the specific ideas to be learned.

Increased interest or disillusionment : Many of the teachers expressed increased interest in and enjoyment of mathematics as a result of more recent curriculum changes, saying things such as: "I enjoy teaching! The interactive nature of today's maths education means far more relating to students, discussions about content, context, historical bases, philosophy, new ideas and always opportunities to reflect on and evaluate the above. I love it!" (Teacher 38).

It was clear from others' comments that not everyone shared the same joy and satisfaction. Instead, they saw weaknesses in the approaches or had become disillusioned with them. Examples of comments made in this regard are: "I think new Maths teaching methods don't allow for a full understanding of the subject, too fragmental" (Teacher 24). "Senior Maths classes have little background at Maths due to new courses diluting the difficulty level. Even junior classes often see the hands on as 'playing', even though it does help them with concepts" (Teacher 25).

Teaching to a diverse student population: Amongst the wide range of factors associated with a diverse student population, including language, cultural, social, economic, ability and attitudinal factors, what teachers commented on most were the difficulties that arise in catering for the least able and most able students. Many felt that when resources are limited, more effort is put into assisting students of lower ability, putting at risk the full development of the more able students. They felt that making mathematics more accessible to all students had resulted in a loss of mathematical rigour, and an injustice for more capable students. They expressed concerns that more capable students were overlooked, it being assumed that they had the ability to achieve with or without challenge or assistance. For example, their comments included the following: "We have in many cases disadvantaged the more able students. Assumption that bright ones can continue unaided" (Teacher 22).

Amongst the other comments expressing the teachers' thoughts on the issue of high and low achievers were the following: "Should encourage high achievers. This is not elitism. All should have equality of opportunity" (Teacher 33). "Anyone who enjoys and has potential should be facilitated. Resources are wasted because the system is aimed at mediocrity. Very weak students need a more integrated program. System cannot handle the variations" (Teacher 1). These issues were discussed at the interviews and can be summarised as:

- some schools have a policy of attempting to encourage equity by ignoring the specific needs of those talented in a particular academic area;
- some teachers felt that the term elitism is used inappropriately, since assisting high achievers should be an integral part of assisting all students;
- being innately able does not prevent poor teaching from reducing a student's achievement;

With few exceptions, all people interviewed regarded it as important that the more mathematically able students should receive an effective level of teaching but, just as their needs should not be disregarded, neither should they benefit at the expense of their less able peers. These points once more present a picture of the pressure that teachers work under in trying to use limited resources to meet the very diverse needs of their students.

Pre-Service and In-Service Experiences

Teachers in general seemed to find their pre-service training of limited value and they stated that later experiences had been more helpful and influential in their teaching. They appreciated that professional development activities can and should be undertaken within schools and professional groups, but also felt strongly that the Department of Education has a major role to play. This was seen as being particularly true when major changes occur and it was suggested by some that a more effective use of the resources of the Northern Territory University could be beneficial. Those whose preparation for teaching did not equip them to teach mathematics felt more so than others that in-service professional development had been inadequate in meeting their needs. Continuing professional development was therefore seen as the key component of any change efforts. That is, to ensure curriculum change is effective, professional development must be provided: "If department wants this done properly then in-service activities must be done" (Teacher 27). There was also a perception that failure to provide necessary assistance in situations of curriculum change could have a negative effect on students' confidence and capabilities.

Teachers employed in the larger urban schools generally felt the assistance they received from colleagues could go a long way to supporting their professional development. Those employed in smaller or more remote schools lack this opportunity, in addition to the other disadvantages of their remoteness. Since a possible consequence could be that their students might be offered a less adequate mathematics education than those in the urban schools, the long term disadvantages to these students could have a social cost greater than the monetary cost required to offer the appropriate professional development activities and resource materials.

The question of the time interval between a decision to introduce a change and the actual date of full implementation was an issue that was often raised by the teachers. If this interval is of a suitable duration, there is time for adequate Departmental support, and time for teachers to develop networks, between as well as within schools, to provide the necessary opportunities to update knowledge and resources.

Conclusions

In summary, it can be said that although in general teachers welcomed change and saw it as both necessary and appropriate, they were critical of the manner in which change was often introduced: lack of consultation with teachers, inadequate preparation of teachers, and absence of appropriate evaluation or modification of changes. Criticism was also aimed at changes made without the decision makers providing evidence of likely improvement in educational outcomes, but rather, in relation to political issues.

This study raises a number of noteworthy issues related to effective curriculum change. What is of most significance from these findings are those issues which the key participants in change, the teachers, saw as essential factors in any changes: the opportunity for them to participate in change related decisions at system as well as classroom levels, the gradual introduction of change supported by appropriate and ongoing professional development activities, and the need to adapt changes to meet the needs of students in diverse language, cultural, social, economic and geographical situations. Finally, a major issue that emerged in this study's findings that is in need of much further study because it questions the actual impact of any changes on student learning, is the fact that teachers expressed strong opinions that the most important factor related to student learning was the personality and personal teaching style of the teacher.

In so far as there will always be a time delay, it seems important to ensure that when new programs are implemented they have been subject to rigorous prior scrutiny through trialing or some similar process. It is also important to ensure they are implemented by teachers who are appropriately prepared. Secondary teachers in the Northern Territory do not perceive either of these points as having occurred. It is therefore important that a continuing evaluation process be conducted to ensure that gradual change to adapt to changing needs replaces the previously experienced radical change in a climate of under preparedness.

Above all, given the importance of mathematics and the fact that it is unlikely that in the near future the Department of Education of the Northern Territory, in common with other education authorities, will be able to recruit sufficient appropriately trained mathematics teachers, acceptance of a need for continuing and thorough professional development at all levels is an issue which must be given adequate consideration.

In a more general sense, the findings of this study have importance internationally for the insight they can offer into effects of curriculum change as they pertain to a variety of educational and cultural situations, be that between one country and another, or within a single country composed of diverse educational and cultural needs. More specifically, it is clear that curriculum changes cannot be transferred in a 'ready-made' format from one educational context to another. Although this is not a 'new' claim within the research literature on educational change, it is highlighted here in a context previously under examined - that of the Northern Territory. This study highlights specific needs within the Northern Territory for provision of professional development of secondary mathematics teachers with little background preparation to teach mathematics, or who work in remote regions where there is little opportunity for professional development interactions with colleagues. Any curriculum changes in the Northern Territory should therefore also involve more extensive consultation with 'local' educators, and a more feasible time frame in which to educate for changes in philosophy, teaching or assessment methods. Resources made available to support networking of teachers, or opportunity for further in-service programs with the Department of Education or the Northern Territory University, would also be beneficial in assisting isolated teachers in their professional development, particularly in relation to curriculum changes.

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The effective use of calculators in the teaching of numeracy

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Current debates about numeracy in the UK focus on the availability of calculators but although standards of numeracy are poor, they have not changed significantly since pre-calculator days. It is the way numeracy is taught which is the problem, with or without calculators. Effective use of calculators should be associated with the concurrent development of students' number sense and mental methods. Calculators can enhance the teaching and learning of mathematics if used as a focus for discussion and reflection on learning.

Definitions of numeracy vary from the "mathematical literacy" of the Crowther report (1959) to the more mundane emphasis on basic arithmetical computation implied by current UK government proposals (Reynolds, 1998). To be numerate suggests both an ability to cope with the practical mathematical demands of everyday life and also an "at-homeness" with number (Cockcroft, 1982) which requires an understanding of the mathematical potential of situations. Numeracy thus demands not only a familiarity with number facts, associated processes and an appreciation of the interconnections between them but also an awareness of the power of number and a predilection to seek numerical explanations to situations.

The Mathematical Association (1992) suggested that a numerate person has "the ability to solve simple everyday problems involving number, by using effectively the knowledge and skills that they possess" and that this effective use should include being able to choose and to devise appropriate strategies for calculations (ibid, p71). Numeracy here requires both mathematical knowledge and skills and, in addition, an awareness of this knowledge base so that effective choices can be made. The choice of an effective strategy for a problem is dependent not only on the knowledge which has been learned but also on one's awareness of that knowledge and the realisation that its use would be appropriate. To devise a strategy requires confidence, an at-homeness maybe, and a view of mathematics as a subject in which students can create their own methods.

To be numerate then is not merely to have secure knowledge of numerical facts and processes, numeracy also includes the capability and disposition to construct personal approaches to the solution of problems which are appropriate to the context and are based on knowledge of individual strengths and weaknesses. To be numerate is to be able to mathematize situations using techniques and processes which are confidently known to generate a secure answer. Numeracy therefore involves an interaction between mathematical facts, mathematical processes, metacognitive self knowledge and affective aspects of mind including self confidence and a disposition to construct personal methods.

The role of the calculator with respect to numeracy is contentious. Girling (1977) equated numeracy with the ability to use a calculator sensibly but the Numeracy Task Force suggests that "numerate primary pupils should: ... recognise when it is not appropriate to use a calculator" (Reynolds, 1998, p7). Girling's suggestion implies that the use of a calculator is but one strategy among many, to be chosen or not as considered appropriate for the task. However, the assertion that there are occasions when it is not appropriate to use a calculator implies that in some way calculators can impede mathematical development and that to labour over pages of arithmetical calculations is intrinsically good - an aspect of mathematical character building!

Mathematics teaching in the UK seems poised to return to the technological dark ages. The use of calculators is to be "discouraged as far as possible" until the start of secondary education (Reynolds 1998, p 25) and availability heavily restricted in examinations . At the same time working parties have been established and targets set to improve students' standards of numeracy. The assumption underlying such a stance is that the use of calculators leads inevitably to a lack of mental fluency and a decline in basic arithmetic skills. However, the research evidence suggests that this is not the case (eg: Shuard et al, 1991; Hembree & Dessart, 1992; Groves, 1994; Beaton, 1996).

The impact of calculators on mathematical development

A recent research study (Jones & Tanner, 1997) explored the effects of calculator use on the basic arithmetic skills of Welsh secondary students. A questionnaire sent to all secondary schools in Wales probed the ways in which calculators were used in mathematics. 126 questionnaires were returned (63%) and were analysed to examine the mode of calculator use by Year 7 (12 year old) students. Three groupings emerged:

1 - *Available*	Calculators were allowed in the majority of lessons and their use was not discouraged.
2 - *Discouraged*	Calculators were restricted to certain topics, and their use was discouraged.
3 - *Restricted*	Calculators were restricted to certain topics, and their use was not discouraged.

The distribution of schools across the groups (1-3) was in the ratio 4 : 4 : 3. A stratified sample of eleven schools was then taken to reflect this distribution. From the national league tables of examination success at age 16 the median and interquartile points were determined. Schools close to these points were then randomly selected to reflect the proportions of the groups.

A test of basic arithmetic skills was developed which included questions similar to those used in studies which pre-dated the widespread use of calculators in schools (APU: Foxman, 1985; CSMS: Hart, 1981). Calculators were not allowed in the test. A short questionnaire at the start of the paper probed students' attitudes towards calculators. The test was given to all students (n>1500) in the selected schools at the start of Year 8.

Comparison of basic skills: There were three sections to the test: number, decimals and fractions. Overall the response rates for the number and decimals sections were comparable with the previous studies but facilities were worse in the fractions section. Data from the questionnaire suggest that, in line with the National Curriculum for England and Wales, fractions are no longer taught extensively at this age.

Impact of mode of calculator use on test scores: If calculators do have a detrimental effect on arithmetic skills then students from group 2 might have been expected to outperform those from group 1, with group 3 in an intermediate position. Alternatively, if calculators have a beneficial effect then group 1 should outperform the others. Contrary to all such conjectures, no significant differences were found between the groups. The extent to which the use of calculators was restricted or discouraged had no significant impact on the students' basic skills (F probability = .82)

Analysis of test score by mode of calculator use (ANOVA)

Group	No. of students	Mean	S.D.	S.E.
1	562	27.77	16.67	.70
2	483	28.40	15.32	.70
3	500	28.02	16.23	.73
Total	1545	28.05	16.11	.41

The groupings were derived from the description by the head of department of the mode of calculator use in year 7. However it is possible that variations occurred between individual classes. To explore this from the students' perspective, responses to statements on the students' attitude questionnaire were analysed. Test scores were compared according to the extent to which the students considered that they used calculators in mathematics. The students who claimed to use calculators outperformed those who did not.

T-test on test score for modes of calculator use identified by students
Statement: I use a calculator in most of my maths lessons

Variable	No of cases	Mean	SD	SE
Agree	380	31.55	17.21	.88
Disagree	922	26.68	15.60	.51

Mean Difference = 4.87 2-tailed significance = .000

Thus, although the mode of calculator use as identified by the head of department was associated with no significant difference in the students' test scores, the students who reported that they used calculators performed better than students who did not. Two possible explanations are offered. It could be that only the more able students are allowed to use calculators in their mathematics lessons or alternatively, use of calculators has a beneficial impact on students' arithmetical skills. The direction of causality can not be inferred from the data. A similar correlation based on analysis of data from the Third International Mathematics and Science Survey is reported in SCAA (1997 p13).

It is also worth noting that only a modest percentage of students reported a free or frequent use of calculators (23% and 41%). This supports the data from the heads of departments and contradicts the impression of uncontrolled access given by the media.

Impact of calculator use on students' misconceptions: The use of calculators might have been expected to produce an improvement in students' understanding of "reversals", - the non-commutativity of subtraction and division, but mode of calculator use identified by the head of department made no significant difference to the students' responses. This suggests that the potential offered by calculators to enhance students' understanding of mathematical structure and symbolisation was not always being used to best effect.

Mental arithmetic: Only half of the heads of departments reported mental arithmetic practice to be a regular feature in their year 7 classes and this frequency of practice declined sharply after year 8. There was no significant difference in the frequency of practice by the mode of calculator use. This suggests that the increased use of calculators is not associated with a parallel increase in emphasis on students' mental strategies.

The mode of calculator use identified by the heads of department had no significant impact on students' basic skills although those students who reported free and frequent use of calculators did significantly better in the test. The survey identified the availability of calculators but did not indicate how they were being used. It would appear then that the issue is not whether to allow the use of calculators but how best to use them in combination with other strategies to improve the teaching and learning of mathematics.

The effective use of calculators

Similar conclusions have been reached by other studies. A meta-review of research into the effects of calculator use (Hembree and Dessart, 1992) concluded that, from the weight of evidence available, calculators enhanced the learning and performance of arithmetical concepts and skills, problem solving, and the attitudes of students. They suggested that future research should focus on ways to implement and integrate the calculator into the mathematics curriculum. Similarly in the UK it would appear that guidance is needed on how best to use calculators:

> Only a very small proportion of schools submitted detailed policies that offered teachers clear guidance about the principles, purposes and practice of calculators use ..." (SCAA, 1997 p10).

In our Welsh study, no variation was found between the different modes of calculator use and the practice of mental arithmetic. In contrast, a common feature of the research and development projects which allowed unrestricted use of calculators at primary level has been the emphasis on the development of students' mental strategies and "number sense" alongside the availability of the calculator. The Calculator Aware Number (CAN) project (Shuard et al, 1991) required the children to explore "how numbers work", to develop their mathematical language and their confidence in talking about numbers. The project also emphasised the importance of mental calculation and the children were encouraged to share their methods with others. Through this CAN found that the children developed a wide range of strategies for computations without a calculator. Instead of replacing mathematical thinking, the calculator gradually came to be used as a source of ideas and starting points. Classroom talk emerged as a crucial feature through which children shared, extended and elaborated their ideas.

Similarly, the Calculators in Primary Mathematics (CPM) project (Groves, 1994) aimed to develop a learning environment which supported the development of "number sense". Teachers considered that they made more extensive use of discussion and the sharing of children's ideas. The project children did not become reliant on calculators, rather they outperformed others on a range of estimation and computation tasks.

There is little evidence to suggest that calculators are detrimental to mathematical development. What does emerge from the research is that the role of the teacher is crucial. Where teachers had training and support in ways to use calculators through their involvement in research projects they placed a greater emphasis on the development of students' own strategies and mental methods through the encouragement of classroom discussion. But why should this enhance learning?

Articulation

There are four major reasons why numeracy might be developed through articulation and discussion of students' own mental methods. They are associated with constructing knowledge and testing its viability; the development of corporate meaning; the development of reflective awareness or metacognitive knowledge; and the development of a disposition to construct.

The processes involved in the construction of knowledge suggest that opportunities should be presented for students to articulate their tentative constructions, to test them for viability against corporate knowledge. However, articulation does more than provide an opportunity for the student to test for viability against corporate meaning, it also contributes to the generation of corporate meaning by providing a further opportunity for construal to other members of the group. During the social process of negotiation, individual subjects arbitrate between rival construals of corporate meaning, using a continual process of representation and reflected thought (Clarke, 1994). The re-presentation and articulation of mathematical thoughts and explanations can thus be a significant factor in self generated learning as re-presentations may be combined in hypothetical situations to generate thought experiments such as: "What would happen if...?". Thought experiments can be a powerful tool for learning and have been seen to be within the capabilities of school students (Brown & Walter, 1992).

Articulation is also significant in the objectification of explanation. The act of verbalisation is associated with bringing the subconscious into the conscious and hence the development of reflective awareness and conscious control (Prawat, 1989). This is characteristic of the metacognitive knowledge required for higher levels of thinking.

> When students begin to consider the adequacy of an explanation for others rather than simply for themselves, the explanation itself becomes the explicit object of discourse (Yackel & Cobb, 1995, p269).

To control a mental function, Vygotsky (1962) claims that a student must be conscious of it, but suggests that unconscious self regulation should precede conscious self regulation, presumably appearing first on the social level between people (interpsychological) and then inside the child (intrapsychological) in an unconscious form (cf: Vygotsky, 1978). The shift to reflective awareness and deliberate control of cognition would then be achieved through a transition to "verbalised self observation" which denotes "a beginning process of generalisation of the inner forms of activity". This is a shift to a higher type of inner activity opening up new ways of seeing things and new possibilities for handling them (Vygotsky, 1962, p91).

> In perceiving some of our own acts in a generalising fashion, we isolate them from our total mental activity and are thus enabled to focus on this process as such and to enter into a new relationship with it. In this way, becoming conscious of our operations and viewing each as a process of a certain kind... leads to their master. (Vygotsky, 1962, p91-92)

When reflective discourse is encouraged within a classroom, teachers can be pro-active in encouraging construction, focusing the attention of students on significant aspects of the discourse for collective reflection. "What was previously done in action can become an explicit topic of conversation" and thus "participation in this type of discourse constitutes conditions for the possibility of mathematical learning" (Cobb et al, 1997). The social character of the discourse may be arranged to lend social status to "the disposition to meaning construction activities" which is a "habit of thought" that can be learned (Resnick, 1988).

The significance of articulation or verbalisation in making processes explicitly available as objects of thought is confirmed by research. Several studies have included articulation by students as significant aspects of their approach (eg: Gray, 1991; Wheatley, 1992; Cobb et al, 1992; Tanner & Jones, 1994; 1995; Galbraith, 1995).

Cobb et al (1992) describe a quasi-experiment involving second grade students following a course guided by a constructivist view of learning which facilitated mathematical dialogue between students and emphasised mathematical argumentation. The teacher's role included focusing group attention on and implicitly legitimizing aspects of students' explanations and redescribing them in more sophisticated terms. The teacher summarised parts which were thought to be shared and drew attention to critical points which were not yet understood.

Experimental classes and control classes gained equivalent scores in questions of an instrumental nature, whereas the experimental classes were significantly better in questions of a relational nature. It is claimed that the teaching approach facilitated the construction by students of "increasingly conceptually sophisticated operations". Furthermore it is claimed that students' "fundamental beliefs about mathematics and themselves as learners" were influenced and the development of "intellectual autonomy" was encouraged whereas the usual course fostered "intellectual heteronomy". It is suggested that, for the control students understanding meant following procedural instructions, whereas for the experimental students understanding had a "more extensive conceptual aspect" (Cobb et al 1992, p498-500). The development of numeracy in the sense of a capability and disposition to construct personal approaches to problems and individual mental methods is likely to require such an attitude.

Wheatley (1992) reports the success of third grade students following a course which demanded that students study problems together in small groups reaching consensus through negotiation before reporting back to the class in the "sharing time". In the sharing time, the class was obliged to try to understand explanations, perhaps requesting clarification and initiating negotiations chaired by the teacher. In this way students learn how to discuss within themselves and in the process of telling others how they think about a problem, they refine their thinking and deepen their understanding.

The discourse between students and teachers should involve a form of flexible scaffolding based on the selective emphasis of children's verbalisations and if

> ... the dialogue between the pupils and with the teacher involves listening and sense making by all participants then we expect the dialogue to include within it the intertwining of more and less sophisticated conceptions, formulations and explanations as each tries to make sense of the other. (Williams, 1997, p13).

The discourse should provide also opportunities for reflection. The teacher should manage the interplay of social norms and patterns of interaction to create opportunities for students to reason for themselves and "engage in reflective thinking or reflective abstraction". These two processes are exemplified by "conceptual reorganisations which occur while experiencing perturbations which are initiated by another student's reaction to their reasoning" and also motivated by a desire to participate in a discussion, students may reflect on and evaluate another's thinking and reasoning (Wood, 1996, 102-103).

The effectiveness of flexible scaffolding and reflective discourse for metacognition and mathematical development in the secondary school was demonstrated in The Mathematical Thinking Skills Project (Tanner, 1997). The most successful teachers utilised approaches based on a form of dynamic scaffolding. This required participation in a discourse in which differences of perspective were encouraged and group attention was focused by the teacher on useful constructions suggested by students.

When this sort of dynamic scaffolding was used in combination with techniques to develop a reflective discourse, for example in a group reporting and peer assessment session, accelerated mathematical development was observed. In reflective discourse students focus on the processes of mathematisation, abstraction and generalisation in the service of understanding structure, thereby encouraging the development of a mathematical disposition (Tanner, 1997).

Conclusion

To be numerate in the sense advocated in this paper requires students to develop a mathematical disposition. Students must be willing to construct individual solutions to problems based on their own self-knowledge, as well as having a secure understanding of number facts and processes. The development of numeracy is not determined by the extent to which calculators are used. Under certain conditions the use of calculators has been shown to enhance students' numeracy. However, allowing the use of calculators is not sufficient in itself and it certainly does not remove the need for good teaching. The research reviewed here indicates the importance of articulation, argumentation and dynamic scaffolding within a reflective discourse for the development of students' mathematical and numerical abilities. The metacognitive knowledge and skills developed during reflective discourse are necessary for the mathematisation of situations and the identification of strategies which are confidently known. When learned in this way, mathematics should also be perceived by students as a subject in which construction of their own methods is not only possible but required.

It is conjectured here that a crucial feature of effective numeracy teaching is likely to be the use of dynamic scaffolding within a reflective discourse. Calculators can be used as a stimulus to provoke mathematical argumentation within such a discourse. If calculators are made freely available to students to check answers obtained through the use of their own mental methods, they may develop the self confidence to articulate their strategies and challenge those of others in public debate. If calculators are used as a focus for such a reflective discourse, they might be expected to lead to the development of numeracy in the wider sense intended in this paper.

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New knowledge/new teachers/new times: How processes of subjectification undermine the implementation of investigatory approaches to teaching mathematics.

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The research presented in this paper confronts head on the difficult question of teacher change. It was carried out with preservice teachers in a mathematics methods course in the second year of their teacher education program. Collected data reveal how students' prior experiences of institutionalised mathematics reveal patterns of subjectification which actively undermine the future implementation of more investigatory methods of teaching. I use the poststructuralist concepts of knowledge, positioning and subjectivity: initially as analytic tools to expose the coercive operation of the mathematics discourse; and collectively, as a conceptual base from which to think about possibilities for change.

Introduction

One does not have to go far to find policy and curriculum documents in mathematics education (*A National Statement on Mathematics for Australian Schools*, 1990; *Curriculum and Evaluation Standards for School Mathematics*, 1989; *Mathematics Counts*, 1982) extolling the virtues of investigatory or inquiry methods of teaching mathematics. This is in keeping with a view of mathematical knowledge as an active personal construction, where learning refers to "the creation or building up of relationships in the mind of the individual" (*A National Statement on Mathematics for Australian Schools*, 1990, p.17). However, the changes in teaching methods required have not impacted classrooms as readily as many would have hoped. Price and Loewenberg-Ball (1997) believe that any changes have been mainly cosmetic and what students learn and how they learn it remains largely unaltered.

Despite all the talk of mathematical empowerment, of students and teachers together searching for mathematical patterns and connections and communicating these with confidence to enhance and develop mathematical ideas, teachers continue to teach largely as they were taught (Foss and Kleinsasser, 1996). It appears that they hold on tenaciously, even though they may not admit it (Schuck, 1996), to a view of mathematical knowledge as facts, skills, rules and procedures to be transmitted, and to the absolute authority of teacher and text. Any creative and investigative impulses are negated when confronted by established notions of mathematical knowledge and how it should be taught.

In this paper I use a poststructuralist view of knowledge to theorise the difficulties preservice teachers experience in attempting to implement inquiry or investigative approaches when teaching mathematics. The interconnected concepts of power/positioning, knowledge/storylines and subjectivity are used as analytic tools to explore the processes of subjectification experienced by five preservice teachers in institutionalised mathematics classes. My aim is not to reject or replace prior views of knowledge, but to overlay and deepen these with a view of knowledge which explains how preservice teachers are made subjects such that they *know* at an intuitive, or visceral level, what mathematics is and how it should be taught. This has implications for future teaching practice because the poststructuralist conception of knowledge holds that "in our *action* is our *knowing*" (Lather, 1991, p.xv: my emphasis). Future classroom practices will be based on students' knowing about themselves (their subjectivities) and about mathematics, neither of which may include the investigative processes encouraged in teacher education programs.

From the data presented I attempt to explore both the possibilities and limitations of what might reasonably be accomplished in teacher education, given the processes of subjectification these students have experienced prior to entering the program. They have lived and know an institutionalised discourse of teacher and text as authoritative keepers of mathematical truths and their mathematical dispositions have been constructed within the regulatory power of the mathematical discourse. It is in making this constructedness visible that they may be able to find ways of moving beyond teaching-as-usual to search for alternative strategies that "invert, invent, and break the bonds of existing discourses" (Davies, 1994, p.44). The argument I pursue in this paper is that agentic new teachers for new times, knowing the coercive nature of all discourses, will be enabled to change classroom uses of language and practices which potentially disenfranchise students. That is, in recognising the problems with practice based on competition for the one correct answer, they may initiate more investigatory approaches which should enfranchise more students mathematically.

Subjectification

Mathematics is a discourse which comprises "socially organised frameworks of meaning that define categories and specify domains of what can be said and done" (Burman, 1994, p.2). Relationships of power/knowledge/subjectivity constantly circulate throughout the discourse producing or suppressing numerate behaviour in students. The environment or context for learning is never neutral. This is a central concern of this research.

I have grouped the data collected into three(3) segments which constantly bump into each other and overlap. However, for ease of analysis the data appear under these headings: knowledge/storylines; power/positioning and subjectivity. I will briefly summarise the poststructuralist interpretation of each of these concepts as I use them to analyse the data. Each of the five(5) preservice teachers spoke for one half to one hour on recollections of past experiences in mathematics classrooms. The recollections were privately spoken into a tape recorder and later transcribed. There was no interview as such. The data were collected in 1997.

Knowing Mathematics

Within the discourse of mathematics, students construct what we might identify as mathematical knowledge. Simultaneously, they are themselves constructed by the power laden language and practices of classrooms which authorise and maintain mathematical and pedagogical "truths", or knowledge. From the data below, we can see that the preservice teachers have learned what counts as authoritative mathematical knowledge and they have learned that as learners of mathematics they are collectors or gatherers of information and procedures, reproducers of others' knowledge, rather than creative initiators or seekers of new and personally relevant knowledge.

> **Hashi**: I remember doing lots of tables, we had to learn our tables off by heart. I remember long division, and working out angles. The teacher would get up in front of the class and teach it to us. There was hardly any discovery learning whatsoever, in fact I can't ever think of a time when we discovered a formula by ourselves. In primary school there was only one way to learn something, and that was the teacher's way. And too bad if you didn't understand it. That was bad luck.
> **Alice**: When I was at primary school the basic facts were really, really important. We used to learn them by rote and say them every day. We, ah, had a test on them every day and your mark would go up on a chart, so it was all recorded on the wall, so it was a really competitive structure... I can just remember the teacher would put problems up on the board, for example 54 subtract 25, and just...about 10 problems, 10 um 10 subtraction problems, and then 10 multiplication problems, then 10 division problems and they'd just be all up on the blackboard of a morning, and we'd just have to do them, and that's basically how it was.
> Um, because we could work through the workbooks in high school, we just worked through at our own level. I'd like, finished the curriculum,

> um, in Grade 9, so I didn't have to do mathematics at all in grade 10, and so I was pretty happy, because I had a free subject. But now I look back, and I think that's really bad because for a whole year, I wasn't doing mathematics, whereas I should have been. I mean, just because I'd finished the curriculum. I don't know, I think that they should have given me different work, um like the next level. I don't know. I just find it hard to believe that I didn't do any maths in grade 10 now that I think about it.
>
> **Melody**: I remember tables, pain at night, competition, quizzes, rods, long division, multiplication, testing, setting out properly, "speed" maths. I remember making patterns with the rods at lunch time and the teacher coming in and being very angry that I had been playing with them. I had to pack them all up into their little green cases.
>
> Maths was always right or wrong, it was a tick or a cross. It could never be nebulous.
>
> **Cathy**: I went to school in a small country town in Scotland, and I would say that learning things by memory seemed to be important. We used to learn our times tables off by heart, even in Grade 1. I remember in Grade 1 we used to stand up and recite our times tables, and if we got them wrong, we got rapped over the knuckles...

The students have come to know mathematics as tables, rules, procedures to be transmitted by the teacher and learned by rote. Once you had learned enough of these you could stop, and have spare periods as Alice did. However, it is the gaps and silences in what the students say that signal possible problems ahead for teaching mathematics in "new" times. Silences in these recollections point to the fact that students are not able to speak the "truths" of mathematics as they have been redefined for the twenty-first century. The skills and knowledges they have developed, to whatever extent, are now redundant. They do not mention exploring mathematical patterns and the various connections between mathematical ideas, the pleasures of conjecture and inquiry, the wonder of exploring mathematics from minority cultures or the excitement inherent in using technology to explore mathematical concepts. These preservice teachers have come to know mathematics as transmitted facts and procedures, and as Schuck (1996) says preservice teachers are not much interested in alternative views of knowledge more appropriate to the world of today and tomorrow.

Power/Positioning

Constructivist notions of knowledge, stressing active and collaborative involvement of students in supportive contexts, have recently become very popular in mathematics education (Klein, 1996; 1997). However, the poststructuralist concept of subjectification does not allow the passive/active binary. It recognises that the environment of learning including relationships of power/knowledge is not external to, but constitutive of, all participants. We can see in the following excerpts that the students are actively engaged in learning, even though they are in "transmission" type classrooms. They are learning through how they are positioned by classroom practices, textual and teacher authority.

> **Josh**: I cannot recall one 'math activity' in my years of schooling. It was strictly 'chalk and talk', 'pop quizzes' (lots of those), homework to hand in, and examinations. Each course had a textbook, the rest was pencil and paper, though I faintly remember recitals of tables, in front of the class by students.
>
> Mathematics was entirely direct teaching. Math was like spelling. The object was to get the convention 'right'. I can not recall there being any

> 'remedial' groups in the schools I attended in Indiana. If students didn't 'pass' a grade, they either went to summer school or repeated the grade. This was not uncommon.

The pedagogy was very teacher directed. Students were positioned as "unknowing" and they learned that the teacher and text guarded authoritative "truths" which you had to access to do well. You had to play a certain game, where, as **Josh** stated:

> The storyline was simple as I saw it. We had to go to school. The teacher was the 'knowing' one. Pay attention, do the work, you get a decent grade. Don't pay attention, don't do the work, you flunk. Too simple!

The pedagogy was not questioned. If you did not come up with the correct answers most of the time you were simply not "paying attention" (Josh) and were "rapped over the knuckles" (Cathy). There was no indication in the data of students' being encouraged to behave autonomously, to find their own way, to explore. Melody found that making patterns with the rods was "wrong", Alice experienced mathematics as getting the correct answer to tables and operations and Melody learned the "correct" way of setting out.

Two influential "frameworks of meaning" structure classroom practice and position students as totally dependent upon the teacher. The first is developmental psychology and the notion of the "norm" or "normal" in development and behaviour. Classroom practice is premised on constructed binaries of right/wrong answers, correct/incorrect behaviours, and competent/incompetent students. Those not achieving at what was considered to be the norm were categorised as "developmentally slow" or "slow learners" and grouped accordingly (see Melody below). A second framework of meaning is liberal humanism, which assumes a unitary individual who can choose or not to be motivated and competent in the classroom. Where students fail it is seen to be because of individual personal lack, and nothing to do with oppressive positioning or inappropriate classroom practices. Josh's comments above demonstrate how liberal humanism might impact on his future practice: "Don't pay attention, don't do the work, you flunk".

Melody reflected that she was classified as a "low achiever" and sent outside the classroom with a teacher's aide to pick up some mathematical content she had missed:

> **Melody**: I can remember them saying: 'OK those in the low group go outside'. We would have to go and sit outside and often it was just going over things we had looked at before. I have strong memories of the old gestetner prints, you know how they have that very strong smell, that's the smell I associated with maths most of all. Doing pages and pages of those things that we had to repeat over and over again. And I remember looking in through the windows at the rest of the class, at my friends and wanting to be in there...I remember it as a time of desperately wanting to be somewhere else than where I was.
> **Melody**: I was made to feel that I could do better if I wanted to and it was my choice that I wasn't doing well at maths. I was the one who was responsible for it and all I had to do was to change my attitude. So the problem was with me...

These preservice teachers have been positioned in such a way in classrooms to know absolutely the unquestioned authority of teacher and text. They have come to know that some students are better at mathematics than others: they have come to know that where one fails it is one's own fault, or one's family or culture. These knowledges are not conducive to the implementation of investigatory pedagogies, where each student must be given a speaking voice, enabled to take oneself up as respected and authoritative while developing mathematical knowledges and applications.

Subjectivities

Delving below the surface of the words that the students are speaking, there is a sense in which, by involvement in the discourse, students learn about themselves according to whether or not they are able to establish themselves as empowered mathematical subjects. "Discourses", states Weedon (1987, p.108) "are more than ways of thinking and producing meaning. They constitute the 'nature' of the body, unconscious and conscious mind and emotional life of the subjects which they seek to govern". The students were positioned subserviently to the, usually, nurturing and supportive teacher, and students such as Melody who attempted to create some knowledge for herself immediately found that making patterns was the "wrong" thing to do with the coloured rods. Alice, on the other hand, appears to have been able most often to do the "right" thing, get the right answers so that she has a whole year in which she doesn't have to do mathematics. However she, as much as Melody and the others, relies on the teacher to judge her competence.

Melody, Hashi and Cathy's recollections suggest that do not consider themselves "good" at mathematics. It would appear that many past experiences have not constituted them as capable users of mathematical discourses.

> **Melody**: I felt terrible that I couldn't do better at maths. I couldn't understand why I did so well in every other subject and did so poorly at maths. I remember thinking that it must be a mind set that you're given and that if you don't have that mind set well then you can't do maths...
> those who were really good at maths got to go out early. In high school they got to use the computer...I absolutely hated computing because I saw it as just another part of maths and I never wanted to touch a computer because I just *knew* I would be bad at it because I was bad at maths.
> **Hashi**: I didn't really like maths very much, because I didn't feel that I was very good at it. I didn't feel that I truly understood the concepts of what was being taught, I only knew it on a superficial level, I only rote learned things and consequently, when I got to high school, I had problems doing process questions.
> **Cathy**: I had a maths teacher in Sydney and he was very chauvinistic, and um...it was definitely that boys did better at maths, because I mean like, every time there was a screech of brakes going past the classroom, it was like, you know, 'Bloody women drivers', see, you could always tell exactly how he felt about women and girls in general, you know, that we were a bit silly, and you know, waste of space really, and...so I can't say that I learned anything, it was just, I just wanted to pass the subject, and I did, I got a 'sound', and then I avoided maths at all costs, in everyday situations.

I interpret the data to read that Josh and Alice, on the other hand, have taken themselves up as agentic mathematical beings.

> **Alice**: I really enjoy mathematics. I always have a sort of positive feel about maths.
> **Josh**: I was a 'high' achiever, according to my teachers, but I took the same lessons and tests as all the other students.

It is interesting to speculate on which of these students will be most able to implement investigatory processes in their classrooms. Alice and Josh managed to get most of the answers to the traditional mathematics questions right, but does this necessarily mean that they will be competent to teach the "new" mathematics, which depends on knowledges both mathematical and pedagogical. Indeed, they may not be keen to interrupt a set of knowledges and practices which they consider to have served them well.

The problem is compounded when these preservice teachers are out in schools, as they are not encouraged to try investigative approaches to teaching. As practising teachers, they are positioned once again as "not knowing" and not enabled to learn for themselves from trial and error.

> **Hashi**: I did suggest doing things like um, tell me everything you can about the number 35, but the teacher changed it, and wanted quick mentals done instead, which is fair enough, its his class...
> It's hard to go out in someone's classroom, and um try and introduce constructivism, especially with teachers who are used to 'talk and chalk', and its unfair to walk into their classroom, and to suddenly impose your ideas on them. It's hard too, if the children have never done this sort of thing before, you can't teach it to them in two weeks, they have to I think, learn some basic skills with cooperation and setting limits on their own ideas, and you've got a teacher marking your prac report, it makes...can behaviour. And also, too, if you walk in there and start imposing your own make life very, very uncomfortable for you if you don't abide by their choices, or their beliefs.

Competence and Agency for Preservice Teachers

The major problem we face is that students so constituted through past discourses do not readily know how to position themselves as teachers with agency who know how to teach "against the grain". They have learned some content, but it is content that can nowadays be done more efficiently by machines. Their knowledge and the context of schooling-as-usual make it difficult for them to speak the mathematical knowledges considered relevant (Australian Education Council, 1990: Department of Employment, Education and Training, 1989; National Council of Teachers of Mathematics, 1989) to the twenty-first century. They have not experienced conjecture, exploration and inquiry as important elements of knowing mathematics as a social and intellectual practice. Even where these ways of knowing are encouraged in preservice teacher education they are resisted by students who do not consider them to be important (Schuck, 1996).

A further problem ensues when these teachers begin teaching. The currently established discourses in mathematics texts of problem solving approaches to teaching, sense-making on the part of students and the use of games and concrete materials remains as rhetoric rather than becoming constitutive of practice. I believe this is because these high ideals are incorporated into teaching-mathematics-as-usual based on competition, the one correct answer and teacher authority. Thus problem solving often becomes finding the correct answer as quickly as possible, sense-making means making the teacher's sense for a tick or good marks and technologies such as games and concrete materials are used in standardised ways to practise or consolidate some skill or procedure.

Preservice teachers need to be competent mathematically and they need to be agentic. I concur with the new pedagogical discourses on the construction of mathematical ideas, connections and relationships and recognise as problematic the fact that this discourse may have been absent in their schooling. I also take seriously the notion that they must be encouraged and supported in learning to orchestrate an investigatory discourse; they must develop the skills of questioning which keep the mathematical conversation alive and which do not cut off inquiry by asking closed questions. Just as importantly, to be agentic teachers of change, they must know how the discourse of mathematics currently operates to disenfranchise learners, how they themselves have been caught up in its operations, and how classroom uses of language and practices might be changed in ways that prove to be empowering for more students. Davies (1991. p.51) states: "Agency is never freedom from discursive constitution of self but the capacity to recognise that constitution and to resist, subvert and change the discourses themselves through which one is being constituted". Preservice teacher education is implicated here.

Making a Beginning in Preservice Teacher Education

Within teacher education, as we continue to teach basic concepts in constructivist ways and learn the skills of communicating these ideas, it is crucial that we also make processes of subjectification visible. We need to look at how, in interaction, we collectively manage to categorise and classify each other into marginal or authoritative positions within the mathematics education discourse. Discussion could focus around the following concepts (adapted from Davies, 1994) which together try to encapsulate the kind of context participants actively create for one another: positioning, subject positions made available, and storylines that are made relevant. For example,

> POSITIONING: How are the participants, the lecturer and students, mature aged students and school leavers, positioning each other? Where does authority lie? Are prior experiences made relevant? Are any gender/race/class divisions visible?
>
> SUBJECT POSITION/SUBJECTIVITY: What subject positions are made available? Are students receiving messages that they are "good", "bad", "slow", "dumb", "a competent student", "an unmotivated student", or "remedial" by either the words spoken, looks given, or processes such as briefness of time to answer, relative difficulty of questions asked. What subject positions does the lecturer assume? Is s/he constituted as authority, transmitter of knowledge, nurturer or perhaps uninspiring pedagogue? If the lecturer does not teach the knowledges the students consider essential, s/he may be positioned by students as out of touch with reality and too theoretical.
>
> STORYLINES: What storylines weave themselves through students' experiences of mathematics education subjects? Does this knowledge include such unwanted storylines as: it is important to understand what the lecturer wants so that this can be regurgitated at exam time; males are better at handling the video equipment and the computers; members of minority groups should not be asked challenging questions; problem solving is all about getting to the answer as quickly as possible; lesson planning has to follow a linear format and lesson outcomes must be established before the lesson has taken place?

Once students are aware that power relationships do exist in all pedagogical collaborative encounters, they might be encouraged to examine how taken-for-granted classroom practices constitute children in mathematics.

If these teachers are to teach in investigatory ways, they will need to recraft their eyes to recognise the limitations of teaching-as-usual: to see how filling in worksheets, hearing tables, doing irrelevant problems where there is one correct answer, streaming, practising algorithms and formulae, position the student as always unknowing and the teacher as sole authority. Furthermore, because these classroom practices are all premised on the regulatory establishment and maintenance of constructed binaries of right/wrong answers and competent/incompetent students they effectively undermine any investigative impulse. Preservice teachers need to work together to think about classroom processes that might better facilitate investigation.

Many would argue that it is the large structures of society and the school that need to change to bring about pedagogical and social change. I have argued, as has Davies (1996) that we ignore subjectivity at our peril and that the larger structures will change only when we have our schools staffed with agentic teachers of vision and voice who, with eyes recrafted to recognise potential marginalisation and oppression and agency at the local level, act to ensure a positive learning experience for as many of their pupils as possible. Subjectivity, I would suggest, as well as constructed cognitive knowledge, significantly influences practice.

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Affective factors and the improvement of algebraic problem solving

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Problem solving is an important activity in algebra, as all mathematics, and research has shown that affective factors can be crucial to mathematical progress. This research study considered 105 15 year old students, and investigated the relationship between their affective domain and algebraic problem solving ability. Analysing students performing at different ability levels, our test and interview data have enabled us to describe a preliminary model of the interaction between self-concept, self-perception, enjoyment, interest, motivation, anxiety and problem solving ability, describing how they may be influencing students' learning.

Background

Problem solving is important in mathematics, and since the National Council of Teachers of Mathematics (NCTM, 1989, p. 23) announced that "Problem solving should be the central focus of the mathematics curriculum", the role of affective factors in mathematical problem solving has been the subject of a number of research studies (e.g. Macleod, 1992; Lester & Kroll, 1990; Mendler, 1989). Results indicate that the affective domain plays a crucial role in mathematics achievement, influencing the individual's ability to solve problems. Some research studies (Mendler, 1989; McLeod, 1992) have concentrated on how the affective variables influence mathematical problem solving, while others have considered the role of affective variables such as self-concept, interest, anxiety, self-perception, motivation, enjoyment and usefulness and the relationship between them (e.g. Marsh, 1989; Fennema, 1989; Mitchell, 1993; Skaalvik & Rankin, 1995). Marshal (1989) noted that repeated episodes of errors while solving problems may lead to a sense of frustration, a distrust of the child's own skills, resulting in strengthening or weakening of the nodes formed among the problem situations. The nodes thus formed may play an important role in the conscious control of both cognitive and affective information processing, which, according to Mendler (1984) allows one's meta-cognitive process to direct efforts to find a solution to a problem. The transition from arithmetic to algebra is a difficult passage for many students, and, as McLeod (1989, p. 33) emphasises "evidence gathered from secondary school teachers suggests that some students who have got on well in mathematics in the past will suddenly lose confidence when they have to address a more difficult problem than they are accustomed to". Thus knowledge of the relationship between affective factors and achievement in algebra problem solving would be very helpful for educators seeking to assist students' learning.

This study was based on the hypothesis that a relationship between mathematical problem solving achievement and the affective domain exists, and can be described, and we have previously reported (Kota, 1997; Kota & Thomas, 1997) some gender-based differences in the relationship between affective factors and algebraic problem solving performance during adolescence. More recent analysis of all our data, in which we studied separately students performing at two different levels of ability, has led us to believe that we may have identified how some of the factors we measured might be influencing problem-solving ability. We present the evidence in this paper along with a preliminary model describing the role of certain affective factors in algebraic problem solving.

Method

The data described and analysed in this paper were collected from 105 form 4 students (15 years old) drawn from 7 secondary schools, both single-sex and co-educational, in the Auckland region of New Zealand. One form of average ability students from each of forms 3 and 4 in the schools was randomly selected to form the subject group and the students were taught by their normal specialist mathematics teachers. All the schools were following the recent New Zealand curriculum published by the Ministry of Education, and in particular, its algebra section.

There were two different types of instruments used in this research. Firstly, there was a group of tests designed to measure the seven affective factors, self-concept, self perception, anxiety, interest, enjoyment, usefulness of mathematics and motivation, using self-descriptive Likert scales. Secondly, an instrument, comprising five word problems constructed from three pilot algebra tests, was designed to measure algebraic problem solving ability. Full details of the tests used can be found in Kota & Thomas (1997).

In the first half of 1996, before the algebra syllabus was taught for the year, students from both forms 3 and 4 in the schools were given both questionnaires, on affective factors and algebra, during their regular mathematics classes. They were also given the same two tests at the end of the school year, in November 1996. Only 105 of the form 4 students completed all of the tests and so only the results of these students are analysed here. Ten of the students, representative of the groups identified, were later interviewed for about fifteen minutes, and asked to relate their feelings regarding mathematics and their work in the subject. These interviews, conducted by the first named researcher, were tape recorded and later transcribed.

Results

The algebra test comprised five questions, each assessed on a scale of 1 to 5, enabling a finer measure of the students' progress, both in terms of understanding and correctness, to be obtained, and giving a range of possible scores from 5-25. Using the median score of 15 we divided the students into two distinct groups, those scoring below 15 were designated lower achievers (LA), and the others, higher achievers (HA), on that test. To keep track of which group each student was in on each test we added I for the earlier and II for the later test. Thus, LAII indicates those students below the median in test II. Figure 1 shows that, of the 42 students in LAI, 22 were also in LAII, while 20 students improved, and were in the higher achieving group, HAII. Similarly, of the 63 students in HAI, 54 were also in HAII while 9 were in LAII. We sought to examine, and identify, any changes in the affective factors of the students over the period between the tests and to try to relate this to their algebra performance, outlining any patterns discovered.

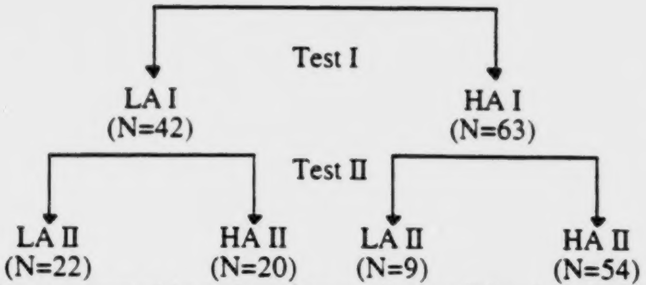

Figure 1: The four outcomes of the students from test I to test II based on their algebra problem solving scores

To accomplish this we used the possible outcomes to compose four groups of students, designated as LA I to LA II, LA I to HA II, HA I to HA II and HA I to LA II.

Table 1 shows a summary of the test results for the students in HA I to LA II (N=9), that is those who did significantly worse in the second algebra test. Although the number in this group is small, and it is difficult to attach weight to the results, the significant decline in their algebra score is matched by a significant decline in 3 of the factors, and weak evidence of significantly lower self-concept and interest, with the remaining factors, usefulness and anxiety, also apparently lower.

Table 1: The affective factor and algebra results for the HA I to LA II group (N=9)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	95.1	12.03	89.6	11.44	1.92	<0.1
Int	69.7	9.45	63.7	11.2	1.48	=0.16
Anx	27.2	4.24	26.7	4.42	0.35	n.s.
SP	22.9	1.81	19.8	3.70	1.98	<0.05
Use	52.0	6.76	50.0	6.44	1.12	n.s.
Mot	50.9	10.39	44.9	9.57	2.31	<0.05
Enj	41.6	8.46	37.2	6.59	2.10	<0.05
Alg	19.0	1.73	11.6	3.21	5.25	<0.001

SC=Self-concept, Int=Interest, Anx=Anxiety, SP=Self-Perception, Use=Usefulness of Mathematics, Mot=Motivation, Enj=Enjoyment, Alg=Algebra Problem Solving

We considered that the above group, which did significantly worse in the algebra test II, would be most closely related to the LA I to LA II group, containing students whose algebra score remained below the median in both tests. Table 2 shows that, for this group too, interest, usefulness and enjoyment declined significantly, with some evidence that anxiety did too. We had also expected to see self-concept, significantly down for this group, but it is important to note that their self-concept, measured at 80.9 in the first test, was already significantly below all the other groups.

Table 2: The affective factor and algebra results for the LA I to LA II group (N=22)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	80.9	12.06	79.0	12.14	0.71	n.s.
Int	65.9	12.79	59.6	8.77	2.89	<0.01
Anx	24.4	3.69	23.2	3.63	1.57	=0.13
SP	19.3	3.45	18.1	3.22	1.31	n.s.
Use	47.2	4.33	43.8	7.33	2.89	<0.01
Mot	41.6	10.84	40.8	10.72	0.51	n.s.
Enj	35.1	8.22	31.9	7.66	3.36	<0.005
Alg	11.6	2.66	11.4	3.06	0.22	n.s.

These two sets of results indicate the possibility that the level of some of these factors might be linked to performance in the algebra test, with both a relatively low, or declining, algebra score, accompanied by a corresponding pattern in the affective factor levels. Any causal link between the two was not apparent at this stage.

If this was the case then, we hypothesised, we might expect the other two groups of students to display a different pattern of results, without this decline.

Table 3: The affective factor and algebra results for the HA I to HA II group (N=54)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	93.4	11.12	93.4	13.66	-0.67	n.s.
Int	67.2	12.05	66.4	11.89	0.61	n.s.
Anx	26.4	4.14	27.4	4.14	-1.99	<0.05
SP	20.7	3.12	21.3	3.46	-1.04	n.s.
Use	47.8	6.60	47.6	6.37	0.37	n.s.
Mot	44.1	10.1	45.9	10.04	-1.65	<0.1
Enj	36.7	8.18	37.0	7.70	-0.42	n.s.
Alg	21.9	2.99	22.0	2.84	-0.2	n.s.

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What did an analysis of the results of those students who either significantly improved their performance, or maintained it at the higher level indicate? The analysis of the test data for the HA I to HA II group in table 3 confirms that, for these higher achieving students, in contrast to the previous two groups, not only was there no significant decline in any of the affective factors, but they are clearly very stable, except for anxiety and motivation; the former having *increased* significantly and weak evidence that the latter had too. It is the final group, LA I to HA II, those who improved significantly in their algebra score, from a mean of 12.25 to 19.55, which in many ways should be the key group. Since one would hope to improve student achievement in algebraic problem solving, these students, who had managed that very well, might hold the key. We see that, again, none of the affective factors shows a decline but all are remarkably stable in their values, with the exception of enjoyment, which fell significantly. This is a surprising, anomalous result which requires further investigation.

Table 4: The affective factor and algebra results for the LA I to HA II group (N=20)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	85.1	13.17	84.1	15.16	0.49	n.s.
Int	64.2	10.05	63.1	11.2	0.72	n.s.
Anx	24.9	3.98	24.6	3.51	0.37	n.s.
SP	19.9	2.86	19.5	2.74	0.99	n.s.
Use	46.7	7.25	46.5	7.08	0.19	n.s.
Mot	45.3	9.81	44.5	10.01	0.77	n.s.
Enj	37.9	7.96	35.4	7.89	2.02	<0.05
Alg	12.3	2.17	19.6	2.30	-9.91	<0.0001

Comparing the factor levels of those students in the LA group at each test (not the same students), with those in the HA group (see table 5), reveals that, for these form 4 (year 10) students, those who are achieving the higher algebra standard have higher affective factors scores. Whilst initially the difference is significant only for self-concept, self-perception and anxiety, at the end of the year, when the HA group had increased in size from 63 to 74, it is significant for all the factors except usefulness and motivation.

Table 5: A comparison of the affective factors for the lower and higher achievers groups

	TEST I				TEST II			
	LA I Mean (N=42)	HA I Mean (N=63)	*t*	*p*	LA I Mean (N=31)	HA I Mean (N=74)	*t*	*p*
SC	82.9	93.6	4.47	<0.001	82.1	90.9	3.11	<0.005
Int	64.8	67.5	1.29	n.s.	60.8	65.5	2.23	<0.01
Anx	24.6	26.5	2.36	<0.01	24.2	26.7	2.81	<0.01
SP	19.6	21.1	2.37	<0.01	18.5	20.8	3.06	<0.005
Use	47.0	48.4	1.17	n.s.	45.6	47.3	1.06	n.s.
Mot	43.3	45.1	0.83	n.s.	42.0	45.5	1.61	n.s.
Enj	36.4	37.4	0.60	n.s.	33.5	36.6	1.9	<0.1

Clearly, the measurement of affect is very difficult, with beliefs, feelings and emotions constantly changing, and what we have sampled is merely a snapshot of this shifting landscape. However, on the basis of the above data, we hypothesise that maintaining high levels of the affective factors, and in particular self-concept, self-perception, anxiety, interest and enjoyment, is strongly associated in form 4 students (age 15 years) with higher performance in algebra problem solving, as measured by our instrument, and alternately declining levels of these factors are related to a lower algebra problem solving performance. We were very interested in examining the relationship between algebra performance and these affective factors further, particularly for evidence of a causal link, and for this we used the data from the interviews with the students.

Modelling a possible causal relationship

The interviews with the students were very revealing about their perception of the links between affective factors and their performance in mathematics lessons and we were able to add to those measured in our tests, two other factors which seemed important for modelling the causal relationship, one external and one affective. First we will consider some excerpts from the interviews with the HA I to LA II students and examine how they describe the role of their feelings in mathematics.

> *. . . Interviewer: Has there been any change in how you feel over this year?*
>
> *R1: I used to enjoy maths.... but it got harder now.*
>
> *. . . Interviewer: When do you enjoy it?*
>
> *R1: When I like it I understand it more and enjoy.*
>
> *. . . Interviewer: Does any thing prevent you from learning maths?*
>
> *R1: . . . I am not sure. . . no not much, but I used to enjoy maths. . .*
>
> *. . . Interviewer: When do you feel confident?*
>
> *R1: I get confidence through the teacher, get ideas, concepts on different ways to approach things. . . I think just not having the security of a good teacher, the teacher really affects me and how they teach . . . It affects my ability to work and my ability to understand. .*
>
> *Interviewer: How does your confidence affect your work?*
>
> *R1: Umm. . . it just makes me a bit better, yeah more sure of myself*

So R1 feels a lowering of her enjoyment, as her test score predicted, and she thinks that this has affected her understanding. The first of the other factors, an extrinsic one, mentioned by a number of the students, was the role of the teacher. R1 clearly thinks that the teacher gives her more confidence, which in turn improves her understanding and hence the quality of her work. Student C1, also links the lowering of her performance to her feelings and to the teacher:

> *Interviewer: Has there been a change over the year?*
>
> *C1: Yep from 3rd form to 4th form yep there had. . umm. . my maths dropped*
>
> *Interviewer: What is the change in your feelings towards maths?*
>
> *C1: I think it is just that I was not comfortable with the teacher I did not like the teacher so when I was in 3rd form it was more interesting, I think I used to enjoy more. In third form it is just working it was easier and in fourth form you got to involve more and is hard*

C1 knows she is not doing as well as previously and, like R1, relates this to being 'comfortable' with the teacher. She also gives a lowering of interest, enjoyment and effort as relevant factors. Many students' comments described the concept of greater study effort as being promoted by feelings such as enjoyment and anxiety. Here, for example, R2 links confidence and enjoyment to focus in the classroom and states that lack of confidence causes her not to 'work as much':

> *. . . R2: Feel confident, concentrate more, do better*
>
> *Interviewer: What are the feelings that help you?*
>
> *R2: Confidence, my enjoyment, just my focus in the classroom, umm. . .*
>
> *. . . Interviewer: Is there any specific feeling that helps you to do that?*
>
> *R2: When I am confident, my confidence. . . I just feel not confident umm. . .that I can not answer the question or like that and do not do work as much when I do not know how to do*

She also recognises her falling standard of work, linking it to the teacher, loss of interest and to her lack of enjoyment:

> *Interviewer: How do you feel about your ability to do maths?*
>
> *R2: About average, umm... I was average at first and I dropped*
>
> *Interviewer: What could be the reason?*
>
> *R2: Probably the teacher, not concentrated, lost interest*

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Interviewer: What is the change you find from beginning to the end of the year?

R2: Umm. . . I used to enjoy it better, I do not feel so now

R2 feels a lowering of enjoyment and self-confidence, confirmed by her tests, and these like many others are associated with her teacher. It may be 'obvious', but teachers clearly play an important role in improving students' beliefs, attitudes and feelings (e.g. Forgasz & Leder, 1996), which in turn influence their ability to work at, and understand, mathematics. The importance of the teacher is confirmed by the comments of M1 and V, who moved from LA I to HAII. They maintain that their feelings have influenced their ability to work and understand, mentioning in particular, improvements in confidence, enjoyment and interest as helping them achieve.

Interviewer: Has there been any change over this year?

M1: Maths is much better now. . . when I had Mrs [A] it was good, her teaching was pretty good. . . My maths is getting better because of Mrs [A]. . . I am much better in my algebra

. . . Interviewer: Does that enjoyment do anything for you?

M1: It gives me. . . more of a . . . umph. . . to do algebra

Interviewer: Are you confident of your ability?

V: Umm...halfway yeah I am not that confident I guess I have improved from last year

Interviewer: What do you think is the relationship between your feelings?

V: Umm...I am interested it helps me to improve what I want to improve

The need for concentrated and focused effort again comes through in V's comments and, interestingly, as we found from the tests, the idea that anxiety can be a positive influence, causing one to work harder:

Interviewer: What do you think has helped you to improve your performance?

V: Umm,.... just put my mind into I guess and I concentrate

. . . Interviewer: Do you ever feel worried in maths class?

V: Umm... yes I do. Sometimes I do not get what other people do - that sort of puts me down

. . . Interviewer: Does it make you work better?

V: Yes

Students C2 and M2, who were in the higher achieving band on both tests, and showed improvement between the tests, similarly linked their performance to greater confidence, understanding, increased effort and enjoyment of the mathematics.

Interviewer: Has there been a change over this year?

C2: Umm.... No not really, I've had to work a bit harder this year

Interviewer? What change did you find from the beginning to the end of the year?

C2: Change in feelings on stuff. I think I enjoy maths now. Last year I didn't like it that much

. . . Interviewer: How do you feel when you are stuck on a problem?

C2: Umm... pretty much annoyed

Interviewer: Does this help you?

C2: Yeah I do more work and do more and more 'til I solve it

Interviewer: Has there been a change in how you feel over this year?

M2: A little, I understand it a little better. . . umm. . . it's a bit more fun

Interviewer: What do you think the relation between your confidence and your performance is?

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> *M2: Well like, when I feel good I get better marks and my performance is better, but if I don't feel very well then it just doesn't work. . . I just find it more enjoyable and it just makes me want to do more because I like it so much*

Conclusions

We believe that this study has provided some evidence linking the maintenance in adolescence (15 years of age) of high levels of self-concept, interest, enjoyment, anxiety, motivation and self-perception with a higher standard of performance in algebraic problem solving, but a falling off in these factors with a lowering of this performance. In addition, two further important factors which emerged from the interview comments were the students' level of effort and an external contribution to their understanding, primarily from their teacher. Students' comments clearly showed that they linked the teacher with their understanding and confidence, confirming results found by Forgasz & Leder (1996), who describe the influence of the teacher on student beliefs. Student understanding then, raises or lowers self-confidence (whilst we measured self-concept and self-perception, these are factors which find there expression in the more commonly understood term self-confidence), which, in turn, influences levels of interest, enjoyment and motivation, leading to greater or lesser effort and hence performance. The following comments from three of our interviewees sum up particularly well some of these relationships, emphasising the driving role of the affective factors.

> *V: Umm. . .well I feel really good about maths and that is I like doing it, so I do it, and I get the best results I can*

Here V explains how her confidence and enjoyment affect her work rate, which she sees as influencing attainment.

> *M1: I feel happy and I enjoy myself at what I do. That gives me more encouragement to do some more because I understand it. . . now I understand it and feel more confident to do what I am doing*

M1 too perceives enjoyment of mathematics as promoting greater effort and understanding as increasing confidence.

> *C1: If you don't feel like doing it, you won't do it and you won't do as well and you won't achieve as much, but if you know you can and you are feeling happy about what you are doing then you just carry on, keep on going*

C1 describes how, for her, greater motivation and enjoyment lead to more effort, and hence better results, or doing well.

Analysing the interview data in this way has thus led us to propose a preliminary model of the relationship between these factors as seen in Figure 2.

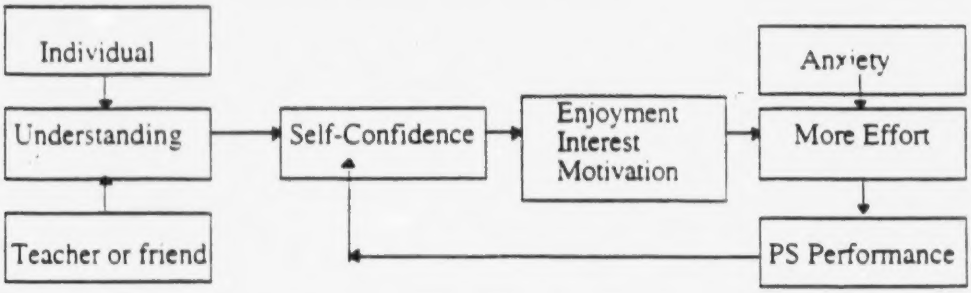

Figure 2: A Preliminary Model of the Relationship Between Some Affective Factors and Mathematics Problem Solving (PS) Performance

At first sight this description does not seem surprising, but there are several points which are worthy of note:

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- the entry point for the cycle appears to be via understanding, and students look outside themselves for assistance with this, especially to their teacher
- self-confidence appears to play a major role and raising this should be a priority
- the students' perceived level of effort is a key influence; anxiety may raise this
- the process is cyclical, with improved work and achievement increasing confidence, but with the affective factors driving the cycle, not the performance

We have recently completed an analysis of corresponding data from 90 form 3 students (14 years old) to test the robustness of this preliminary model we have proposed. Very similar results are emerging which appear to support the findings presented here.

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Tertiary mathematics: Perceptions of school leavers and mature-age students[1]

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As part of a study focusing on mature age students in tertiary mathematics courses, a survey was administered to a large number of first year mathematics students. Data on personal background, self-perceptions as learners of mathematics and perceptions of the tertiary learning environment were gathered. Responses from school leavers and mature age students were compared. Differences were found on several background factors and some studentsí attitudes and beliefs. Generally, the mature age students were more satisfied with the tertiary learning environment.

Introduction and Previous Research

For some time it has been recognised that participation rates in mathematical studies at all levels are not equitably distributed among groups within populations. Almost without exception females are found less likely than males to study the most demanding mathematics courses offered and to persist with mathematics to the highest degree levels (Leder, Forgasz & Solar, 1996). The identification of other disadvantaged groups appears to be context dependent and can vary from country to country. In Australia, socio-economic status and ethnicity/cultural background (often characterised as non-English speaking backgrounds [NESB]) are common grouping categories of disadvantage (see Yates & Leder, 1996).

Much of the work on factors contributing to differential participation rates in mathematics courses has centred on school-aged children. Using a variety of methods and theoretical perspectives, previous research exploring gender differences has identified a complex of contributing variables: cognitive, affective, and environmental or contextual (e.g., Burton, 1990; Fennema & Leder, 1993; Leder, Forgasz & Solar, 1996). In Australia, higher participation rates in optional school-level mathematics courses have been found for students from higher socio-economic backgrounds [SES] (Ainley, Robinson, Harvey-Beavis, Elsworth & Fleming, 1994; Teese, Davies, Charleton & Polesel, 1996). Lamb (1997) found that gender differences in participation rates were related to the interaction of SES and positive attitudes and beliefs about mathematics.

At the tertiary level, factors influencing participation rates and learning outcomes have been of increasing interest as mathematics enrolment numbers have declined in Australia as elsewhere (J•rgensen, 1997). Forgasz (1996) found that the attitudes and motivations of a small group of mature-age[1] mathematics students were more functional, (that is, more likely to lead to success and future participation) than those of their school-leaver counterparts. It was argued that more mature-age students should be accepted into tertiary mathematics courses. Perceived levels of discrimination by gender and ethnicity, Forgasz (in press) contended, had the potential to impact negatively on the decisions of some students to persist with tertiary mathematics study.

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[1] Students who have not necessarily completed the formal academic pre-requisites required generally, or for specific courses, and who are 21 or over on March 1 of the year in which tertiary entry

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Mature age students and mainstream[2] tertiary mathematics courses

Various policies have been put in place to encourage young Australians to proceed to tertiary education. These changes seem to have been of particular benefit to two groups: females and older students. Females now substantially outnumber males at university (e.g., Australian Bureau of Statistics, 1996) and the greater flexibility shown by tertiary institutions has resulted in increased enrolments of mature age students. The pattern does not appear to have been uniform across different subjects or courses. While national data are difficult to extract, an exploration of the data file for Australia-wide university enrolments in 1996 revealed that 35% of all students who commenced a bachelorís degree were aged 21 or over (Leder & Forgasz, 1997). A smaller proportion of this age cohort appears to enrol in tertiary mathematics. Limited to data from three Australian universities, Forgasz (in press) found that of 1072 (58% male) undergraduate mathematics students surveyed, 11% (15% of males; 5% of females) were mature-aged.

As part of a larger study focussing on the experiences of mature-age students studying mainstream tertiary mathematics, a large scale survey was administered to first year students enrolled in mathematics courses at five Australian universities. Differences between mature-age students and school leavers were of particular interest. Within each group, the data were also examined by gender, SES, and student ethnicity. Some of the findings are reported in this paper.

The Study

The instrument and analyses

The survey instrument used was a slightly modified version of one previously administered to undergraduate mathematics students by Forgasz (in press). The modifications allowed for a sample population of first year undergraduates and for a focus on mature-age students. Items provided the following data on students:

- biographical and background information including: sex, age, ethnicity (e.g., language/s spoken at home), SES indicators (e.g., receiving financial assistance to study), school attended for grade 12 (e.g., school type)
- self-perceptions as a learner of mathematics including: enjoyment of mathematics, perceived achievement levels, confidence, attributions for success and failure
- perceptions of the learning environments: quality of teaching, course difficulty, workload, assessment, availability of help, and discrimination

Students responded to some items on 5-point Likert-type scales (1 = strongly disagree to 5 = strongly agree). For other items they selected among categories (e.g., yes/no/ sometimes). Responses were analysed using $\mathrm{SPSS}_{\mathrm{WIN}}$. For some variables, independent groups t-tests and chi-square tests were used to explore for differences among the various groupings of students. Statistical significance was set at the .05 level.

The sample

Sample sizes differed across universities but were representative of the relative sizes of the institutions (see Table 1). It can also be seen on Table 1 that the proportions of males, mature age students, and students of non-English speaking background [NESB] comprising each university sample varied considerably. These institutional differences are worthy of further investigation. However, for the purposes of this paper, the university samples were combined. Table 2 shows frequencies for the entire sample by gender, age and language breakdown.

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[2] ìMainstreamî does not include ëbridgingí or similar courses

Table 1. Sample sizes by university, gender and student age.

	University					
	1	2	3	4	5	Total
Total	75	94	195	201	246	811
M[1] (%)	37 (50)	86 (92)	100 (52)	121 (60)	150 (62)	494 (62)
F	37	7	91	80	94	309
SL (%)	57 (76)	66 (70)	182 (93)	175 (87)	234 (95)	714 (88)
MA	18	28	13	26	12	97
ESB (%)	55 (73)	35 (37)	128 (66)	130 (65)	157 (64)	505 (62)
NESB	20	59	67	70	88	304

It should be noted that 8 students did not identify their sex, 2 did not indicate whether they were ESB/NESB

[1] M = male, F = female; SL = School leaver, MA = Mature age; ESB = English speaking background, NESB = non-English speaking background

Table 2. Sample size by gender, age and language background

	Student Sex		Age		Language	
	M	F	SL	MA	ESB	NESB
Total N=811 (%)	494 (61)	309 (39)	714 (88)	97 (12)	505 (62)	304 (38)
Males (%)			428 (60)	66 (69)	307 (61)	186 (62)
Females (%)			280 (40)	29 (31)	193 (39)	115 (38)
School leaver (%)					455 (90)	257 (85)
Mature-age (%)					50 (10)	47 (15)

The data on Table 2 indicate that of the total sample:

- males comprised 61%
- mature-age students represented 12%
- NESB students made up 38%

Compared to their proportions in the total sample:

- there was a higher proportion of male mature age students (69%)
- mature age students comprised a slightly higher proportion of the NESB (15%) group

These data are consistent with Forgaszí (in press) sample break-up and suggest that the students surveyed were representative of the Australian tertiary mathematics cohort and that the composition of cohorts has not varied markedly over the last few years.

The composition by age of the mature age cohort was as follows: 63% were between 21 and 25 years of age when they commenced their current courses; 37% were older.

Results

There were three groups of variables on which comparisons were made between the school leavers and the mature age students:

- personal characteristics
- beliefs and attitudes
- perceptions of the learning environment

The findings for each cluster of variables are presented below.

Personal characteristics

Data were gathered on various aspects of studentsí backgrounds and personal status. A summary of selected variables on which the school leavers and mature age students appeared to differ are shown on Table 3.

Table 3. Personal characteristics: School leavers and mature age students

Variable/Characteristic	% School leavers	% Mature Age
Full time students	99	87
Receive Austudy[3]	31	52
Never married/no permanent partner	97	73
Have children	-	16
Grade 12 in Australia	95	65
Australian-born	68	50
Single-sex school for grade 12	28	22

Personal characteristics for which it might have been expected that members of these two groups would differ appeared to be confirmed by the data shown on Table 3. Compared to the mature age students, higher proportions of school leavers were studying full-time, single and without children. Interestingly, higher proportions of mature age students than school leavers received Austudy, were not Australian born and/or completed grade 12 studies outside Australia (consistent with their higher representation among the NESB group); a lower proportion had attended single-sex schools for grade 12.

Ten per cent (69) of the school leavers and 73% (71) of the mature age students had not proceeded directly from school to their current university courses. These students were asked to indicate one or more reasons for having had this break in their studies. The data are summarised on Table 4.

Table 4. Reasons for not proceeding directly from school to current university course: Percentages (and frequencies)

Reason	% School leavers	% Mature Age students
had not decided on career	12 (8)[1]	23 (16)
wanted to travel	20 (14)	13 (9)
wanted a break from study	28 (19)	27 (19)
wanted outside experience	19 (13)	14 (10)
family influence	9 (6)	6 (4)
no intention to go to university	1 (1)	7 (5)
not academically qualified	-	3 (2)
studied at another tertiary institution	41 (28)	45 (32)
had to work to support family	-	10 (7)
had to work to support self	9 (6)	32 (23)
other reasons	25 (17)	10 (7)

[1] Frequencies shown in brackets

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[3] Australian government financial support for students. Stringent eligibility criteria apply, including age and parental means-testing.

Again, as may have been anticipated, the school leavers and mature age students differed in the extent to which particular factors might contribute to a time lapse between their attendance at school and for their current courses at university. The mature age students were more likely than the school leavers not to have decided on a career path, to have had to support themselves and/or their families, and/or not intended to pursue tertiary study. The school leavers were more likely than the mature age students to have wanted to travel and/or gain experience outside educational institutions.

Beliefs and attitudes

In several places on the survey questionnaire, items were included which reflected studentsí attitudes and beliefs about themselves as mathematics students. Many similarities were found for the two groups of students. On average, there was no statistically significant difference in their beliefs about:

- potential levels of success in tertiary mathematics that year (SL=3.36; MA=3.5)
- the extent to which likely success in tertiary mathematics was attributed to ability (SL=4.11; MA=4.19), task difficulty (SL=3.11; MA=3.08) and luck/environment (SL=2.55; MA=2.51)
- the extent to which possible failure in tertiary mathematics was attributed to ability (SL=2.84; MA=2.95), effort (SL=4.13; MA=3.93), task (SL=3.43; MA=3.44) and luck/environment (SL=2.37; MA=2.44)

For two of the affective items, statistically significant differences were found:

- A higher proportion of school leavers (33%) than mature age students (17%) indicated that they were studying mathematics because they believed they were good at it ($p<.01$)
- Mature age students (4.43) scored higher than school leavers (4.06) on attributing likely success at university mathematics to *hard work* ($p<.001$).

The students were asked if they were required to use (a) computers and (b) scientific calculators in their courses. Among those who said ëyesí, it was surprising to find that there were no differences among the two groups in perceived levels of competence with either technological instrument. It may have been expected that school leavers might feel more competent, particularly with the use of computers. As may have been predicted, students believed they were more competent with scientific calculators (N=533, mean = 4.16) than with computers (N= 342, mean = 3.60).

Enjoyment of mathematics at school and at university

Students were also asked whether they had enjoyed mathematics at school (yes / no/ sometimes) and if they were enjoying mathematics at university. The results for the two groups of students are shown on Table 5.

Table 5. Enjoyment of mathematics at school and university: Percentages of school leavers and mature age students

Enjoyment of mathematics	**School Leavers**			**Mature Age**		
	Yes	No	Sítimes	Yes	No	Sítimes
School	57	9	34	53	16	32
University *	35	19	46	48	15	38

* $p<.05$

The distribution of responses for the enjoyment of mathematics at school was similar for the two groups of students. However, there was a statistically significant difference ($\bullet^2$=5.86, $p<.05$) for responses to the enjoyment of mathematics at the university level. The mature age students were more likely than the school leavers to enjoy their tertiary studies. Overall, there were fewer students who enjoyed tertiary mathematics than school mathematics, but the decrease in enjoyment levels was greater among the school leavers. Compared to their responses about school mathematics, there was also an increase in the number of school leavers who did not enjoy mathematics at the tertiary level; this trend was not evident among the mature age students.

Perceptions of the tertiary learning environment

The school leavers and mature age students differed significantly on several items aimed at gathering impressions of the tertiary learning environment. Students were asked to indicate the extent to which they agreed (strongly disagree = 1 to strongly agree = 5) with each item about ìuniversity mathematicsî. The results are shown on Table 6.

Table 6. Perceptions of the tertiary learning environment: Mean scores for school leavers and mature age students

University mathematics	School leavers	Mature age	Significance levels
easy	2.73	2.67	
enjoyable	3.05	3.38	t=2.90, df=779, p<.01
challenging	3.91	4.13	t=2.47, df=777, p<.05
interesting	3.23	3.64	t=3.56, df=778, p<.001
useless	2.24	1.88	t=-3.36, df=772, p<.001
well taught	3.27	3.66	t=3.52, df=763, p<.001
understand work	3.50	3.62	
lectures boring	3.44	2.65	t=-6.53, df=769, p<.001
lecturers approachable	3.50	3.87	t=3.43, df=772, p<.001
tutors helpful	3.91	4.09	
enough personal help	3.21	3.41	
tutorials useful	3.84	4.00	
assessment fair	3.82	3.87	
too much work	2.85	2.51	t=-3.04, df=770, p<.01
confident of passing	3.73	3.76	

The data on Table 6 indicate that compared to the school leavers, the mature age students found university mathematics to be more enjoyable, challenging, interesting and well-taught. They also considered lectures less boring and lecturers more approachable. Compared to the mature age students, the school leavers thought mathematics was less useful and that there was too much work.

Discussion

The data presented in this paper suggest that mature age students and school leavers constitute distinct cohorts along a number of dimensions. Their age differences alone revealed differences in life experiences. The mature age students, for example, were more likely than the school leavers to have family responsibilities and to come from

NESB backgrounds. Given the large proportion of Australians now undertaking tertiary studies, the higher proportion of mature age students than school leavers who did not proceed directly from school to university because they had to support themselves or their families suggests that they may be from lower socio-economic circumstances than their school leaver counterparts. That a higher proportion of mature age students was also in receipt of Austudy appears to support this inference. On the other hand, it may reflect their high levels of motivation in that they are prepared to make financial sacrifices in order to return to study.

The data also revealed many similarities but also some important differences in the two groupsí attitudes and beliefs about themselves as learners of mathematics. There were no differences in the two groupsí beliefs about their likely performance levels for the year or their perceived competence levels with scientific calculators and computers. The findings with respect to technology were somewhat surprising. Considering the relatively recent and increasing exposure to technology for mathematics learning now experienced by school students, it was anticipated that mature age students would feel less certain about using the equipment.

Compared to the school leavers, fewer mature age students indicated that they were studying mathematics because they believed they were good at it. This seems consistent with the slightly higher proportion of mature age students who said they had not enjoyed mathematics at school and with the groupís higher mean score for attributing success to effort. Yet, the mature age students indicated that they were enjoying tertiary mathematics to a greater extent than the school leavers. Forgasz (1966) noted that mature age mathematics students were more highly motivated than their school leaver counterparts. Despite the longer time period since the mature age students last studied mathematics and their poorer experiences at school with mathematics, perhaps their higher motivation and their greater likelihood to be studying tertiary mathematics by choice partially explain the differences in the two groupsí self-perceptions.

There were also differences between the two groups of students in their perceptions of the tertiary learning environment Overall, the mature age students were more positive than the school leavers. They were less critical of the quality of teaching, of access to staff and of the availability of assistance, and seemed more stimulated by the mathematical content and its worth. These findings may be related to their greater motivation and maturity and/or their life and work experiences prior to commencing their present tertiary courses. This is not to say, however, that the mature age students were fully satisfied with their tertiary mathematics programs. There would still appear to be room for improvement.

The greater dissatisfaction with the tertiary mathematics learning environment evident among the school leavers should be of concern to tertiary mathematics educators. As they constitute the majority of the cohort, tertiary mathematics departments would be ill-advised to ignore the concerns expressed. It should also be noted that the overall decline in the enjoyment of mathematics from school to the tertiary environment replicated earlier findings (see Forgasz, in press). Perhaps students are responding to the different teaching and learning styles they have experienced at school and in the tertiary setting. The data appear to support contemporary efforts for all tertiary teaching to take greater cognisance of educational theories on effective student learning. There were several aspects of the tertiary mathematics learning environments in particular about which students were either ambivalent or fairly negative. The studentsí sentiments may be taken as indicators of areas within mathematics departments that invite closer examination.

Final Words

Previous large scale research on mature age students in Australia has focussed on students enrolled in Arts and Education faculties (e.g., Hore & West, 1980). As a group mature age students have been found to be successful and highly motivated. From interviews with 23 tertiary mathematics students, of whom 13 were mature age, Forgasz (1996) made similar conclusions. The data from the large scale survey reported in this paper appear to corroborate these earlier findings.

The next stage of the present study will explore in greater depth the experiences of mature age students of mathematics and will make comparisons with those of school leavers. The findings may provide answers to the issues raised by the large scale survey data reported in this paper.

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Using technology to enhance children's spatial sense

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Abstract

This paper examines the way in which two young children engage in a computer-based problem solving activity that required a degree of visual-spatial reasoning. The case study traces the children's spatial-visual understandings and looks at the way in which these understandings were activated and developed through the particular activity.

Literature Review

Computers make it possible to represent visual and spatial mathematics with an amount of flexibility not offered by other media. Graphic programs, for example, allow the user to represent images accurately as either two- or three-dimensional representations. These images can then be transformed or rotated with precision. With appropriate software the computer can become a powerful tool that enables children to manipulate spatial arrangements and construct visual images which would usually be limited by their own drawing capabilities. As Clements and Battista (1992) commented "the potential of such drawing tools lies in the possibility that children will internalize such functions, thus constructing new mental tools ... Interaction with certain computer environments may help students build less restricted concept images" (p. 452). Moreover, an increased familiarity with relevant software allows the user to investigate and construct mathematical understandings at increasingly sophisticated levels of development. Such claims are particularly relevant to the development of visual and spatial understandings. As Noss, Healy and Hoyles (1997) commented:

> the computer can incorporate the visual into every stage of the activity. It can externalise visual images and at the same time render them manipulable...Computer screen representations of visual objects and even visualised relationships can be acted upon directly and we can observe the ensuing changes in the represented relationships. (p. 209-210)

Importantly, these representational changes provide immediate feedback for the user and foster valuable learning environments. Provided the computer environment is flexible, children are able to explore, predict, become more adventurous, and be more responsible for their own learning. It may be the case that visual images are evoked when a user engages in the use of, and interacts with, particular computer software. Nemirovosky and Noble (1997) argued that visual representations "in the mind" take place when using a tool requires the user to analyse external images (on paper or on a computer screen etc).

Interactive programs allow the user to explore and develop 2D and 3D environments that closely simulate "real-life" experiences. We are able to take a *Magic schoolbus* ride through a human's body by navigating a path through various body systems. The more advanced technology becomes, the greater the visual impact it will have on the user. It could be argued that such technology is under utilised in educational settings (Lowrie & Hill, 1996a) and that even when it is used, it is not developed to its full capacity.

Visual imagery can be a powerful source of reasoning in problem solving (Antonietti, 1991; Lowrie, 1998; Lowrie & Hill, 1996b; Kaufmann, 1990) and such reasoning is linked to the development of spatial concepts. Imagery is often viewed as particularly important in the initial stages of the problem solving process (Kaufmann, 1990; Lowrie, 1996; Pirie & Kieren, 1992). Computers simulation programs, despite their obvious advances, do not always make these representations easy to understand. Children often find it difficult to transform their understanding of real life experiences to the representations created on the screen.

For children, and particularly young children, some form of direct instruction or external interaction may help to support learning experiences that emerge from using simulation based software. Moreover, if guidance and appropriate questioning take place mathematical understandings that are not necessarily made explicit in the program can be developed. The interactive relationship between the user, the teacher and the technology may develop a sound model for processing information. Osborne and Wittrock (1983) suggested that problem-solving performances are likely to be fostered in situations where procedures for focusing children's attention—such as providing motivating experiences, questioning, assisting students to clarify ideas, and providing alternative experiences of a particular concept—are adapted. Moreover, initial discovery stimuli should provoke personal feelings and understandings so that children are able to relate new ideas to previous concepts. If these experiences evoke a range of verbal cues, in an interactive manner between student and teacher, important understandings can be clarified to help represent the problem more clearly.

Lived experiences are visually represented on a computer screen, with the user able to manipulate these representations with freedom and flexibility. New insights about these experiences are generated through the interactiveness of the technology . The teacher also plays a vital role in this process. As Brown & Wheatley (1997) commented, the concrete representations offered by the technology may evoke, in the user, visual representations that are crucial in doing mathematics . In this paper, it will be argued that children's spatial sense can be further enhanced through directed questioning from the teacher.

Methodology

A case study methodology (Yin, 1994) was used to investigate the way two young children, Dylan and Rhiannon, used visual-spatial understandings to solve a computer-based simulation activity. Data were collected from audio-taped interviews, annotated transcripts, children's work samples and their responses to open-ended questions. In some instances the researcher was positioned behind the student in the role as a participant observer, whereas on other occasions the researcher interacted with each student as they solved the task. The study had two main purposes:

1. to determine the nature of the visual-spatial understandings used by the children completing the task; and
2. to examine the extent to which the interactions between the student, the researcher and the computer-based activity created an effective learning environment.

The participants

Both case study participants were competent computer users for their age. Dylan, aged 7, used computers once a week at school and almost everyday at home to play a variety of 'educational' and 'entertainment' games. Rhiannon, aged 8, did not use the computer as much as Dylan but tended to use it for a variety of writing and information purposes. Both children were used to reflecting upon the thinking processes they used to solve problems.

The computer software program

The computer program used for the investigation was Electronic Arts (1996) *Madeline thinking games*. The interactive program, which is based around a popular children's cartoon character, has twelve problem solving activities. The *Let's decorate* activity was selected for study. The challenge is to create and decorate a bedroom. There are a selection of wallpaper and floor designs, furniture, flower bouquets and toys that can be selected from icon boxes positioned at the bottom of the screen. Three walls of a bedroom are used as the template to introduce perspective. Many pieces of furniture, pictures and windows are tilted in a certain direction so that they can fit against the walls of the bedroom. The main objective is to select an appropriate item to be positioned on walls in the room.

It was postulated that a range of visual-spatial reasoning skills or understandings were required to solving this problem. Specifically, ideas associated with position, dimension, perspective, and angle seemed likely to be encountered. At no point did the investigation attempt to evaluate the "educational value" or relevance of the program. Computer programs of this genre are becoming more popular in schools and are often catalogued in book club advertisements. This program was selected because it was (a) already being used in the school, (b) suitable for young children, and (c) perceived to encouraged a degree of visual-spatial reasoning.

Initially, the children were exposed to a template of the bedroom. A voice, of Madeline, then invited the user to select a bedroom scene. The first time the children played the game the researcher did not interact with the students, but took notes of the individual's reactions and comments. In subsequent sessions, the researcher encouraged the children to describe what they were thinking about, posed questions in relation to the movement of objects, and attempted to establish the type of images the children constructed "in their mind." In these sessions the researcher aimed to determine the nature and type of visual-spatial understandings used by the children, and to consider the strategies they used in applying this knowledge. Dylan's representation of the problem is presented in Figure 1 with Rhiannon's in Figure 2.

Results and Discussion

Spatial-visual Processes and Understandings

The template of the bedroom provided the children with an opportunity to use their knowledge of spatial relations to conceptualise the representation. Although the problem was actually represented for them, they were still challenged to interpret the three-dimensional representation. Dylan knew that the template was a representation of a bedroom and could actually imagine what you would see when you walked into the room.

INT: What is this [the image on the screen] a picture of?
Dylan A bedroom
INT: What is in the bedroom?
Dylan Carpet and walls with wall paper on top.
INT: Where would the door be?
Dylan Back here somewhere [indicating that the door is in front of the screen]
INT: But it is not on the screen?
Dylan Because we are facing out that way [gesturing that we are looking in to the room with his hands] and as soon as you walk into the room you would see this [the image of the screen].

He thought carefully about where the door might be and attempted to locate it in front of the screen. In other words, the space "outside" the screen was related to the space "inside" the screen. Dylan, however, found it difficult to fully understand the relationship between the screen image and that of a bedroom. The compression of space—from a life sized bedroom to a representation of it on a computer screen —was not easily interpreted. When asked to identify were a point in the middle of the front wall would be he was able to say "just above the top of the drawers." When asked to do the same on the right hand wall he replied "near the plant" (see Figure 1). This response was obviously not correct. He commented that:

> you can see five squares that go across the wall (see Figure 1). Halfway would be in the middle of the third square (Dylan).

His argument may seem logical but he has not yet come to terms with the way space is represented on the screen. Mathematical understandings associated with perspective, diminishing line, and the representation of 3D space in a 2D environment have not been considered in his response.

Rhiannon was more intent on explaining why the representation actually looked like a bedroom. She uses terminology including "straight" and "diagonal" to describe understandings related to proportion and perspective. Although she did not describe the side walls in terms of a set of parallel lines, she used her hands to replicate the top and bottom lines of the side walls.

> Because it is 3D it makes it more real that way...The lines on the front are straight and the lines that way [points to the screen] are diagonal. When the lines are diagonal it actually looks like we are standing there... The diagonal make this part of the room further away than this part. Up here [points to the front] it is smaller because it is further away than those walls. Because we are closer to here it appears to be a lot bigger. (Rhiannon)

At her level of spatial development, Rhiannon was beginning to consider perspective and 3D to 2D representation. In her explanation she used phrases like "actually looks like we are standing there" and "it appears to be a lot bigger" to describe the simulated world. Interestingly, she commented that she could imagine herself walking around in the room.

> I could see myself walking through the door, walking up to the front wall, turning around and walking back out again. (Rhiannon)

Her vivid use of imagery was in contrast with her analytic description of why the 3D representation appeared to be real. Moreover, it appeared that she could move between real and imaginary environments.

Rhiannon's use of both visual and analytic reasoning is also evident in her justification for putting furniture or other objects on specific walls.

Rhian.: If I were to put it over here [on an inappropriate wall] it looks like the window is floating in the air.
INT: Why does it look good on that side?
Rhian.: Because that's were the angle goes.
INT: Is there any other reason?
Rhian.: Because this side of the wall is bigger than the side with the corner and the picture goes the same way.

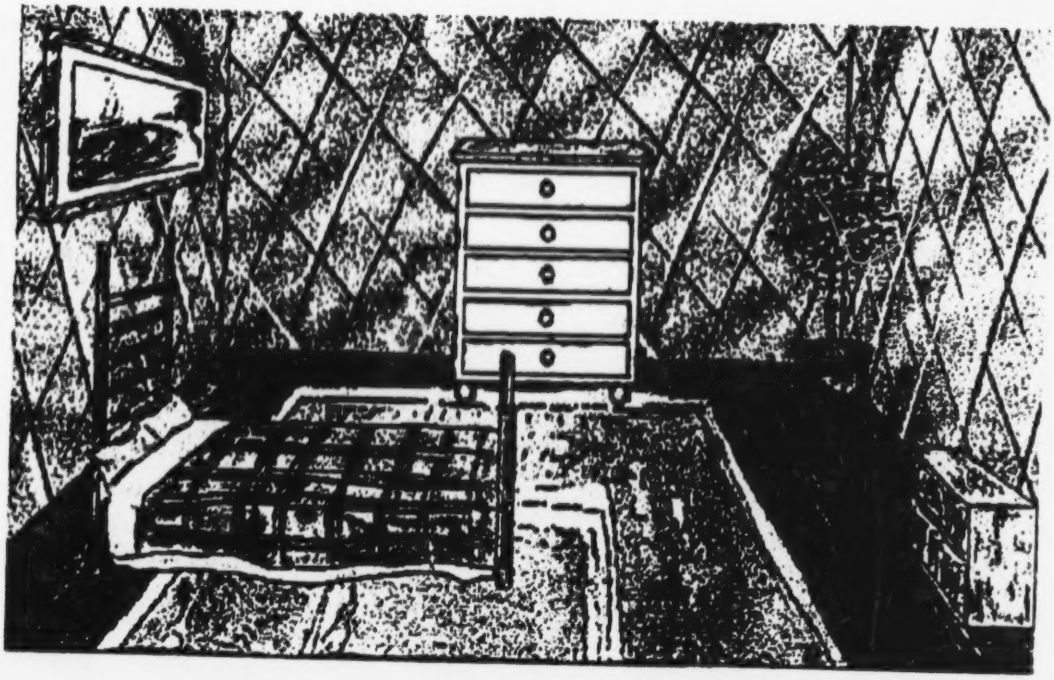

Figure 1: Dylan's Representation of the Problem.

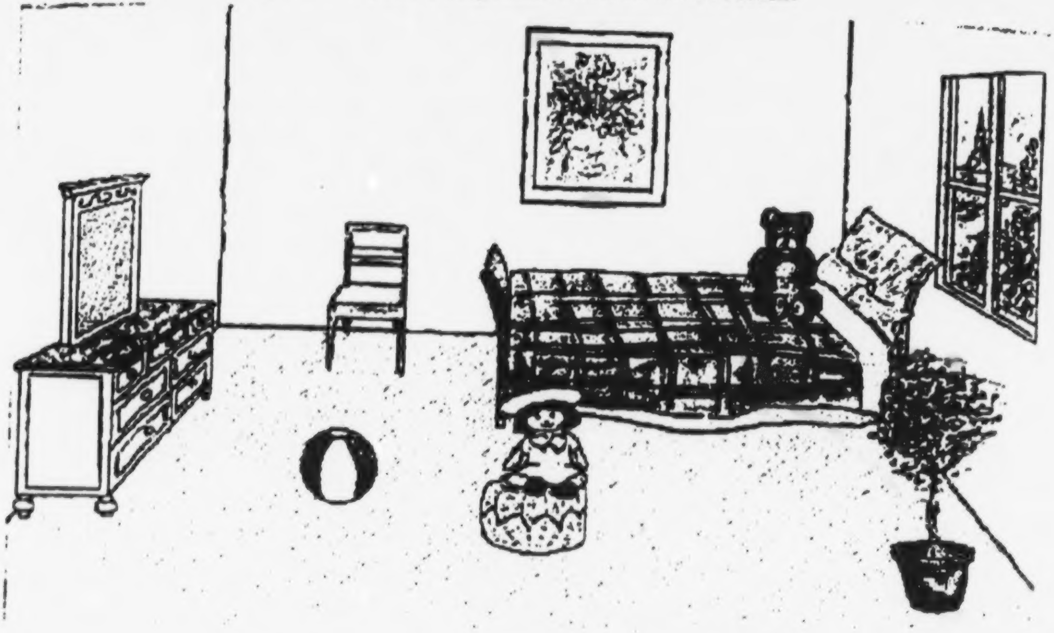

Figure 2: Rhiannon's Representation of the Problem.

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The Interaction Between the User, the Teacher and the Technology

It could be argued that simulation games such as the one presented in this paper do not really develop visual reasoning skills. This would be based on the premise that children have the opportunity to use trial-and-error strategies, in a somewhat random manner, until the objects are placed in appropriate positions. The user would not make reflective judgements about their decisions, or critically evaluate their solutions (the completed picture of the bedroom). On the contrary, the two children in this study carefully selected the furniture to be positioned around the room and would often change selections when additional furniture was placed in the room. This may have happened because someone was watching them. Rhiannon commented that "the picture needs to be further forward now that the mirror is on the same wall." Although this may have been for aesthetic value, the decision was probably influenced by ideas of converging parallels and vanishing points. Nevertheless, the program did seem to challenge the children's mathematical understandings more frequently when they were encouraged to reflect upon decisions they had made.

The three way interaction provided the children with opportunities to verbalise their thought processes in what Vygotsky (1962) claimed were opportunities for complex thinking. Dylan, for example, formulated his decisions by analysing what he could see on the screen. This changed when he was challenged to think about his responses.

INT: Show me where halfway along this wall would be [interviewer points to the side wall]?

Dylan: About here.

INT: How did you know that?

Dylan: You can see five squares that go across the wall [and] halfway would be in the middle of the third square .

INT: Is this wall shorter than the front wall?

Dylan: No, but we can't see it all.

INT: Why not?

Dylan: Because it can't fit on the computer screen. The wall would go out to about here [demonstrated space outside the screen with his hand].

INT: So, where would halfway be?

Dylan: Oh, it would be about here —above the bookcase [see Figure 1]. I forgot that you have to think about the parts you can't see.

Ideas associated with perspective and representation were now considered. At the end of the interview Dylan was asked what he had learnt from the experience. He was able to make statements about the direction of the lines that were based on mathematical understandings.

INT: What have you learnt today?

Dylan: The way the lines run. On one side of the wall the lines run up and on the other side they run down.

INT: Show me what is up.

Dylan: [He positions his hand close to the left side of the screen and tilts his fingers down]. It goes the other way on the other side.

His "analytic" observations were supported by effective visual reasoning processes. Zazkis, Dubinsky and Dautermann (1996) described situations were problem solvers move back and forth between visual and analytic strategies and that this is sometimes more effective than using one type of representation in isolation.

Directed questions were also used to monitor the thinking processes used by Rhiannon and to encourage her to reflect upon particular mathematical understandings as they arose. In one instance a comment she made ("it doesn't matter where you pout the plants") was challenged and she seemed to become confused with the ongoing questioning. She was then instructed to select a similar object to use as a baseline measure to compare its shape with other pieces of furniture. She made the connection almost immediately.

Rhian.: It doesn't really matter where you put the plants.
INT: Why is that?
Rhian.: Because it could go here or over there because it doesn't have an angle.
INT: What is the difference between the plant and other shapes in the room?
Rhian.: I don't know what you mean.
INT: All right, select another plant and place it on the screen for me. Now talk about this shape in comparison to the window or the bed.
Rhian.: [the plant] doesn't have an angle. The angle doesn't matter.
Well it's like the seat [She selects a seat and puts it on the screen].
I can put it anywhere too because they haven't got any angles.
INT: What other words could you use to describe the differences?
Rhian.: Well it's rounder.
INT: What about the corners do you know what they are?
Rhian.: A corner is when two edges come to a point and they come to an end.
Yes, most of the pictures [the objects] have sharp corners but the round shapes do not. That is why they can go anywhere.

The flexibility offered by the computer program allowed the interviewer to help the user construct ideas and understandings as the problem was being solved Additional examples could be selected to reinforce patterns and relationships among the objects. The computer was particularly useful in these situations. Both children developed understandings of representation and perspective by using the program. I would suggest, however, that the ongoing interactions between the user, the teacher and the program stimulated the children's thinking and provided them with opportunities to develop complex spatial understandings associated with perspective and representation.

Conclusion

This paper described the way in which two young children constructed spatial understandings associated with perspective and representation when using computer software that aimed to develop problem-solving skills. The metacognitive interactions that took place during the activity provided opportunities for the children to not only represent the problem—but to also assess possible success (Garofalo & Lester, 1985) as they responded to specific questions. Importantly, negotiation of meaning between the teacher and the student took place on a regular basis (Bauersfeld, 1991) with the student free to abandon, correct or change developing ideas. As Gagné and White (1978) suggested, quality questioning techniques can assist students to process information more effectively. The flexibility of the computer environment enhanced these interactions.

In the present study, the young children found it difficult to move between and link 'life-like' three-dimensional space with images presented in the simulation program. Although the screen images appeared to be representative of the walls of a bedroom, these images were not always immediately apparent to the children. On several occasions, however, the students were able to modify their spatial understandings as they verbalised their thought processes. Being able to think carefully about the problem representation helped the children to develop mathematical understandings associated with perspective, proportion and angle. Moreover, they were able to transfer their understandings of real situations to a representation of reality on the screen. The interactive environment helped the students engage in a range of visual and spatial understandings in a stimulating problem-solving environment.

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Pre-service teachers' understandings of word problems

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Mathematics curricular statements refer to problem solving, in particular word problems, as a means of relating the curriculum to "reality". Research indicates that pre-service primary teachers have little knowledge of word problems despite the strong emphasis on their importance within the curriculum. This study reports on the understandings of word problems types of pre-service primary teachers. A number of activities were given as part of a mathematics education method course. Educational implications of these findings are discussed and recommendations proffered as to the amelioration this situation.

Introduction

The Australian Curriculum and Standards Framework: Mathematics (Board of Studies, 1995) details the learning outcomes for all students in the compulsory years of schooling in this key learning area. This framework advises teachers that by the completion of Level 3 (Year 4 students - about 10 year old children) students are expected to be able to "generate word problems using specified numbers and operations" (p. 54). The American Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) (NCTM) similarly advises teachers of elementary classes that students should have multiple experiences in solving realistic word problems.

Numerous researchers (e.g., Stacey & Groves, 1978; Verschaffel, De Corte & Borghart, 1996; Woodward, 1991) have investigated the role of word problems in mathematics education method courses for pre-service teachers. Stacey & Groves (1987) introduced problem solving as part of their first year method class. Their students, working in collaborative groups, were introduced to a wide variety of problems including word problems. They found that many pre-service teachers were somewhat troubled by the lack of a "tight" course structure; others had difficulty working collaboratively; whilst only a few "thrived on the freedom allowed" (p. 340). Verschaffel et al. (1996) looked at the artificiality of many word problems and the inability of a number of pre-service teachers to use their knowledge of reality to solve a series of problems, half of which were problematic problems. Problematic problems require students to view the problems in context and to use their knowledge of real situations to obtain an accurate solution. They point out the necessity to change "the teachers' own conceptions and beliefs about the importance of real-work knowledge in arithmetic word problems solving" (p. 4-394). From a somewhat different perspective, Woodward (1991) asserted that teachers will not teach problem solving until they become proficient problem solvers themselves. Pre-service teachers experienced a variety of strategies (such as pattern identification, table building, graphic representations) as a means of solving a diverse range of problems. He found that pre-service teachers gained confidence in their ability to teach mathematics problem solving through this experience. Fennema, Carpenter, Levi, Franke, & Empson, (1997) in their Cognitively Guided Instruction course use a teacher's knowledge of word problem categories as a means of gaining a full understanding of their students' mathematical thinking.

If, as research has shown, teachers are required to have an understanding of word problems to effectively teach their students to become competent problem solvers, then they should have an understanding of the categories of word problems their students are likely to meet. This study was conducted in order to ascertain whether these pre-service teachers had gained an understanding of problem categories from their experiences in the methods course they had undertaken.

The Study

This study involved 42 pre-service primary school teachers taking a mathematics method as part of their Graduate Diploma of Education.

A 1-3-6 method of building a collaborative decision regarding the categorisation of word problems was employed. Because of space constraints, this methodology is explained in the activities below rather than as a separate section. The response sheet for each student was collected in their final group and analysed to determine their categorisation and generation of word problems.

The activities described below enable an analysis of individual and groups of pre-service teachers' ability to come to grips with word problems. The findings should provide an indication of additions may be needed to method courses to enable these pre-service teachers cope with the coming demands of the curriculum they will be required to teach.

1. Students were initially presented with 36 unclassified word problems and asked to classify them into twelve different categories. They were told that there were six problems involving either addition or subtraction and six involving either multiplication or division. Students were asked to work individually and write down the number of the problems which they considered belonged in the same category. Although no category headings were provided to the students, these are shown in Table 1.

Table 1. Categories and example of word problem

Type of word problem	**Category**	**Example of word problem**
Additive	**Joining**	*Mary has F5¢ in her pocket. On the way home she found 10¢ in the grass. How much money does she have now?*
	Separating	*Richard had 15 marbles. He lost 7 of them in a game. How many marbles has he left?*
	Equalising-add on	*Jessica has 15¢. Alexander has 35¢. How much more does Jessica need to save to have the same amount as Alexander?*
	Equalising-take away	*Irene has 25 marbles and Peter has 17 marbles. How many does Irene need to lose to have the same number as Peter?*
	Part-part-whole	*In the purse there is a 10¢ coin and a 20¢ coin. How much is in the purse?*
	Comparison	*In his bag Igor had 25 small marbles and 16 large marbles. How many more small marbles are there than large marbles?*
Multiplicative	**Comparison multiplication**	*Margaret has written 12 lines of her story. James has written three times as many lines as Margaret. How many lines has James written?*
	Comparison division	*Jillian has already written 46 lines of her story so far. This is twice as many lines as David has written. How many lines has David written?*
	Cartesian product	*Elizabeth has 3 summer skirts, 4 blouses and 2 pair of shoes. How many different outfits can she wear?*
	Partition division	*Ms Vormann bought 15 chocolate bars to give to the children at the party. There were 5 children at the party. If each child received the same number of chocolate bars, how many chocolate bars did each child receive?*
	Quotition division	*Mr. Tsitas bought 15 chocolate bars to give to the children at the party. Each child was given 3 chocolate bars. How many children were at the party?*
	Equal groups	*Each week Ms Bye bought 5 pieces of fruit for each of her three children's school lunches. How many pieces of fruit did she buy for all of the school lunches each week?*

2. Following this, students were asked to form into groups of three to compare their categories and to identify and reflect upon the criteria they used to classify the problems.
3. The third activity required each group to join with another group forming groups of six students. Working collaboratively these larger groups required students to compare their categorisation criteria and to generate a similar word problem to illustrate each category.
4. Finally students were required to present their findings to the whole class and be prepared to justify their categorisation system.

In all, four interrelated tasks were given, viz

1. To classify the 36 given word problems into twelve different categories.
2. To generate criteria by which word problems may be categorised.

3. To generate a word problem reflecting these criteria for each of the twelve categories. To justify, in their own word, their choice of categorisation system and to explain how their own problems validated these criteria.

Results

Due to space constraints, the data for the six-student groups are only presented. Individual and three-student group responses parallel these findings.

Task 1: Categorisation. The initial task required students to categorise the 36 word problems into 12 different additive or multiplicative categories according to their own criteria. Table 2 shows the number of students who, working collaboratively, were able to correctly categorise the 36 word problems into categories similar to those shown in Table 1.

Table 2. Number of categories successfully classified by students.

Number of categories successfully categorised	Number of students n = 42
12	12
11	0
10	6
9	0
8	6
7	0
6	0
5	0
4	6
3	0
2	0
1	0
0	12

Table 2 indicates that even though 24 students (57%) were able to successfully classify 24 or more word problems, fewer than one-third of the students (29%) were able to correctly classify all word problems into twelve categories. Six students (14%) were able to correctly classify only 12 word problems and another six (14%) were unable to successfully classify any problems into categories. Anecdotal evidence suggests that these two groups of students lacked sufficient motivation to constructively participate in the activities.

Task 2: Criteria. This task involved students developing criteria by which the twelve groups of word problems may be categorised. Criteria used as a standard by which to evaluate student responses were based on Carpenter & Moser (1984) for additive word problems and Maguire (1996) for multiplicative word problems. These criteria arise from a semantic analysis of word problems. A full description of the semantic analysis of word problems is presented in Carpenter, et al.. (1984) and Maguire, (1996).
The criteria generated by students were classified as:

- Operation only - using only the arithmetic operation in order to classify the word problems. This would generate only four categories.
- Operation with additional features - using the arithmetical operation together with additional terms to differentiate between different categorisations involving the same operation.
- No criteria detailed - where the groups detailed no overt criteria by which the word problems could be categorised.

- Overtly stated criteria similar to those used as standard semantic features as outlined by Carpenter & Moser (1984) and Maguire (1996).

Table 3 illustrates the criteria developed by the students to classify the word problems into the twelve categories.

Table 3. Type of criteria generated and the number of students using these criteria.

Criteria	Number of students
Operation only	0
Operation with additional criteria.	30
Standard criteria	0
No criteria detailed	12

This table shows that no students relied solely on the arithmetic operation. The majority of students (30 students) used the arithmetic operation together with additional criteria to differentiate between the word problems which may be solved using the same operation. Twelve students detailed no criteria by which they classified the word problems into the twelve categories. No students were able to state criteria according to the standard semantic features of the word problems.

Intuitive criteria involving both an arithmetic operation and additional features to differentiate between problems requiring the same operation included such criteria as:

- addition in stages - first add one quantity and then following quantities;
- addition of identical objects - addition of "ducks" and "ducklings";
- addition of different objects - addition of "boys" and "girls";
- subtracting different things - subtraction of "boys" from "children";
- multiple groups of containers - seven boxes each containing three balls;
- dividing between people - number for each person.

Some students were able to categorise Cartesian Product problems as probability whilst others identified some word problems as missing addend.

Task 3: Word Problems: Students were required to generate a word problem which reflected the criteria they developed to classify the 12 different categories of word problems and to report their findings to the class.
Table 4 illustrates samples of student generated word problems.

Table 4. Student generated categories and an example of student generated word problem for these categories.

Category (as described by students)	**Example**
Add like objects	*Mary has 5 tazos. She won another 10. How many tazos does Mary now have?*
Adding different objects	*Lonnie has 5 rings and 2 necklaces. How many pieces of jewellery is Lonnie wearing?*
Subtraction to find difference	*Sena has 8 cocktails and Aranda has only 2. How many more cocktails does Aranda need to have to drink as much as Sena?*
Subtraction - how many (different items) more	*Irena has 100 guests for her wedding on the bride's side. There is 34 on the groom's side. How many more guests are on the bride's side?*
Comparative subtraction	*Jane has 6 boyfriends. Helen has only one boyfriend. How many boyfriends does Jane have to dump to have the same amount of boyfriends as Helen?*

Subtraction to find remainder	*Susanna had $50. She spent $35 on a Sportsgirl top. How much money does she have left?*
Probability	*Paula has 4 tiaras, 3 diamond rings and 2 ball gowns. How many outfits can she make altogether?*
Multiplication of lots	*Julia has 10 boxes of Lindor Balls and there are 6 balls in each box. How many Lindor Balls does she have?*
Comparative multiplication	*Jena has completed 2 of her assignments and Suzie has completed twice as many. How many assignments has Suzie completed?*
Division to find a number of people	*Jackie is having a party. She has 100 buritos. Jackie doesn't want any. Each guest gets 2 buritos. How many guests at the party?*
Division to allocate a number of items	*Judith is dating 10 Carlton footballers. She has 20 chocolate bars. If they are distributed equally how many bars does each footballer have?*
Comparative division	*Ira has 12 husbands. This is 3 times as many as Julia. How many husbands does Julia have?*

As can be seen from the above table this group of students was able to generate word problems for each of the 12 categories. Although the terminology differs from that of Table 1, they were able to create categories very similar to those shown in Table 1 and generate an example to illustrate each of their categories. Although the issue of gender within the social context in which word problems were generated was not a factor considered in this study, the very striking references to specific gender roles is worthy of further consideration in a later study. The group which generated these word problems was an all female group. Their word problems appear to reinforce a particular view of gender which may be considered counterproductive to acceptable pedagogical practice.

Task 4: Reflection and discussion. When a spokesperson for each group explained the group criteria and justified their choice to the class, much of the discussion revolved around the diversity of word problems presented. All students were surprised to discover that there were so many different categories of word problems. They were more surprised to learn that the sample was by no means exhaustive.

Discussion

It is apparent that although most students were unable to classify all the presented word problems into appropriate categories (30), those students (12) who were able to successfully cope with the demands of the first task were able to use the categories to devise intuitive criteria. These students were able to follow up their classification with the generation of suitable word problems using their intuitive criteria. It is of concern that no students were able to explicitly state their criteria in terms of a semantic analysis of the word problems. In stead students tended to rely upon the operations they saw as leading to a solution to the word problems. Where necessary students looked for additional features to further refine their criteria. These features (such as "more", "less") were extracted from the context of the problems.

It is quite obvious from Table 1 that all of the word problems presented to the students were those which could be solved using a straight algorithm without having to consider a realistic context within which the problems were situated. Had the word problems provided for these activities included more "realistic" or *problematic* contexts the students would have been introduced to problems more in tune with those likely to be met outside the classroom. This point will be further explored below. The results are in line with those found by Verschaffel et al. (1996).

The present study allowed pre-service teachers to consider word problems as a normal segment of the mathematics curriculum. As stated above, the pre-service teachers taking part in this study were not aware of the diversity of word problems available for inclusion in implementing their curriculum. This is in spite of the fact that word problems are referred to in the well publicised curriculum which they will be required to teach following graduation (Board of Studies, 1995). It is a curriculum outlined in the statements provided for teacher guidance by the Department of Educational, Victoria (Board of Studies, 1995).

Educational implications

There are two main implications of these results, viz

1. The continuing need to acquaint pre-service teachers with the role of word problems within the implemented curriculum.
2. The need to adopt what Verschaffel et al. (1996) refer to as "problematic" word problems so that students can learn to discover the mathematics of realistic situations and not regard school mathematics as something divorced from reality.

The role of word problems within the implemented curriculum

The Curriculum and Standards Framework: Mathematics (Board of Studies, 1995) (CSF) has detailed learning outcomes for students at each level of schooling. Within the CSF the following are instances of word problem learning outcomes identified:

- generate word problems using specified numbers and operations (CSF, p.54);
- generate further problems from familiar mathematical situations (CSF, p.104);
- ask questions to clarify the essential nature of a problem (CSF, p.104);
- identify key information in a problem and represent it using models, diagrams and lists (CSF, p.104).

Although the intended curricula of many countries state explicitly that problems solving in general and word problems in particular have an important part to play in the development of mathematical competence, anecdotal evidence gathered from my own teaching experiences suggests that neither teachers nor their students like generating word problems. Indeed teachers find it rather difficult to generate word problems (Maguire, 1996a), particularly within the primary school system, and many students are so unused to generating their own word problems that they have great difficulty in so doing (Maguire, 1998). In order to overcome these difficulties not only should mathematics methods courses include greater emphasis upon problem solving and generation but continual professional development resources should be devoted to training teachers in the use of problem solving within the classroom. This would allow both pre-service and their practising colleagues to become problem solvers themselves and, hopefully, allow them to benefit from research into problem solving already available.

Problematic word problems

Verschaffel et al. (1996) draws attention to an often neglected aspect of word problems i.e. a lack of real contexts within which text - and teacher - generated word problems are presented to students. An analysis of teacher generated word problems (Maguire, 1996a) indicates that not only had this author (as a class teacher) ignored many of the categories of word problems, and reinforced many of the misconceptions of mathematical operations to students (Greer, 1992), but also almost entirely neglected to include "problematic" word problems within the 200 word problems generated for this author's primary school class. This situation has the effect of divorcing school mathematics from real mathematics. Once again the amelioration of this unsatisfactory situation lies in the introduction of more "realism" into word problems provided to both pre-service and practising teachers.

One can posit the question "Is it any wonder that pre-service teachers are unable to elucidate classification criteria in order to categorise word problems?". The answer to this is "No!", particularly if they are not taught to see the mathematics of everyday situations within the classroom. It is evident to this researcher that school mathematics is not seen as real mathematics by many teachers or their students. The role of mathematics as a community skill is threatened by the generally poor results gained by many English speaking communities as shown in the recently released reports of the Third International Mathematics and Science Study (TIMSS) (Beaton et al., 1996). Even though Victorian primary school children scored higher than their cohort in the U.S., England and New Zealand, they were a long way behind the "leaders". The TIMSS report shows that

Australian children are falling behind their Asian neighbours in their mathematical competences. In order to overcome this lag in achievement additional research is required to determine whether knowledge of word problem categorisation (Carpenter & Moser, 1984; English, in press; Greer, 1992) and practice in problem generation and solving facilitates the understanding and incorporation of word problems into the repertoire of beginning teachers.

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Protasis: A technique for fostering professional development

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The term *protasis* is used to describe an epigrammatic expression which, in an appropriate context, stimulates questioning of assumptions and behaviour. By extension, the word protasis is applied to a disciplined technique for fostering professional development through the use of protases, and a theory is put forward of when and why protasic techniques work.

1 Definitions

Protasis is a noun meaning *premise* or *assertion*, derived from the Greek word *tasis* meaning tension. It refers specifically to the first statement of a syllogism which is usually a generality ('All men are mortal'). It is followed in a syllogism by the particular ('Socrates is a man') from which a conclusion is reached. In the context of professional development, a *protasis* is the action which takes place when an epigrammatic assertion of generality is juxtaposed with particular experiences and challenges assumptions by catching and focusing attention on a significant inner tension, with the potential of leading to self-questioning, and ultimately, to greater clarity or even change. By extension, the word is applied to a concomitant technique for exploiting protases in professional development.

The method is simple enough. A number of brief assertions are used in conversation, displayed on walls, or introduced in some way. When someone 'objects' or otherwise shows interest, they can be invited to offer specific examples which illustrate their point of view, and they can offer or be offered examples which challenge it. From this, discussion and investigation may develop.

A statement is effective as a protasis when it is epigrammatic, laconic, and provokes disagreement or amazement. It seems to demand explanation or refutation. Often it arouses opposition, which is a sign that an inner tension has been generated. Something inside objects, rebels, or is at least quizzical. To be effective, the tension needs to be sufficient to be challenging and not easily dismissed, but not so great as to be avoided.

The power of the use of protases lies in the internal tension experienced, which needs to be allowed to generate an internal action. By niggling away at the back of the mind, a protasis serves to upset standard ways of thinking, to disturb established patterns and reactions, and thus helps to open the way to change and development. As the Zen saying goes "A good explanation never explains everything" (Shigematsu 1981, number 341) and similarly, in the Sufi tradition it is said that "A solved problem is as useful to the mind as a broken sword on the battlefield" (Shah 1970 p119).

Thus it is part of the protasis technique to resist temptation to expound and explain in detail, because the whole idea is that the individual temporarily acts both as if the assertion is wrong and tries to locate counter examples, and as if the assertion has validity, and tries to locate confirmatory examples, before elucidating consequences. In either case they are working away at extracting the thorn which has somehow disturbed, penetrated or exposed their assumptions.

Any general assertion can serve as a protasis for a syllogism, so the term *protasis* can most effectively be used to refer to the action generated when the individual juxtaposes specific examples from their past experience with the generality to reach conclusions. The context in which the assertion is encountered has significant influence on how much attention is given to considering the protasis in the first place. It is the combination of context and source, the assertion itself, the experiences of the individuals, and the way of working with the effects which together constitute the protasis technique.

Clearly not every assertion has the effect of inducing questioning: a wise aphorism such as "We soon believe what we desire", or good advice such as "The end suggests the means" (Polya 1957) may make perfect sense, but usually evaporates almost immediately, precisely because it does not have a hook or barb to catch attention. Similarly, absurd assertions such as "grass is purple" can be dismissed out of hand.

And yet, in the right conditions, even such an outlandish remark as *grass is purple* might lead someone to question why it is that grass is green, and hence to enquire into the scientific basis of chlorophyll and photosynthesis. It might also lead a more philosophically minded person to enquire into how it is that they know that grass is not purple, and hence into a consideration of knowledge and epistemology. Such an assertion, and hence perhaps almost any assertion, can act as stimulus to enquiry. This example may seem farfetched at first, but it illustrates the point that protasic tension depends on the perceiver as well as on the assertion.

2 Background

The protasic technique has been used for centuries in different contexts, but its use in education makes most sense when coupled with a perspective expressed by "You can't change other people; you *can* work on changing yourself." An immediate reaction *against* an assertion such as the last one is often a sign that something of significance has been touched. A classic response to this particular protasis is to look for counterexamples, for examples in which someone has, or appears to have changed someone else. The political arena is often chosen for such purposes, but the existence of the assertion suggests questioning the appearance of change as having been caused by others. For example, it may be that while many forces acting on a person do come from outside them, the person themself is the one who actually makes any changes.

In other words, you may be able to train other people to behave in certain ways under certain conditions, but if you want them to develop a well-spring in themselves from which behaviour stems, so that they can cope with changing conditions and contexts, then it is necessary to employ all three aspects of the human psyche: behaviour (enaction), emotion (affect), and intellect (cognition). Put more forcefully as a protasis, "Only behaviour is trainable; Only emotion is harnessable; Only awareness is educable." The third assertion in this triple comes from Gattegno (1970), but placed in the context of the other two, extends his meaning considerably. The three-fold structure corresponds to an ancient metaphor for human psyche based on the horse-drawn chariot (Rhadakrishnan 1953, Katha Upanishad I.3.3-5). The chariot is the body, seat of trainable behaviour through tinkering with structure; the horse is the emotional force, harnessable but not always easy to control; and the driver is awareness, educable but impotent without connections to the other two. The driver is linked to the horse by reins, and the horse to the chariot by shafts, each of which have interpretations in the structure of human psyche. The terms cognitive, affective and enactive are modern psychological terms for more or less the same structure.

Notice that the preceding paragraph plays a double role. It expands a little upon a perspective which contributes to the effective use of protases, but it also offers an opportunity to consider the conjecture that sometimes elucidation has a deadening (*it's all worked out*) effect, while in other circumstances a little elucidation can intrigue, providing a starting point for further work.

If you can't change others, what can you do? You *can* foster and sustain conditions which make change, or development, possible and desirable. You *can* offer situations and challenges which will bring people up against awkwardnesses and inconsistencies in their own thinking; you *can* establish an atmosphere which fosters self-questioning and exploration of deeply held beliefs. Protasic techniques are a good way of doing this.

It is extremely tempting to try to tell people things, to try to design courses to support professional development in which experts come and tell what they know, or in which people are cajoled into reading what others have written and perhaps then writing summaries of what they have read. These vestiges of Victorian authoritarianism are still to be found in many institutions, because habits die hard, or put more protasically, "Force of habit is a force of habit." Despite Swifts' observation (1726), summed up as "Man is not a rational animal, but an animal capable of reason" we carry on trying to inflict words on people in an attempt to change their behaviour and their thinking. This paper is a case in point. It is incredibly easy in this text-processing age to be drawn into contributing to the ocean of print. The protasic approach is quite the antithesis of more and more words. It depends on a very few words which cut through peoples' defences and start them thinking. Examples have already appeared, and probably make the point most clearly, since if you reflect back on what has been said so far, it is almost certainly the case that the protases are what stand out and are remembered. The purpose of this article is to outline and develop various aspects of a disciplined use of protases.

3 Protasic Techniques

A Conjecturing Atmosphere:

It is crucial to the protasic technique that people receive useful support from colleagues. This is essential for several reasons. First, it is very easy to get wrapped up in a world of your own, and to be blissfully unaware of habitual forms of thought and interpretations. Second, colleagues can be tremendously useful as sounding boards. Third, human beings seem to have a natural propensity for seeking resonance from other humans. We all like to feel that we are making sense, not just to ourselves, but to others as well. A conjecturing atmosphere is established when people are ready and eager to try out ideas when they are unsure, and to listen and question ideas of others, not with the object of proving them wrong, but simply of trying to clarify what they mean. The natural propensity to seek confirmation in others is exploited later in the protasic approach to validation and epistemology.

Explanation and Elucidation

A succinct assertion or premise is just words. In order to make sense of a protasis, it is necessary to locate meaning, not just in individual words, but in their juxtaposition as well. Dictionaries are usually of little use, because the whole point of a protasis is to challenge accepted meanings. By being interpreted in different ways, or placed in different contexts, the words can release different insights. A protasis which admits several different meanings, often at different levels of outer and inner experience, is more likely to have a prolonged and invigorating life than is an unambiguous or clever assertion whose challenge quickly wears off. It is often tempting, especially in the face of a remark such as 'that statement means absolutely nothing to me', to break into explanation. It has been found much more effective however to build up a collective meaning in a group by working on a protasis in the following way.

First, savour the assertion

Let the words roll around in the mind, attracting, as they almost certainly will, various associations and memories of past events. Some will appear to offer counter-examples, and others will appear to be supportive. It is often illuminating to observe your own propensity (to seek counter-examples or to seek confirmations), and then to try to change that tendency by seeking the other. Of course, neither counter-examples nor confirmations are enough in themselves, since the scope of validity of an assertion depends on context and personal meaning (confirmations), bounded by counter-examples.

Second, seek connections, not explanations

After a period of contemplative seeking of connections and associations lasting as briefly as a few seconds or as long as several weeks, participants offer brief-but-vivid accounts of particular events which illustrate for them some aspect of the assertion. The essence of a brief-but-vivid account of some event or situation is that everyone recognises it - not in specific detail, but in form. Most human experience is universal. Even if we have not been in the same specific situation, there are usually elements which can be recognised. If this were not so, then stand-up comedians and popular psychology tracts would have only a limited circulation. In fact they translate well across cultures. It does take some discipline to learn to describe succinctly, without excessive embroidery, typical events which others also recognise, and to avoid theorising, deducing and explaining. The less these energy sapping behaviours are employed, the better.

Before you try to account *for* something, first agree an account *of* it. It is all too easy and attractive to launch into generalities and explanations before agreeing on the details, but if you are trying to *account for* something, it is important that there is agreement first on the phenomenon being accounted for. So start with brief-but-vivid *accounts of* incidents. Although others may appear to resonate with your generalisations, there will be much more likelihood of resonance and communication if you first share specific vivid details of typical events and situations. Generality emerges most profitably through widespread resonance with other peoples' experience rather than through the uttering of abstracted explanations.

Third, negotiate examplehood and interpretations

As various people contribute brief-but-vivid accounts of situations which are resonated in them by some assertion, it usually turns out that there is a wide variety of interpretation. It is then up to the group to negotiate whether a brief-but-vivid account constitutes an example of some aspect of the protasis or not. Negotiating usually takes the form of recognising in an account-of some aspect which helps to elucidate the protasis, or deciding that the particular account-of is not sufficiently illustrative to be helpful. Some accounts may be rejected, others may be sought in order to add substance to aspects of the protasis not yet illustrated or illuminated. This of course takes time.

Fourthly, maintain a questioning stance

As brief-but-vivid accounts-of are accumulated, the protasis begins to take on richer meaning. There is then a danger (any technique which has potential must have a corresponding danger) that the effect of the protasis will be reduced, even eliminated. An essential part of the discipline is a frequent return to the original assertion, to seek alternative interpretations and meanings. A protasis is effective to the extent to which someone finds themself feeling appropriately uncomfortable and stimulatingly challenged. An easy resolution may equally well be a defensive response to a deep-seated tension, or an indication that there is nothing of import. The effectiveness of a protasis resides partly in the assertion, partly in the person, and partly in the context in which it is encountered.

Don't look for instant results

A successful protasis works over a long period of time. It usually happens that as time goes on, your perception of meaning changes and develops, becomes richer and more complex, and thereby opens up new aspects for detailed attention. Thus, it is not to be expected that a single meeting will move from first encounter to elucidation. Rather, the technique of sharing and discussing accounts of relevant situations is in order to establish a reasonably harmonious basis of meaning. As further observations are made, the same technique of presenting brief-but-vivid accounts will help colleagues to modify and enrich their meanings of the assertion.

4 Psychological Aspects

Slogans

A slogan, such as appears in advertising, appears to say a lot, but on inspection, actually says very little. Their essential aim is to cue the audience at some time in the future (when making a purchase or voting, for example). They are limiting and enthralling (in both senses). By contrast, a protasis initially may appear to say little, but on inspection is found to say a great deal, once an action takes place internally. It has the potential to inform not just a single act in the future, but future practice.

Protases can easily become slogans if they are bandied about without reference to or contact with the experiences to which they are linked, but then any words can become superficial jargon if divorced from experience.

Metaphors

There are several common metaphors to describe the working of the protasic technique, with some if not complete accuracy. Since language is based on metaphors which help us to form abstract ideas based on our experience of nature, it is useful to explore explicit metaphors which help to describe the protasic technique.

The grit of sand which certain oysters turn into pearls suggests thinking of a protasis as something which disrupts a smooth flow and which requires working on, or around, in order to return to equilibrium. Often the result of such work is a pearl of great price, in the form of a revised or modified belief structure and corresponding practice. It may also simply amount to a wrinkle, but this too represents growth through adaptation. One result of adopting the perspective on change suggested in section 2 is that there is no certainty that a particular person will elect to develop and grow along predetermined lines. Accommodation of grit may take a variety of outward forms and shapes.

Grit is a form of disturbance, and any metaphor of disruptions or turbulence could be applied to the protasic technique. Images such as a stream diverted by a rock, a sailboat functioning because of the turbulence it sets up in the wind, or the profusion of activity at the boundary of fields and forests, of water and land, can be used as texts for reading aspects of the effect and nature of protasis. It is at the edges, where Vygotsky's zone of proximal development lies, that change and development, creativity and experiment, are to be found.

The garden metaphor for teaching seas teachers planting ideas as seeds in pupils minds, and nurturing their growth. The same could be applied to the protasic technique, with the protasis as a seed which may germinate if the conditions are right. Certainly people often report that only after many years do they begin to see what the assertion might be suggesting, as if it had lain dormant yet somehow active all that time, a seed waiting for the correct conditions to germinate and so become visible.

A protasis can be likened to an itch which simply has to be scratched. Scratching corresponds to trying to elucidate it, and to reconcile it with other beliefs. The compulsiveness of the itch speaks to the disturbing, inescapable nature of the tension set up by an appropriate protasis. A Zen form of scratching is particularly catching because of its reversal of the usual experience: "Scratch first, itch later". (Shigematsu 1981, number 1171)

Three Factors

It was pointed out earlier that the effective working of a protasis depends on three factors the assertion itself, the person encountering it, and the source and context in which it is encountered. To be effective, the assertion must have some feature which attracts attention. Common devices include juxtaposition of unexpected ideas ("Sleep is to hunters as excitement is to students" Shah 1970 or "Wounds are to a patient as assessment is to students"), reversing a cliché ("I see what you are saying, but are you saying what you see?"), imposing a surprisingly severe restriction ("Only awareness is educable"), drawing attention to an implicit relationship ("The subordination of teaching to learning" Gattegno 1970), and combinations of these.

A protasic action is set up most effectively when the particular assertion is met in a context which has some importance for the individual, or comes from a source in whom the person has some investment. If you encounter a slogan on a wall which you disagree with, you are likely to dismiss it. If you read an epigram in a book which impresses you, or by someone you respect, then you are much more likely to try to make sense of it. If someone whom you respect draws your attention to it, and seems to imply that there is something worthwhile in it, then a tension is set up between your respect and your reaction. Sperber and Wilson (1988) build a whole theory of communication around the single observation that we expend energy trying to construe only when we think that someone is trying to tell us something. Protases form a class of stimuli which depend on investment in the source to justify the effort needed to make sense of them.

Typical Responses

There are four types of typical responses to protases, based on pairings between immediate reactions and longer term effects.

Rejection/Rejection

The most obvious case is an immediate, and a long term rejection. This occurs where the assertion is too much at odds with the person's own perspective, and where it comes from a source in which the person has little or no investment. In these cases, the person is usually quiet about the matter, and slips away almost unnoticed. It is possible that with continued exposure, a person might begin to consider the assertion, but that is only likely if respected colleagues or leaders also take it up.

Rejection/Consideration

This is a well known phenomenon, particularly in examples of religious conversion and major shifts of perspective. At a more mundane level, it is often the people who react and complain most strongly at first who are likely to go away and work on an idea. Their initial reaction is merely a manifestation of a penetrating thorn or embedded piece of grit.

Enthusiasm/Consideration

There are some people who react enthusiastically right from the start, and who do go away and consider the assertion over the longer term. Typically, such people find that the assertion crystallises thoughts or feelings that they have already been having, however ill-formed and inchoate. Initial enthusiasm is most usually not a sign of potential active consideration in the future. Most frequently it is an indication of respect for the source rather than an indication of having encountered a significant idea, and so will soon die away.

Enthusiasm/Rejection

It is typical that an initial enthusiastic response is followed by rejection, not in the sense of a conscious act, but rather in the form of forgetting all about it. In other words, the enthusiasm acts as a barrier against truly encountering the challenge implicit in an assertion (e.g.. "Sleep is to hunters as ... "). Energy generated by an initial challenge is dissipated in enthusiasm rather than focused on elucidation. Where the assertion is not found challenging, then it is quite likely to be forgotten, simply because it makes no lasting impact, being too quickly integrated into the old perspective.

Energies

As indicated already, the effect of a protasis, its source and context, and the perceiver, is to generate energy. There is no guarantee what people will do with that energy, but the suggestions made in section 3 have been found to contribute to productive use of energy released. The aim of those techniques is to minimise the opportunities to dissipate energy in criticism, judgements and easy dismissal, by drawing upon experiences and looking for common themes. In any discipline it is essential to have an interplay between the particular and the general; so too in professional development it has been found to be critical to avoid generalities until particulars have been negotiated. Otherwise a lot of energy disappears in making and rejecting general comments which are so interpretable in so many ways as to block all attempts at precision, and simply exercise previously established prejudices.

5 Validation

The aim of this section is to outline an approach to validating both protasis as a phenomenon and the protasic technique as a means of enhancing or supporting the use of protases. The method proposed for validation is consistent with, and an extension of, the discipline involved in the protasic technique itself, and is just one component of a complex supportive environment for fostering professional development of any kind.

Epistemology

What does it mean to say that the protasic technique is effective? One approach would be to undertake psychological studies of large numbers of people, subjecting them to various kinds of assertions under various conditions, to observe their subsequent behaviour, and to interview them and their colleagues in order to try to detect traces and effects of the particular protasis. This detached approach is, however, out of keeping with the spirit of the protasic technique as outlined here, because it is based on a cause-and-effect view of how actions come about. Protases are but one component of an action which may or may not take place. Techniques for working with protases can at best enhance the possibility that an action will take place, but cannot guarantee it.

It is difficult to see how to describe a protasis, the state of the recipient and the relation between recipient and source as predictors of protasic effectiveness in terms which are not ultimately tautologous. Since the concern here is with its effectiveness as a technique for fostering professional development (in conjunction with many other techniques), it is appropriate to suggest that validation of effectiveness should be undertaken in a manner consistent with the discipline to which it contributes.

That discipline is based on noticing opportunities to react creatively, and choosing how to behave in the situation. It has been elaborated in some detail elsewhere (Mason 1984, 1986, Mason and Davis 1987, Mason 1996, 1997). What is relevant here is the underlying epistemology which supports and justifies the discipline, and gives rise to the methodology for validation.

The prime assumption in this epistemology is that people try to make sense of the world. They do this by telling themselves stories, by trying to account for the things that they notice. Feyerabend (1991 p141) put it that "All you can do, if you really want to be truthful, is to tell a story", and Bruner (1996) sees it as the core and essence of being human. What do we tell stories about? We tell them about the fragments of our experience, that is, about what we notice, about things which stick out in our minds for some reason. We account for them, we weave them into a narrative which helps us integrate them, helps remove the sharp edges and inconsistencies, and reinforces our sense of continuity. In short, it helps make sense of experience. It is obvious that we can notice only that which we are attuned to notice, and it is clear that we cannot use or build on what we fail to notice. In a sense then, what we know at any given time, is, effectively, what we are attuned to noticing together with whatever metaphoric resonances and metonymic triggers which take place. Our brains of course form impressions of aspects which we do not consciously notice, and these form the basis for noticing in the future.

The effectiveness of a protasis for a particular individual at a particular time resides in the extent to which the protasic action, which consists of questioning, conjecturing, observing, and experimenting, assists the person to notice more, or more sharply, and as a result of that noticing, to be able to choose alternative responses to common situations.

To validate the protasic technique consistently with the discipline proposed here, it is necessary to look back into your own experience, to see if you recognise times when some assertion got under your skin in some way, either as a child when some adult's remark really got to you, or when a colleague criticised you in some way and you reacted at first and then gradually came round to questioning your behaviour, or in some other way. If you can work with colleagues, describing situations briefly but vividly, in an account-of rather than with explanation and justification, so much the better, since then the protasic method is likely to become meaningful much more quickly for you.

Another activity is to look out for situations in which you find yourself caught, even just momentarily, by some challenging remark. The idea is to catch yourself reacting, and to bring to the situation your sense of the protasic action. As a result, you may find that you are able to allow the protasis to work away inside you instead of rejecting the disturbance out of hand.

Another activity step is to work with a group of colleagues, taking some controversial assertions from a respected source, then working away at them as outlined above.

Having undertaken this discipline for a period of time, acting-as-if the protasic technique has some substance, you may be in a position to judge whether you think that there is something in it for you. It may be that for some reason, it doesn't speak to your experience, or that you are unable to interest colleagues in co-operating. It may be that the protasic technique is not suitable at present, or that you have not come across a sufficiently challenging assertion (though there should be several in this paper!). The most that can be deduced from observations like these is that protasis as you construe it from what I have said is not relevant to you at present. Thus the technique can be falsified only by individuals and groups at particular times in particular situations.

6 Sources

The roots of the technique are as old as recorded history, for it is the catchy but slightly ambiguous or enigmatic protases which tend to survive. The writing on Nebuchadnezar's wall, proverbs and parables of the prophets, Vedic hymns, Egyptian and Greek myths, Aesop's fables and the 'foolish' tales of Mullah Nasruddin and Til Eulenspiegel, Zen koans and modern folktales all have a protasic element. One-liners such as *Religion is the opiate of the masses* , and koans like *the sound of one hand clapping* find their reflection in the stock in trade of stand-up comics. The ones that stick are the ones that set you thinking.

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Towards a psychology of knowing-to

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While considerable attention has been focused on forms of teacher knowledge (eg. Shulman, Elbaz, Petersen, Carpenter *et al*) as essential components of expert and effective teaching, but knowing-that, knowing-how, and knowing-why are insufficient to enable a teacher to act upon that knowledge. What is critical is that teachers know-to act in the moment-by-moment unfolding of a mathematics lesson. We distinguish different types of knowing, and following Gattegno, relate these to levels of awareness. Our methods use descriptions and illustrations such as a fine-grained study of one teacher involved with innovative materials for teaching algebra to 13 year-olds to help the reader experience distinctions which we have found fruitful. Furthermore, the situation is self-referent, for what we want children to learn is to to know-to act themselves when formulating or solving problems. Drawing on the study, on our experience, and on the experience of others, we move towards a psychology of how knowing-to arises and can be supported.

Background

As a teacher, I know (theoretically, in that I have used them) many many tactics for use when teaching mathematics, but in the moment I do not always know-to use them: they do not always come to mind. As a student of mathematics, I know many many techniques, heuristics, and processes, (again theoretically in that I have experienced them and even written about them), but in the moment when working on a problem I do not always know-to use them: they do not come to mind.

Whitehead (1923 p7) described the central problem of all education as 'the problem of keeping knowledge alive, of preventing it from becoming inert' (p7). He forcefully asserts that 'this evil path represented by a book or set of lectures which will practically enable a student to learn by heart all the questions likely to be asked' (p7) 'culminates in a uniform examination [which] is so deadly' (p8). He wants theory to be applied within the curriculum, but points out that how this is done will depend on the 'the genius of the teacher, the intellectual tyope of the pupils, their prospects in life, the opportunities offered by the immediate surroundings of the school, and allied factors' (p7-8). The notion of inert knowledge has been taken up and examined by a multitude of authors seeking both explanation for it and strategies to over come it (see Renkl *et al.* 1996 for a survey). We are trying to probe what it is like from the inside when requisite knowledge does come to mind, and means for assisting that process.

Despite passing tests and exams and so indicating the presence of knowledge (however short term), it is very possible to get into a situation and not to have relevant 'knowledge' come to mind. For example, Schoenfeld (1988) describes in detail how students who were known to know all requisite facts did not (Schoenfeld says 'were unable to') employ them to achieve a geometric construction.

Shulman (1987) found that a teacher with a broad subject knowledge was the most 'conceptual' in his teaching, and a teacher with little subject knowledge was the most 'rule-based'. This fits with experience, for when faced with a problematic situation, people will move to where they are confident, controlling uncertainty by specialising, restricting, and falling back on confidence-inspiring strategies and entities (Mason 1993).

But broad subject knowledge in itself is insufficient to make an effective teacher. Dewey (1902) distinguished between the expert immersed in advancing a subject, and a teacher who is concerned with how 'the subject matter may become a part of experience' (p22). He called this, the *psychologising* of the subject matter:

> 'to see it is to psychologise it ... It is the failure to keep in mind the double aspect of subject-matter which causes the curriculum and the child to be set over against each

> other . . . The subject-matter, just as it is for the scientist, has no direct relationship to the child's experience. ' (p22)

The *transposition didactique* (Chevellard 1985, Kang & Kilpatrick 1992) has become a well used label for the transformation of expert awareness (subject matter as it stands to the specialist) into instruction in behaviour (as it is construed by the student). Dewey, and every committed teacher and author, constantly struggles against this transformation.

Awareness

Gattegno (1987) used the term *awareness* in order to approach Dewey's concern and the *transposition didactique*. For him, an awareness need not be conscious, because it may have been integrated into functioning. He referred to awarenesses which enable people to act, and to awareness of those awarenesses, which creates a discipline. We go further, for it is awareness of discipline, which is necessary in order to teach. In order to keep the levels and forms of awareness to which Gattegno draws attention clear, it will be convenient to speak of *awareness-in-action* to refer to awarenesses which make it possible to perform actions, but which we need not be aware of explicitly in order to carry out those actions. The term draws metonymically on Vergnaud's notion of *theorem-in-action* (Vergnaud 1981, Binns & Mason 1994). Thus,

> You can count without being aware of one-to-one correspondence, you can add, subtract, multiply, and divide, without being explicitly aware of your awarenesses-in-action of numerals, place-value, routines, the role of order, etc. which make that arithmetic possible.
>
> You can form and detect patterns and locate formulae that generalise specific cases without being explicitly aware of your awarenesses-in-action of same and different, relatedness, induction, stressing and ignoring.
>
> You can combine fractions according to rules, without being aware of how fractions relate to decimals and to integers, and how they generalise number, without being aware of the slide between operator and object, without relating them to a number-line.

Awarenesses-in-action include the natural powers we all possess for making sense of the world, manifested in specific ways when mathematics is the content. The behaviours to which awarenesses-in-action contribute can to some extent be trained without explicit reference to awareness, for it is not necessary to be explicitly aware as long as situations remain routine and do not require innovation, novel interpretation, or creativity. Thus, short term success can be achieved in getting children through assessment hurdles, or teachers through in-service events on reform topics such as investigations, co-operative learning, structured lesson plans, constructivism, and whole-class teaching. But education is about more than training of behaviour in routine actions on symbols on paper, and teaching is about more than carrying out sequences of instruction. Teaching is about inducting children into disciplined forms of thinking and perceiving, and these emerge when awarenesses-in-action are made explicit and formalised.

Gattegno defined a discipline as the study arising when people become aware of awarenesses-in-action (Tahta 1988 quoting Gattegno): "Sciences are born when someone states that what occupies his mind *is*, and, because of that, is part of reality and worth being considered by others."

Mathematics arises as a discipline when we become aware of the awarenesses-in-action that constitute counting, ordering, classifying, and relating, and start to formalise these in the languages of algebra and geometry. Gattegno suggested that algebra emerges when we become aware of our awarenesses-in-action involved in the dynamics of relationships, and geometry through awareness of the awarenesses-in-action involved in the dynamics of our minds (of which mental imagery is a major part). For example, when we attend to how we order, how we classify (the notion of equivalence, of same and different), how we count (through one-to-one correspondence), and formalise these in general statements we produce algebra, and simultaneously, the rules of algebra. When we attend to our powers of mental imagery to conceive mentally of points, lines, curves, surfaces, and movements and inter-relations between them, when we formalise them into assertions of what *must*, *might*, and *mayn't* happen (Mason 1991), and justify these assertions, we develop the discipline of geometry.

What makes teaching a discipline? In his comprehensive review of teacher knowledge, Shulman (1986) reminds us that the original meaning of higher degrees (masters and doctorate) was recognition of the ability to teach the subject, not just prosecute it as an apprentice (bachelor). He quotes Wheelwright's Aristotle:

> We regard master-craftsmen as superior not merely because they have a grasp of theory and *know* the reasons for acting as they do. Broadly speaking what distinguishes the man who knows from the ignorant man is the ability to teach, and this is why we hold that art and not experience has the character of genuine knowledge (episteme) – namely that artists can teach and others (i.e. those who have not acquired an art by study but have merely picked up some skill empirically) cannot (Wheelwright, 1951, p69).

We are going further and suggesting that to teach requires more than knowing reasons. It requires knowing-to act, prompted by awareness of awareness of awarenesses-in-action, in order to provoke students into themselves knowing-to act.

Knowing

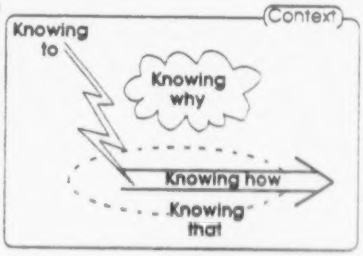
The noun *knowledge* has an unfortunate finality and rigidity associated with it, implying a finished product separated from time and situation. By contrast, the gerund *knowing* carries a sense of dynamic change situated in the moment.

Forms of Knowing

Changing prepositions often provides insightful contrasts. We find it useful to distinguish between

Knowing-that:

- the sum of the interior angles of a triangle is 180 degrees, ...;
- fractions and decimals are names for numbers;
- doing-and-undoing is a major theme in mathematics generally and in algebra in particular;
- stressing and consequent ignoring is the process behind generalisation;



Knowing-how:

- to add two fractions, multiply two finite decimals, find the area of a triangle;
- to stimulate productive discussion amongst pupils such as talking-in-pairs, say-what-you-see, card-sorting, conducting an interactive lesson in silence;
- to provoke disturbance so there is something to conjecture and resolve;

Knowing-why:

- only some numbers with infinite decimal names also have fraction names;
- to divide one fraction by another you flip the second over and multiply;
- discussion with peers can be an aid to learning mathematics;
- a task has been structured in a particular way in a text-scheme;

Knowing-to (in the moment when needed, not just theoretically):

- use some strategy or heuristic;
- draw attention to some mathematical thinking process just provoked;
- draw attention to some mathematical theme that has emerged;

divert from the text-scheme because of genuine class interest and readiness.

Knowing-why means having some 'story' to account for knowing-that or knowing-how, but the story does not have to be valid or true in someone else's theories.

We are not claiming hard and fast distinctions between these categories. Some know-how depends on knowing-that, knowing-to draws upon knowing-how, and knowing-why encompasses knowing-that and knowing-how. But it is often the case that people apparently know-that, yet not how to act on that knowledge; know-how but fail to recognise an opportunity to employ it; know-why something must be the case but do not use it.

It is not surprising that the first three of the *knowing* modes have been used before. For example, Ryle (1949) used them in the context of a discussion of mind, and Miller, Malone, & Kandi (1992) used similar terms.

Skemp (1979) distinguished *knowing that, knowing how*, and *knowing to*, though he described the latter in terms of *being able to* use a technique in a novel situation. That particular choice of language, which is common, is frequently misunderstood as 'able to in the sense of knowing-how, that is, having demonstrated out of context. By using the language of 'knowing-to in the meoment', we can make progress beyond the simple possession of abilities.

Biggs (1994) rehearses Ryle's distinctions and then directs attention to what he considers to be a different way of considering knowledge, in five hierarchical levels following Bruner's spiral curriculum and resonant of van Hiele's levels (1986):

Tacit: manifested through doing without conscious awareness or stories

Intuitive: directly percieved or felt

Declarative: description of how and why expressed in some symbol system that is publicly understandable

Theoretical: abstracted or generalised statements going beyond particular instances

Metatheoretical: knowledge about the process of abstraction and theory building

Biggs includes two additional forms, *Procedural* and *Conditional* which do not seem to fit into his hierarchy. Yet each of these seven forms finds their equivalent in combinations of the types of knowing we have delineated, when coupled with awareness. Our distinctions have the advantage of being based on what it is like as a knower, rather than being analysed from outside.

Jong & Ferguson-Hessler (1996) use a matrix of types against qualities in order to classify research on knowledge. They use categories such as *situational knowledge, conceptual knowledge, procedural knowledge*, and *strategic knowledge*, and qualities such as *level* (as in surface or deep), *connectedness, automatedness, modality* (as in verbal and pictorial), and *generality* (as against domain specific). Again, these categories are based on the same sorts of distinctions we are making, but whereas Jong & Ferguson-Hessler are trying to analyse other people's knowledge, we are trying to get at how it is that people know-to act in the moment.

Renkl *et al.* (1996) review much of the literature on inert knowledge as a common phenomenon. They locate three types of explanation for it: *metaprocess* (there is a disturbance to accessing what is needed), *structure deficit* (there are aspects missing in what is known), and *situatedness* (mismatch between current situation and previous situations). In the last category they note particularly Clancey (1993), and Greeno, Smith & Moore (1993) who see knowledge as relationally defined and not a property of the individual. Greeno *et al.* use the analogy of motion. Motion is not a property of the object because it depends upon frame of reference; motion is a relation between a frame of reference and an object. So too with knowing.

Proponents of situated cognition often write as if they have eliminated questions of behaviourists 'trasnfer', but fail to address the question of how the situatedness in which a skill is employed broadens and becomes more general, more abstract. The language of knowing-to provides an appropriate access to such questions.

Shulman (1987) identified a number of different components of teacher knowledge: subject matter content knowledge; pedagogical content knowledge, knowledge of other related content, of curriculum, of learners, of educational aims, and general pedagogical knowledge. However he made no overt reference to self-awareness, to the dynamic of knowing which co-emerges with the situation, or to what it is like to be a learner (unlike Elbaz 1983). Rather, it is all knowledge about: knowing-that and knowing-how. Shulman located his categories through interviews with teachers about what they thought about and knew about, but, of course, these interviews did not take place in the moment of teaching. The research methodology naturally leads to knowing-that, with room for knowing-how and knowing-why to emerge, but is much less likely to highlight knowing-to, since that involves the internal psychological process of noticing an opportunity (Mason 1994). When teachers are able to re-enter the situation without elaboration or justification, some 'inflight-decisions' are captured. But when teachers elaborate and reason about what they 'must have felt', access to knowing-to is lost.

There is a parallel with teachers asking pupils reflection-intended metacognitive questions such as 'how did you do that?'. Asking questions does not necessarily draw attention to the fact of the question, does not necessarily expose such a question as useful, and the response does not necessarily reveal what it was actually like. Creating an inner teacher (Hyabashi & Shigematsu 1988, Mason & Davis 1989) through metacognitive intervention depends crucially on teacher awareness of their own awareness of their awarenesses-in-action.

Evidence

Our methods involve seeking confirmation and challenge in the experience of others. We aim to convince not by analysing data in an attempt to prove something, but rather by trying to provoke your awareness of the distinctions we ourselves find useful. In the short space available we offer, in addition to our exposition in the previous sections, two brief extracts from a study (Spence 1996) which suggest that the phenomenon we are addressing is recognisable.

Discussion in pairs

The following brief extract has been chosen to highlight both the effect of a knowing-to, and consequences of not knowing-to with respect to generating discussion.

While talkjing with the researcher about students' dependency, the teacher asked what could be done about it. The researcher suggested "I would be inclined at this point just to step back and let them struggle with it for a while rather than you doing all the struggle for them".

In the next lesson on pattern recognition and expression as a formula, the teacher was for the first time observed letting go of his usual firm direction and control of what was said and done by the students. Square brackets are researcher diary notes made during the lesson.

T1: What did you learn from p 25? . . . (little or no response)

T1: Share with someone what you learned from p 25. (buzz of conversation)

[This is unusual behaviour not previously seen. There were periods of children disagreeing with what another has said; of teacher writing at the board with a succession of organising questions.]

T1: Is that a formula for odd numbers? *S*: yes; *T1*: yes it is

[Later: kids are asking teacher "How did you know that?" and "Why does it do that." The atmosphere has switched from a pupil's earlier 'just tell me the answer and I'll be happy' to animated enquiry. The researcher was excited by what she had seen, but as the teacher approached her at the end of the lesson it was clear he was dissatisfied as he threw his hands in the air in a gesture of frustration.]

T1: "Help me. They're just not getting this"

R: "Some of them do. This is the first time they seem to care. They care enough to ask questions."

T1: "I knew I could trust you to make me feel better about this lesson."

The pre-lesson discussion seems to have supported a knowing-to get the children discussing in pairs in in that lesson which continued in subsequent lessons but was not used with other classes. The teacher's request for help can be interpreted as awareness of a missed opportunity, of not knowing-to do something, but also an unarticulated awareness that something was possible. We see the teacher as caught up in his agenda and in his perception and frustration that the pupils are not 'getting formulae' which is where his attention is focused. Yet the researcher could see a marked contrast in pupil involvement from normal practice. The teacher does not seem to be aware of the fact of his formula-focus, nor how it blocks children in working on locating and describing patterns. Without this awareness he is unlikely to be able to suspend it or let go of it as a goal and listen to what the pupils are saying and doing, using his awareness as a guide rather than as path.

Is Zero Even?

During work with sequences of integers (called number-strips because they are displayed in consecutive cells on a strip of paper), the issue arose in a lesson of teacher T2 as to whether 0 is to be considered even. Student S1 remarked that zero was not even as it was only a place holder. For S2, being a place-holder meant having no meaning. Students came up with a variety of justifications: 'It's got to be even because every number that's got a 0 on the end', '−1 and 1 are odd, so ...'. During the discussion, the teacher was seen for the first time to be departing from the material, opening up and sustainging discussion. S1 still insisted 0 was not a number. At one point the teacher says

> S6 triggered something in my mind. What happens when, I think you defined all the even numbers as 2 times the step number. OK. Which means that I also could take the number divided by 2 (writes on chalkboard) and get the step number can't I?
>
> S6: Yeah
>
> T2: So when you take 0 and divide it by 2, is there a remainder or is there not a remainder?
>
> S6: There's not a remainder.
>
> T2: So there's no remainder.
>
> S7: Yeah, but it's not even because ...
>
> T2: So what happens with a negative 2?
>
> S6: It's an even number.
>
> T2: OK. So did we agree that every other number is an even number?
>
> S7: Yeah, but ...
>
> T2: (laughs). Keep thinking about it. We'll play with it. Keep thinking. I want to get out of the way. I want you to go on now [to the next work] ...

[The class is over. Children leave the room while the teacher is still talking and trying to explain. As I leave, the teacher asks me to come tomorrow with ideas about what he can do.]

> T2: I'm not doing too well here.

The teacher has unusually allowed the lesson to follow an unexpected direction, but remains dissatisfied. The researcher saw an opportunity for work on what makes a number a number, to draw upon historically different perceptions (to ancient Greeks, not only was zero not a number, neither was one or two), to engage with children in a perennial philosophical questions about the denotation of 'nothing' (Rotman 1987), and to develop the notion of definition in mathematics. With the requiste awareness it was an opportunity for checking whether something has required properties, and then either accepting it as satisfying the definition, or modifying the definition so as to exclude zero, and for finding that definitions are framed so as to make theorems easy to state rather than having objective existence, in true Lakatosian style (Lakatos 1976). In fact, no reference was made to this issue in the following lesson.

Text-scheme authors have to depend upon teachers seizing opportunities to develop aspects of mathematical thinking and mathematical themes as they arise, for it is not possible to guarantee that instructions written on a page will generate specific discussion, no matter how well the materials are trialled and honed. Successful teaching depends on teacher awareness in the moment, their knowing-that, knowing-how and knowing-to all in one instant; their awareness of mathematical content, themes and processes all coming to the surface at once.

It is much easier to see opportunities from the back of a classroom than when you are caught up in the midst of the lesson. That is why observing others can be as important as having someone observe you. Opportunity can only be seen for yourself if you have the requisite awareness. It is not sufficient to have been through a course which 'covers' such topics, not even enough to be able to answer direct questions about whether zero is a number from someone outside of the classroom. There has to be a match between the situation as it emerges and the ethos in which the teacher works.

Towards a psychology of knowing-to

Consistently with our methodology, we have interrogated our own experience to locate what it is that informs our knowing-to in the moment, as well as observing and talking to others, and getting them to try things out in theoir experience. We have found, as illustrated above, that *grasping* (being caught up in a dominant agenda) can block a freedom to go with the flow guided by mathematical awarenesses. But *going with the flow* without richly developed mathematical awarenesses is as likely to divert attention away from mathematical thinking as towards it. Whitehead and Shulman are correct in stressing that the teacher needs active content and pedagogic knowledge.

In the midst of the flow, something in the situation triggers recognition of a possibility, whether as student or as teacher. We have found that collecting brief-but-vivid accounts of situations which share the possibility to act in a certain way, combined with a label to act as a focus or hub for that act, enhances the possibility that that act will occur to us in the future. Furthermore by imagining ourselves in such a situation and actiung in the desired way, we can further enhance the poossibility of the act coming to mind (Mason 1996).

Knowing-to in the moment is not usually a process of carefully considered options. Rather, something comes to mind as a possibility at the same time that it starts to happen. This requires a sensitivity to the situation, and a rich collection of prepared actions. Labels such as *transposition didactique*, and many others, can serve as hubs in a network connecting situations and actions, allowing sensitivities and then actions to be triggered by situations.

Teachers taking Open University Inservice courses often report that the label-frameworks which form the core of the course have served to remind them to act differently, that is to know-to act. But this only happens when the teacher has been able to link past experience, future actions, and a label, in their own experience.

The principle lesson for us is that training behaviour is useful, but educating awareness is crucial, even if it requires time in the presence of a respected other. Certainly there is need for a community of enquiring, questioning colleagues. As individuals we are trapped in habits, and as the epigram says "Habit forming can be habit forming".

Teachers are traditionally isolated in their classrooms and trapped by fear of betraying their uncertainties in subject matter, pedagogy, and didactics, at all of Shulman's levels (*op cit.*). It is easier to automate and habituate as much as possible so as to reduce strain and energy drain. Unfortunately it also reduces the potential for teaching to stimulate learning.

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School-based assessment in VCE Mathematics: Ten years on

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Since the introduction of the Victorian Certificate of Education in 1989, assessment in VCE mathematics subjects has included school-based tasks undertaken over an extended period of time. This paper reports on an evaluation of the latest of many changes that have been made to these tasks since their introduction and examines whether they have retained their validity for assessing problem solving and investigative project work. Student use of powerful technology is identified as an emerging issue in their future conduct.

History of Change

Since the introduction of the Victorian Certificate of Education (VCE) in Victorian secondary schools in 1989, assessment in VCE mathematics subjects has comprised a mix of school-based assessment tasks undertaken over an extended period of time and traditional end-of-year examinations. Two types of school-based Common Assessment Tasks (CATs) have been used: an investigative project CAT and a problem solving CAT.

The history of the problem solving CAT is reviewed in McCrae and Stacey (1997). Until 1992, students were given two weeks to solve one of three or four centrally-set 'challenging problems' and to prepare a written report of up to 1000 words describing their solution. Because of concerns over the authentication of students' reports (Stephens and McCrae, 1995) the *Challenging problem* CAT was suspended for 1993 and replaced in 1994 by a two-component *Problem-solving* CAT. The first component of the new CAT is similar in nature to the original CAT, requiring students to prepare an 800–1200 word report over a two-week period on one of three centrally-set problems. However, the problems are more structured than the earlier 'challenging problems', each one consisting of a sequence of specific questions about a particular situation. Hence, it is clearer to both teachers and students what is required and so it is possible, for each problem, to set a test of the mathematical techniques used in its solution: this is the second component of the *Problem-solving* CAT.

The one-hour test is conducted a few days after the report is due and has a similar, but not necessarily identical, context to the corresponding problem. It contributes 40% to the CAT grade and provides evidence of the authenticity of students' reports. If a student's mark on the test is significantly lower than would be expected according to his/her mark on the report, the student is interviewed about the report by a panel of teachers. If the student cannot convince the panel that he/she fully *understands* his/her report, the report grade is reduced to that of the test. Students who are found not to have *authored* their reports are disciplined.

McCrae (1995) conducted an evaluation of the first (1994) implementation of the *Problem-solving* CAT. He found clear support among the teachers involved for the continuation of the new problem format. There was general agreement that problems structured in this way are accessible to more students and less time-consuming for students, and that they are easier to assess reliably than the more open problems characteristic of the former *Challenging problem* CAT. However, the teachers tended to disagree with the suggestion that the closed format increased their confidence in the authenticity of their students' reports and tended to agree that it reduced the task's validity as a measure of problem solving ability.

The teachers were firmly of the opinion that the inclusion of the test/interview process improved both the public credibility of the CAT and its credibility amongst the students concerned. They were unsure, though, whether it enabled a more valid assessment of problem solving ability. The requirements that students must submit with their report any draft material and a bound log book containing all working notes, were considered much more important factors than the test/interview process in allowing teachers to feel confident about authenticating their students' reports.

McCrae recommended that in future each problem include a question requiring some aspect of the solution to be generalised to improve the task's validity as a measure of problem solving ability and to ensure less clustering of marks near the top of the scale. Since then, all problems have included a 'generalisation' component. McCrae also noted that test questions should be sufficiently similar to the corresponding problem questions to enable them to be answered by each student using the techniques *which he/she used* in solving the problem.

Stacey and McCrae (in press) analyse the changes which have taken place in the nature of the problems set in the problem solving CATs from 1989 to 1997. They judge that the trend has been towards problems that require less creativity in their solution, are more familiar in nature and offer less opportunity to generalise results. They conclude that, although the CAT is a less valid assessment of problem solving than originally, it still has sufficient validity to support its retention, especially since its existence continues to encourage creative problem solving work in earlier years.

The history of the investigative project CAT, until the end of 1993, is reviewed by Stacey (1995). Students had to submit a 1500–2000 word report of a project they had undertaken over a period of four weeks on a centrally-set mathematical theme. In the first few years there was considerable scope for students to shape their own investigations. They were able to define the path of their investigation, as well as the mathematical techniques to be used, to determine what data or information they needed and to obtain it themselves by experiment or from an appropriate source.

However, widespread concerns about the consistency of assessment across the State of very divergent projects, and about the difficulty of determining beforehand the breadth and depth of investigation needed for a good grade, soon led to a reduction in the openness of the CAT. By 1993, students had to select one of three specified 'starting points' within a theme for their investigation. Each starting point consisted of a number of questions intended to provide clear guidance on the expected direction and extent of the investigation, including in what way(s) it might be extended. There was still scope for students to choose what mathematical methods they would employ, but they were no longer expected to obtain their own data. Stacey concluded that, despite the changes, the CAT was still a valid assessment of investigative work in mathematics.

From 1994 to 1996, the format of the investigative project CAT remained the same, though the starting point questions typically became less open to interpretation and to method of solution and continuing attempts were made to constrain the breadth (if not the depth) of extension work. The main objective of this 'tightening up' was to limit the excessive amount of time that students were spending on the CAT in their quest for high grades. The total time spent on the project was not supposed to exceed 10 periods in class and 20 hours out of class, but many schools devoted all class time during the four week period to it and outside of class students were concentrating on the project to the detriment of everything else. The matter came to a head in 1996, with a large and influential group of schools reporting that, in Mathematical Methods 3/4 (the mainstream VCE mathematics subject) students averaged nearly 90 hours on the investigative project CAT.

The response of the authorities to the workload crisis was to reduce the prescribed time period for the CAT from four weeks to two weeks, to shorten the length of the report to 1200–1500 words and to direct the investigations even more closely with starting point questions that were predominantly closed. This overhaul was accompanied by the introduction of a test/interview process, identical to that associated with the *Problem-solving* CAT, to assist with the authentication of students' reports which could be expected to be much more convergent than previously under the new conditions. Thus, in 1997 the Mathematical Methods investigative project CAT had two components: the report and a closely related one-hour test, conducted three days after the report was due and contributing 25% to the final grade.

The format of the investigative project CAT for Further Mathematics 3/4, a lower level VCE mathematics subject, was not altered for 1997—the time period is still four weeks and there is no test component. Excessive workload has never been a major issue in this case as, typically, Further Mathematics students are not competing for selection into courses that require a high Tertiary Entrance Ranking score.

The 1997 Study

The authors conducted an evaluation of the implementation of the 1997 Mathematical Methods 3/4 investigative project CAT, similar to the 1994 evaluation of the *Problem-solving* CAT (McCrae, 1995). The evaluation was carried out through the use of a questionnaire sent to 126 secondary schools randomly selected, using stratified random sampling, from approximately 500 schools teaching Mathematical Methods. Sufficient copies of the questionnaire were provided to each school so that each teacher of Mathematical Methods could independently complete and return it. A total of 92 schools replied to the survey, a response rate of 73%, with 160 individual teacher responses. The response rates for metropolitan schools (72%) and country schools (75%) were about the same (after some follow up with metropolitan schools that did not reply by the due date). The response rates by sector were: Government 70%, Catholic 79% and Independent 78%; one of two TAFE institutes also responded. The number of students that inform the survey total about 3100. This represents just over 17% of the 17 833 students that completed the CAT.

The questionnaire contained 42 questions covering eight areas: background information, project attributes, report attributes, test attributes, comparing report and test grades, interviews, authentication issues and other issues. Most questions required respondents to select from one of five alternatives on an ordinal scale (eg. strongly disagree, disagree, unsure, agree, strongly agree). For data analysis, a corresponding scale of 1 to 5 was used so that an average rating could be calculated. In a few questions, respondents were asked for a numerical response (e.g. the average amount of time spent in assessing each report). Space was left at the end of the questionnaire for respondents to write comments on any aspect of the CAT and just over half (51%) availed themselves of this opportunity.

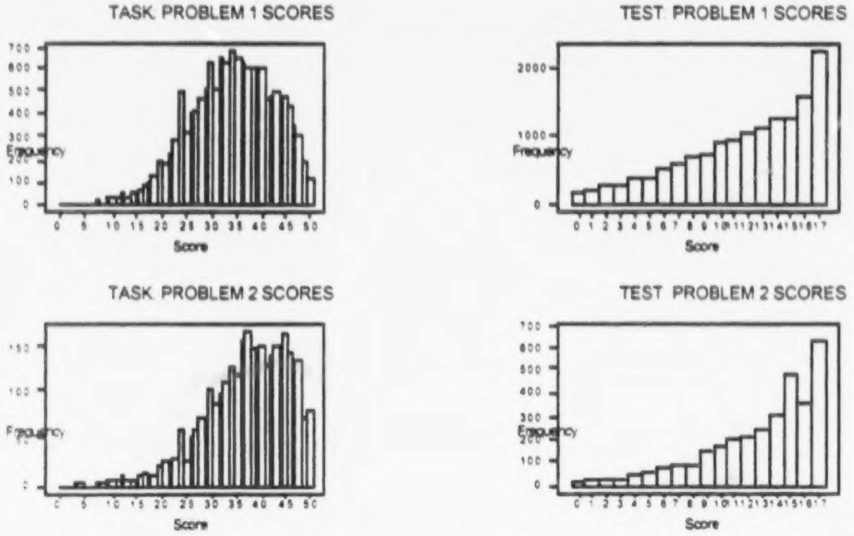
Students had to choose between two starting points, SP1: *Bushwalking with Kim* and SP2: *Blood flow* (Board of Studies, 1997). Respondents regarded both starting points as suitable for students to demonstrate their abilities in project work, with SP1 (average rating 3.8) marginally more suitable than SP2 (average rating 3.5). Both starting points were seen as requiring a reasonable balance of routine and non-routine work, with SP1 (average rating 2.7) tending to involve more routine work than SP2 (average rating 3.0). *Bushwalking with Kim* was by far the more popular choice, being attempted by 82% of students.

The ten criteria used to assess students' reports were judged to be relatively easy to apply and the five-point scale (very low, low, medium, high, very high) was seen to make it relatively easy to discriminate between the reports. Three-quarters of respondents reported that their students spent more than 25 hours on the investigation and teachers spent an average of 43 minutes per report as first markers. About two-thirds of the respondents also spent an average of 23 minutes second marking reports and some complaints about the workload involved in assessing the reports were registered.

Respondents agreed that the test was very closely related to the investigation in both cases (average rating 4.6 for both). However, the tests were criticised for favouring students who had used particular approaches in their investigations and it would appear that the setting panel needs to be more careful in this regard in the future. Just over half of the respondents felt that the level of difficulty of the tests was 'about right', with all but a handful of the rest believing that the tests were too easy.

On average, respondents believed that both tests discriminated reasonably well between levels of understanding of the mathematics employed in the students' reports (average ratings SP1: 3.1, SP2: 3.2). However, each of the five possible responses from 'not well' to 'very well' had a solid group of supporters. The explanation for this volatility lies in accompanying comments such as "The test discriminated well between two standards of students but in the top group there was not much of a spread of results."

an observation which is confirmed by the histograms of scores reproduced in Figure 1. This failing occurred because, in both cases, the hardest parts of the investigation were not tested. This deficiency should be remedied in future years as it also brings the test's effectiveness as an authentication instrument into question.



TASK PROBLEM 1 SCORES

TEST PROBLEM 1 SCORES

TASK PROBLEM 2 SCORES

TEST PROBLEM 2 SCORES

Figure 1. Histograms of report and test scores for Mathematical Methods CAT 1, 1997.

For both starting points, respondents were well satisfied on average with the correspondence between their students' performances on the report and the test (average ratings SP1: 3.6, SP2: 3.8). The population report/test correlations were 0.665 for SP1 and 0.616 for SP2. These are lower than might be expected, given the close relation between investigation and test, and probably suffered from the test deficiencies already identified with some respondents expressing views similar to the following comment:

> My students generally performed better on the test than they did on the actual project .... The test really only tested the routine elements of the question and did not ask any 'thinking' questions to discriminate [between] those who had mastered the routine maths and those who understood the more complex concepts involved.

Respondents advised that interviews were necessary in 173 cases for SP1 (6.9% of those who did that starting point), with 16 (9.2%) of those interviewed not having their reports authenticated. The corresponding figures for SP2 were 25 (4.4%) and 6 (24%). In total, 198 (6.4%) of the students covered by the sample had to be interviewed and 22 (11%) of them—about 0.7% of the student sample—did not have their reports authenticated. Respondents were generally well satisfied with the outcomes of the interviews (average rating 4.6).

Students must provide evidence of progress on their investigation and demonstrate that it is their own work by having at least one consultation with their teacher, based on a draft of their report or their log book contents, during the period allowed for the investigation. Consultation with students was rated as the most important factor in enabling teachers to authenticate students' reports, with 70% of respondents regarding it as an essential condition. The submission of log books, containing all working notes, (average rating 4.0) and draft material (average rating 3.9) were judged as very important, whilst the existence of the test (average rating 3.7) and the possibility of interview

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(average rating 3.6) also were regarded as important, but not as essential as the other factors.

There was general agreement that the report plus test format improves public credibility of the CAT (average rating 4.0) and, to a marginally lesser extent, agreement that the CAT's credibility among VCE students also is improved (average rating 3.8). There was a strong tendency to agree that the new format for the CAT assists teachers to authenticate authorship of reports (average rating 3.6), but a tendency to disagree that it reduces the amount of collaboration that might otherwise occur (average rating 2.7).

Students may 'consult any source material or person' (Board of Studies, 1996, p. 60) in preparing their project report, but are expected to acknowledge all assistance provided and how it helped them complete the task. It is clear from a number of similar written comments that many teachers believe that, despite the presence of the test/interview process and other conditions designed to provide evidence of authenticity, the CAT remains open to abuse by students receiving unacknowledged assistance from sources such as fellow students, teachers, family members, private tutors and commercial seminars. The issue of acknowledgment aside, many respondents expressed concern that students who could not access such forms of assistance because of their family's socio-economic situation were unfairly disadvantaged.

Respondents clearly agreed that structuring the starting points so that the questions required specific mathematical responses makes the project accessible to more students (average rating 4.1) and generally agreed that this should be a feature of future investigative project CATs (average rating 4.0). They tended to agree that it reduces the amount of time that students might otherwise spend on the task and increases the reliability with which they can assess students' reports (average rating 3.5 in both cases). They were unsure as to whether it increased their confidence in the authenticity of their students' reports (average rating 3.1) and tended to disagree that it reduced the task's validity as a measure of students' ability at investigative project work (average rating 2.7).

Some respondents commented to the effect that the redesigned CAT was no longer really an investigative project, but more a problem solving activity akin to the *Problem-solving* CAT. Nevertheless, these respondents, like the group as a whole, tended to agree that the new format enabled a more valid assessment of students' abilities in project work. This apparent contradiction may be seen as an indication that teachers' concerns for their students' results and welfare outweigh all other factors. One respondent observed that it was a "much better CAT period than last year .... Students were less anxious, had more sleep and stayed in better health than last year."

Use of Powerful Technology: An Emerging Issue

The use of technology emerged as an issue in the 1994 evaluation of the *Problem-solving* CAT. Students had relied heavily on the aid of graphics calculators or computer software (such as graph plotters and spreadsheets) in solving one of the problems, but this technology was not allowed in the test. In early 1995, it was announced that students would be allowed to use graphics calculators in the end-of-year VCE mathematics examinations from 1997. Later, it was decided that also from 1997 one of the *Problem-solving* CAT problems, and the related test, would assume that students had access to a graphics calculator.

The use of technology in the Mathematical Methods 3/4 investigative project CAT has been actively encouraged since 1995. At first (1995), this encouragement was restricted to using a graphing package but in subsequent years it has been extended to include the use of a graphics calculator, a computer spreadsheet or 'other computer packages' (Board of Studies, 1997, p. 5). It is doubtful that the excessive workload issue, which caused the CAT's redesign for 1997, would have arisen without the availability of such technology which enabled students to generate graphical representations and analyses not otherwise within their capabilities.

Respondents to the 1997 survey reported that most students had easy if not constant access to a graphics calculator (average rating 3.9) and made reasonable use of it if they did have access (average rating 3.2 for both starting points). Access to computer software such as graph plotters and spreadsheets was quite reasonable (average rating 3.6), though not as constant as graphics calculator access, and tended to be more frequently used than were graphics calculators (average ratings SP1: 3.7, SP2: 3.6). A significant part of this preference for using computer software can be attributed to its ability to produce a better quality product than graphics calculators for direct inclusion in students' reports.

Question 42 of the survey (see Figure 2) presents a partial solution by a supposed student author to one part of *Bushwalking with Kim*. The solution uses the symbolic manipulation capabilities of the Texas Instruments TI-92 calculator to perform most of the algebra involved. It demonstrates that the impact of powerful technology, such as computer algebra systems, on VCE mathematics assessment will not be confined to the substantial impact it is going to have on the examination CATs (McCrae, 1996). (As another demonstration, McCrae (in press) shows that *Bushwalking with Kim* can be tackled very effectively with dynamic geometry software such as The Geometer's Sketchpad and Cabri-geometry.)

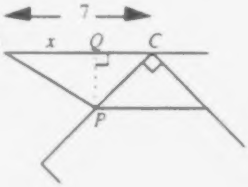
Question 42

Consider the following partial solution to part c. of Starting Point 1. In answering this question, **assume that you have no reason to doubt** that the student author **did** perform the derivations on his/her TI-92 calculator.

a. Would you in the current circumstances have
  - **A.** authenticated the work irrespective of the student's known strength at calculus and algebra
  - **B.** authenticated the work if you knew or could establish (eg by interview) that the student understood all the steps involved, without necessarily being able to perform them by hand
  - **C.** authenticated the work if you knew or could establish (eg by interview) that the student could perform all the steps involved by hand
  - **D.** *not* authenticated the work irrespective of the student's known strength at calculus and algebra

b. If all students had access to a calculator like the TI-92, do you believe that such work should be
  - **A.** authenticated irrespective of the student's known strength at calculus and algebra
  - **B.** authenticated if you knew or could establish (eg by interview) that the student understood all the steps involved, without necessarily being able to perform them by hand
  - **C.** authenticated if you knew or could establish (eg by interview) that the student could perform all the steps involved by hand
  - **D.** *not* authenticated irrespective of the student's known strength at calculus and algebra

Partial Solution to part c.



Let x km be the distance from A to Q as shown on the diagram.
Then, by Pythagoras in ΔAQP,
$AP^2 = x^2 + (7-x)^2$ (since $QP = QC$)
$= 2x^2 - 14x + 49$ where $x \le 7$ and $2(7-x)^2 \le CD^2 = 49$
That is, by TI-92 screen dump 1, *[Screen dumps have been omitted]*
$AP^2 = 2x^2 - 14x + 49$, $7 - 7/\sqrt{2} \le x \le 7$.
Using $t = \frac{d}{v}$, Kim's travelling time, T hours, is given by

$$T = 2\left\{\frac{\sqrt{2x^2 - 14x + 49}}{v_b} + \frac{7-x}{kv_b}\right\} = \frac{2\left\{k\sqrt{2x^2 - 14x + 49} + (7-x)\right\}}{kv_b}$$

By screen dumps 2–6, this has a minimum at $x = \dfrac{7\left(\sqrt{2k^2-1}+1\right)}{2\sqrt{2k^2-1}}$

and by screen dumps 7–9, the minimum time is $\dfrac{7\left(\sqrt{2k^2-1}+1\right)}{kv_b}$.

Figure 2. Question 42 on the Mathematical Methods CAT 1 questionnaire.

The first part of Question 42 asks respondents under what conditions (if any) they would authenticate the student's work in the current circumstances of limited access to such technology. The second part is identical to the first part except that respondents were to assume that all students had access to a calculator like the TI-92. The idea for this question came from Roberts' (1997a) account of the dilemma he faced in assessing the *Problem-solving* CAT of one of his students in 1996. The student had obtained the general case solution to the last part of the problem apparently by using a computer algebra system to solve an equation he had formulated. When interviewed, the student was unable to explain the steps involved in the solution process—though he could explain the equation, and so this part of his report was not authenticated. Stacey (1997) and Pierce and Roberts (1997), all from universities, responded to Roberts and argued that the student's work could have been authenticated, but Roberts (1997b) was not convinced.

The responses to Question 42 leave us in no doubt that most VCE mathematics teachers would support Roberts. A majority (56%) of respondents indicated that in the current circumstances they would have authenticated the student's work only if they knew or could establish (e.g. by interview) that the student could *perform* all the steps involved by hand. Just over half as many (29%) would have authenticated the work if they knew or could establish that the student *understood* all the steps involved without necessarily being able to perform them by hand. Only a small number (9%) of respondents would have authenticated the work irrespective of the student's known strength at calculus and algebra, or not authenticated it irrespective of those strengths (6%).

As to what they would have done if all students had access to a TI-92, there was practically no shift in the attitudes of respondents, with the corresponding percentages being 56%, 29%, 10% and 5%. This should not be taken as indicating that teachers are unconcerned about equity of access to technology at present, but rather it indicates their strong conviction that students should continue to be able to do their own algebra in future. It is difficult, however, to see how this view can prevail as access increases to computer algebra systems that can show each step in a solution for students to copy: the authentication burden on teachers would be immense. If the school-based CATs are to continue in their current form, then it is more likely that the use of powerful technology, such as computer algebra systems and dynamic geometry software, will need to be accommodated in a similar manner to that which has occurred already with other technology, such as plotting software and spreadsheets.

Concluding Remarks

The use of school-based assessment tasks in the VCE has given rise to several concerns over the ten years of their existence. The principal areas of concern have been the consistency and reliability of assessment across schools, authentication of student work and 'cheating', student and teacher workload, fragmentation of teaching and learning, and the usefulness of the results for selection into tertiary courses (Hill, Brown & Masters, 1993). The two school-based mathematics tasks, an investigative project CAT and a problem solving CAT, have both undergone significant changes during this period in response to these concerns and, in their present forms, appear to satisfy the criteria for school-assessed tasks fixed by the recent Committee of Review on the Victorian Certificate of Education (1997, recommendation 25).

In both cases, the price that has been paid has been the sacrificing of much of the 'openness' that characterised the original tasks. This is particularly true of the Mathematical Methods 3/4 investigative project CAT which has evolved into a task that is essentially indistinguishable in character from the *Problem-solving* CAT. In the forthcoming reaccreditation process, careful consideration needs to be given to the future role of project work in VCE mathematics, including its assessment.

Finally, there has been increasing use of technology throughout the history of the school-based assessment tasks. This trend will continue as more powerful technology, such as computer algebra systems, becomes readily accessible to students. The effect this will have on the assessment of the CATs needs addressing in the wider context of its impact on VCE mathematics curriculum and assessment in general.

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A new trilemma.

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One perennial unanswered question for debate among mathematics educators is that of 'what is mathematics?'. Polarised views range from those who point to traditional classical Euro-centric knowledge as the definitive answer to those who wish to widen the definition to include mathematics from every instance of mathematical activity. This paper explores the dilemma thus raised and suggests that the inclusion of a third element may help to clarify the issue.

Introduction

This paper had its origin in an essay for the Culture and Sociology of Mathematics Education paper at the University of Auckland. In the essay I looked back at the impact of a resource book *Kowhaiwhai - Geometry of Aotearoa* (McKenzie, 1989). At the time of its publication I believed that this would be the first of many bicultural mathematics resources in New Zealand, that the book was in line with the ideals of ethnomathematics, and that it would be a step towards changing the ways in which students and teachers, both Maori and non-Maori, would view mathematics. The essay dealt with the problems raised by the use of the resource; problems which highlighted one of the major conflicts within mathematics education - the conflict between traditional school mathematics and the mathematics found outside the classroom.

In this paper I intend to explore this conflict, and to suggest that the dilemma faced by educators wishing to make use of both types of mathematics is an oversimplification of the situation, and that there are at least three distinct types of mathematics available. I have called the issue raised by this third face of mathematics a *trilemma.* This proposal then raises further issues for educators and researchers:

- do all forms of mathematical activities qualify as material for mathematics research?
- can resources based upon differing types of mathematics be equally effective in the teaching of mathematics?
- are mathematical activities with a high cultural content open to all?

Mathematics and mathematics

When *Kowhaiwhai - Geometry of Aotearoa* was trialed at Huntly College in the Waikato region of New Zealand in 1986 I found that a number of students insisted that 'this isn't mathematics'. The course used kowhaiwhai, the rafter patterns which are a feature of the Maori meeting houses in New Zealand, to teach transformation geometry. Sometimes the objection was 'my father says this isn't mathematics!' At the time, I was scornful of such attitudes, on the basis that I was more likely to be in a position to judge what was or wasn't mathematics, but today I am not so certain.

A tension has existed among mathematics educators for a long time between those who incline to the position that mathematics is what is taught in schools and universities, and those who contend that mathematics is much wider than this, and must include all those areas where people use mathematical ideas in their everyday lives. Attempts to enlarge the definition of mathematics have met with considerable resistance and are a cause for debate among mathematical researchers

D'Ambrosio is usually credited with coining the term *ethnomathematics* (1984) and was amongst the first to define the expression. In the early evolution of the term he gave a definition "the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age, professional classes, and so on." (D'Ambrosio, 1985). Such groups could be ethnic, but could also be social, such as the domain of darts-players, whose numerical computations and language of number are particular to those who take part in this sport. Such a broad definition will include many practices not normally regarded as mathematics. This does not mean that the practices do not contain elements easily identified as being mathematical, but that the underlying concepts do not fit into the structure commonly identified as mathematics.

Since D'Ambrosio's early definition, mathematicians have continued to explore the term and what such a word might entail. Barton (1996a) describes ethnomathematics as a field of study which examines mathematical ideas in their cultural context. This involves challenging existing abstractions or concepts of mathematics. He notes that an alternative structure is not of itself ethnomathematical, but that the description refers only to the process of identification. He summarises this in a definition.

> Ethnomathematics is the field of study which examines the way people from other cultures understand, articulate and use concepts and practices which are from their culture and which the researcher describes as mathematical. (Barton, 1996a)

On this basis the *study of kowhaiwhai* is ethnomathematics, because I examined the rafter patterns and attempted to understand what the artist was doing, and was able to describe mathematical ideas. Studies of frieze patterns have also been done by Zaslavsky (1992) and Ascher (1991), the latter including kowhaiwhai as an example.

The Need for a Change

The interest in ethnomathematics is, at least in part, a reaction to what Joseph refers to as an ethnocentric bias (Joseph, 1995), a result of the hegemonic belief in the Eurocentric view of the development of mathematics. This view, he concludes, has lead to the concept of the subject as being deductive, abstract, a-prioristic, with axiomatic foundations. Joseph sees a multicultural approach to mathematics as part of a general strategy to make mathematics more accessible (Joseph, 1990), by countering the traditional view with one which recognises the contributions made by non-European mathematicians in the development of the subject.

Ethnomathematical study can arise for a number of reasons. Gerdes (1994) gives different types of mathematics which might be studied in this way, including:

- indigenous mathematics
- sociomathematics
- mathematics in the socio-cultural environment
- spontaneous mathematics
- oral mathematics
- oppressed mathematics
- non-standard mathematics
- frozen mathematics
- folk mathematics
- mathematics codified in know-hows.

All of these have been studied by a number of different researchers. Of particular interest to Gerdes was the way that mathematics changed in a country (Mozambique) as the political climate changed. Gerdes followed the evolution from an informal system aimed at inculcating obedience to the established tribal authority, through a Eurocentric colonial system which looked towards Portugal for examples to follow, into a revolutionary system where questions used examples of nurses at health posts, guerrilla fighters and literacy classes to meet the educational aims of the new government (Gerdes, 1981). In such case Gerdes noted that the mathematics was not culture-free, but that the culture of the period strongly influenced the content and pedagogy of the education system. Gerdes makes the distinction between the artisan who imitates a known production technique and the artisan who initially discovered the technique. The former, he claims, is not doing mathematics. However the latter (Gerdes, 1988) did do mathematics and was thinking in a mathematical way. Hence he sees the need for the mathematics classroom to be one in which the students re-invent the 'frozen' mathematics, and thus be stimulated.

These statements indicate that Gerdes sees the mathematical activities themselves as being examples of ethnomathematics, whereas Barton, D'Ambrosio and Ascher take the line that the activities provide for the *possibility* of ethnomathematics. In this line of thought, it is the study of the mathematical properties within the activity that is ethnomathematics, not the activity itself. This change in the meaning of the word *ethnomathematics* has been a gradual one, as the field of study has evolved.

The Three Face of Mathematics

It is commonplace nowadays to use the expressions *Mathematics* and *mathematics* to distinguish between the formal mathematics which is taught at school and onwards, and the informal mathematics which is used by people in everyday life and work. A third category *(premathematics)* has been added to these, as defined by Bishop (1988). Bishop lists six fields of activity which every culture develops:

- counting
- locating
- measuring
- designing
- playing
- explaining.

It is easy to see from the previous section how researchers have been so intent on valuing the activities of each culture that it has been an automatic process to seek mathematical content and then describe what is found as mathematical knowledge. Into this category I would place studies by Zaslavsky (1992) on counting and gaming, the extensive study by Lean (1992) on numbering systems in Papua New Guinea, and the recent work by Cooke (1995) on Yolgnu genealogy. These works all describe activities which can be placed into one of Bishop's six fields, and on this account I distinguish these from ethnomathematics.

There is a real danger with this approach of reading into an activity a lot more Mathematics than is actually present. Since each culture engages in premathematical activity, then every researcher who seeks evidence of premathematical activity will find it. If the researcher always finds mathematics, then the researcher who studies this mathematics is always a mathematician. It is not surprising therefore that mathematicians who lean towards the traditional view of the subject are unwilling to see its definition expanded, since the resulting scenario would resemble that sung by Don Alhambra in the Gilbert and Sullivan opera *The Gondoliers:* "If everybody is somebody, then no-one's anybody." But if this premathematics is mathematics in the early stages of development, then the study of premathematics differs from the study of ethnomathematics as described earlier. This is not to say that one form of mathematics is more important, or more valuable then another. This classification recognises the difference in what is being done in each type. Premathematics is the term for the activities done by a society or culture as essential building blocks in the development of that society or culture, mathematics with the small 'm' is the term for the mathematics which individuals and groups developed because of the needs of that individual or group, and a study of mathematical activities is what ethnomathematics does.

This mathematics differs from premathematics in that the areas of development will be largely determined by the environment within which that group is placed. Gerdes' study (1981) on liberation arithmetic is ethnomathematics, because the arithmetic has been used in a context for a new purpose (a political one). Thus a society which lives in a desert will develop spatial skills for survival to a far greater extent than others; a society which has permanent dwellings will develop building design skills to a far greater extent than nomads. A study into kowhaiwhai patterns (McKenzie, 1989) can be given as an example of ethnomathematics, and it differs from the premathematics examples given earlier because of a difference in what is being studied. Premathematical activities arise in every society because of basic needs, and these needs are common to all cultures. Ethnomathematical activities arise because of the interests of a particular culture. The mathematics of building a shelter is premathematical, because a shelter is an essential; the mathematics of systems of decoration is ethnomathematical because there comes a stage at which the artist is using and developing ideas for their own sakes. Ascher (1991) describes the rafter patterns as being formally structured, and as being the work of special craftsmen, part of a tradition passed on through generations. She notes that cultures such as the Maori have no intrinsic need for decorations with such a high degree of internal organisation. The patterns do not enhance the practical function of the adorned object, and therefore indicate an interest in spatial ordering. This move from essential to interest is what moves the activity from premathematics to mathematics, and the study from anthropology to ethnomathematics.

This distinction between premathematics and mathematics was described by Ascher when she moved from the premathematical limitations by saying

> "that if you go out and look for culturally embedded mathematical ideas, what you will find is counting. That is too small, it is much too small. Certainly mathematical ideas of number, logic, and spatial configuration organised into systems, include counting. But counting doesn't include all those wonderful things that are in mathematics and that are mathematical ideas" (Ascher and D'Ambrosio, 1994).

The third category, Mathematics, is that mathematics which we often identify as classroom mathematics, and differs in that mathematics is being studied as an entity, rather than a tool for some other purpose. Mathematics is sometimes as being described as what mathematicians do, and this definition represents a move away from mathematics as part of some larger activity, as in kowhaiwhai artistry, to Mathematics as being something to be studied for itself. D'Ambrosio claims (Ascher and D'Ambrosio, 1994) that you cannot separate a mathematics from the philosophy of the people who create and use it. He illustrates this by consideration of Greek attempts to trisect an angle by ruler and compass. He continues by emphasising that this attitude came because the scholars were interested in something other than solving a problem; it was not that they could not solve it, it was that they could not solve it under the conditions that they themselves had set. This is Mathematics - going beyond immediate needs to another level of reflection.

There is a continuum across the three categories, but some distinctions are clear. Thus a study of premathematics differs from a study of mathematics, which differs from a study of Mathematics.

Summary

If we accept the proposition that mathematics can be divided into three (at least) distinct categories: Mathematics, mathematics and premathematics, then we can give some distinguishing characteristics for each type.

1) Premathematics (as defined by Bishop)
   - restricted to the six areas of activity
   - static development
   - mathematics is used for the building of a society
   - the study is anthropology and is done by anthropologists
2) mathematics (as given in D'Ambrosio's early definition of ethnomathematics)
   - involving the development of mathematics in some area
   - dynamic development
   - users can experiment with ideas
   - mathematics is used to improve the quality of life (e.g. to make surroundings more attractive or to make someone a better trader
   - the study is ethnomathematics and is done by ethnomathematicians
3) Mathematics (as it is traditionally regarded)
   - new ideas are added to an existing body of knowledge
   - dynamic development
   - mathematics is often developed and studied for its own sake
   - the study is called mathematics and is done by mathematicians

While mathematicians who currently research premathematical activities may object to being described as anthropologists, my contention is that the term is a more accurate description of the work being done. I also recognise that many will argue the classifications which I have made in this paper, and there are certainly going to be many studies which will not fall neatly into one classification. These guidelines, however, can serve to make us aware of the distinct differences in the activities being studied. The three types do not form a hierarchy. If there is an ascending order in the sophistication of the mathematics used, that does not mean a corresponding increase in importance. In the past, ethnomathematics has too often been regarded as second class mathematics, (and presumably premathematics would be third class) but the difference is the role that the mathematics plays, not in their importance.

Conclusion

The trilemma for mathematics educators and researchers is to reconcile the differences between the three types of 'mathematics' so as to deliver the best to our students. If the three faces: Mathematics, mathematics, and premathematics are different, then the way in which we can use them will also be different. There is the potential for major problems in the misuse of mathematical and premathematical activities in misguided attempts to expand teaching into new areas within the classroom. When Cooke presented his paper at the Bridging Mathematics Conference at Batchelor in 1995, he asked a group of Yolgnu people who were present to comment, and received no reply; the group was at a loss as to know how to respond (Barton, 1996b). Barton argues that the study of gurrutu (Aboriginal kinship patterns) is not the same as the study of mathematical relationships. He adds:

> It would not be mathematics, but it was not intended to be. Similarly, what Michael Cooke explained to the conference was not gurrutu; it was some aspects of gurrutu which were recognisable as mathematical and which participants could relate to
> (Barton, 1996b).

This consequence of an attempt to merge cultural and mathematical activities can provide us with a clue to the trilemma of mathematics. Just as gurrutu is more than mathematical patterns, so kowhaiwhai are more than geometric designs. It is no easy thing to teach the entirety, to include all aspects of the activity, but it is more serious to focus solely on one facet. My students were nearly right. What I was teaching them was not Mathematics, what I was teaching was mathematics. What they expected, and what their parents expected, was Mathematics. This is not to say that we should, or must, be restricted in our teaching to the traditional Mathematics of old. There does need to be an acceptance of the wider definition to include other forms of 'mathematics', but we need to be aware that all 'mathematics' are not equal in the mathematical sense. We will find mathematical elements everywhere, but should not confuse the activity with the concept of Mathematics as something to be studied for its own sake. Premathematics is important, but not so much for the mathematics which we can find, as for the role it plays in the development of societies. My experience in teaching mathematics (small m) indicated that students were not inclined to accept mathematics but wanted Mathematics.

What is unknown is whether using mathematics is more effective than Mathematics in facilitating student learning. What is unknown is whether premathematics can be used successfully in mathematics education. If the categories are significantly different, it seems quite logical that teaching methodologies would also have to be altered for each type. There is considerable interest among teachers in the use of premathematics and mathematics. The task for researchers will be to determine whether these efforts can be productive and then how to maximise the effectiveness of new approaches to mathematics education.

My own experiences have indicated that the use of mathematical material can have both positive and negative effects. What I see as the next stage is a systematic investigation into the results of using material from the different types to teach and to measure the effectiveness of each. This will be a difficult task and a number of problems surface immediately. It will be necessary to find material from different resources to cover the same topics and to use these under controlled conditions. Parallel classes should provide some good evidence. However, the evidence and the objectives need to be carefully matched. Mathsphobia is part of our reality, and if an approach using mathematics or premathematics gives no better or even slightly worse understanding than a Mathematics approach, it will be important to also measure attitudes to see if some compensatory gains are made.

Care will be needed in choosing material which will not create problems of cultural ownership. Topics such as *gurrutu* and *kowhaiwhai* carry a great deal of cultural significance, and consultation with the cultural 'owners' will be essential before such material can be freely used. Consideration will therefore need to be given to the time required to use mathematical or premathematical activities to avoid 'westernising' the activity by omitting some facets of it to meet classroom time restrictions. Such a research programme may well require the resources of a number of interested parties, but the possibilities of widening the resources used in mathematics education provide good reason for taking this topic further.

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